FAsT-Match: Fast Affine Template Matching

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1 Construction of the Net $\mathcal{N}_\delta$ (a δn_1-Cover)

Once the net's density has been selected, an appropriate cover of the space of Affine transformations can be constructed. In this section we construct a δn_1-cover of all affine transformations from an image I_1 of dimension $n_1 \times n_1$ to an image I_2 of dimension $n_2 \times n_2$. We denote this set of transformations by $\mathcal{A}$. The cover will be a product of several 1-dimensional grids of transformations, each covering one of the constituting components of a standard decomposition of Affine transformations [1], which is given in the following claim.

***Claim** 1.1: Every orientation-preserving affine transformation matrix A can be decomposed into $A = TrR_2SR_1$, where Tr, R_i, S are translation, rotation and non-uniform scaling matrices* [1].

We now describe a 6-dimensional grid, $\mathcal{N}_\delta$, which we will soon prove to be a δn_1-cover of $\mathcal{A}$. The basic idea is to discretize the space of Affine transformations, by dividing each of the dimensions into $\Theta(\delta)$ equal segments. According to claim 1.1, every affine transformation can be composed of a rotation, scale, rotation and translation. These basic transformations have 1, 2, 1 and 2 degrees of freedom, respectively. These are: a rotation angle, x and y scales, another rotation angle and x and y translations.

The idea will be to divide each dimension into steps, such that for any two consecutive transformations T and T' on any of the dimensions it will hold that:

$$\ell_\infty(T, T') < \Theta(\delta n_1) \quad (1)$$

Starting with translations (x and y), since the template should be placed within the bounds of the image I_2, we consider the range $[-n_2, n_2]$. Taking step sizes of $\Theta(\delta n_1)$, guarantees by definition that Equation 1 holds. Similarly, for rotations we consider the full range of $[0, 2\pi]$, and use steps of size $\Theta(\delta)$. This suffices since rotating the template I_1 by an angle of δ results in pixel movement which is limited by an arc-length of $\Theta(\delta n_1)$. Finally, since the scales are limited to the interval $[\frac{1}{c}, c]$, steps in the scale axes of size $\Theta(\delta)$ will cause a maximal pixel movement of $\Theta(\delta n_1)$ pixels.

The final cover N_δ, of size is $\Theta((\frac{n_2}{n_1})^2 \frac{1}{\delta^6})$, is simply a Cartesian product of the 1-dimensional grids whose details are summarized in the following table.

1. arguments are similar for orientation-reversing transformations (which include reflection)

transformation	step size	range	num. steps
x translation	$\Theta(\delta n_1)$ pixels	$[-n_2, n_2]$	$\Theta(\frac{n_2}{n_1}/\delta)$
y translation	$\Theta(\delta n_1)$ pixels	$[-n_2, n_2]$	$\Theta(\frac{n_2}{n_1}/\delta)$
1st rotation	$\Theta(\delta)$ radians	$[0, 2\pi]$	$\Theta(1/\delta)$
2nd rotation	$\Theta(\delta)$ radians	$[0, 2\pi]$	$\Theta(1/\delta)$
x scale	$\Theta(\delta)$ pixels	$[1/c, c]$	$\Theta(1/\delta)$
y scale	$\Theta(\delta)$ pixels	$[1/c, c]$	$\Theta(1/\delta)$

The final result is formulated in the following claim, where the proof follows in a straight forward manner from the above construction: Given the net $\mathcal{N}_\delta$ and an arbitrary affine transformation A in $\mathcal{A}$, there exists a transformation A' in $\mathcal{N}_\delta$, such that A and A' differ by at most $\Theta(\delta n_1)$ (in the sense of the distance ℓ_∞) in each of the 6 constituting dimensions. Now, taking an arbitrary pixel p in I_1 and applying either A or A' on it, the results may not differ by more than $\Theta(\delta n_1)$ pixels, and this can be shown by a sequential triangle-inequality argument on each dimension.

***Claim** 1.2: The net $\mathcal{N}_\delta$ is a δn_1-cover of $\mathcal{A}$ of size* $\Theta((\frac{n_2}{n_1})^2 \frac{1}{\delta^6})$.

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References

[1] R. Hartley and A. Zisserman. *Multiple view geometry in computer vision*. Cambridge university press, 2008.