CS 242 2007

Denotational semantics, Pure functional programming

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Reading: Chapter 4, sections 4.3, 4.4 only
Skim section 4.1 for background on parsing

Syntax and Semantics of Programs

- Syntax
  - The symbols used to write a program
- Semantics
  - The actions that occur when a program is executed
- Programming language implementation
  - Syntax → Semantics
  - Transform program syntax into machine instructions that can be executed to cause the correct sequence of actions to occur

Theoretical Foundations

- Many foundational systems
  - Computability Theory
  - Program Logics
  - Lambda Calculus
  - Denotational Semantics
  - Operational Semantics
  - Type Theory
- Consider some of these methods
  - Computability theory (halting problem)
  - Lambda calculus (syntax, operational semantics)
  - Denotational semantics

Denotational Semantics

- Describe meaning of programs by specifying the mathematical
  - Function
  - Function on functions
  - Value, such as natural numbers or strings

defined by each construct

Original Motivation for Topic

- Precision
  - Use mathematics instead of English
- Avoid details of specific machines
  - Aim to capture “pure meaning” apart from implementation details
- Basis for program analysis
  - Justify program proof methods
    - Soundness of type system, control flow analysis
  - Proof of compiler correctness
  - Language comparisons

Why study this in CS 242 ?

- Look at programs in a different way
- Program analysis
  - Initialize before use, …
- Introduce historical debate: functional versus imperative programming
  - Program expressiveness: what does this mean?
  - Theory versus practice: we don’t have a good theoretical understanding of programming language “usefulness”

Basic Principle of Denotational Sem.

- Compositionality
  - The meaning of a compound program must be defined from the meanings of its parts (*not* the syntax of its parts).
- Examples
  - P; Q

    composition of two functions, state → state
  - letrec f(x) = e_1 in e_2

    meaning of e_2 where f denotes function …

Trivial Example: Binary Numbers

- Syntax

  b ::= 0 | 1
  n ::= b | nb
  e ::= n | e+e
- Semantics value function E : exp -> numbers

  E [[0]] = 0 E [[1]] = 1
  E [[nb]] = 2*E[[n]] + E[[b]]
  E [[e_1+e_2]] = E[[e_1]] + E[[e_2]]

  Obvious, but different from compiler evaluation using registers, etc.
  This is a simple machine-independent characterization …

Second Example: Expressions w/vars

- Syntax

  d ::= 0 | 1 | 2 | … | 9
  n ::= d | nd
  e ::= x | n | e + e
- Semantics value E : exp x state -> numbers

  state s : vars -> numbers

  E [[x]] s = s(x)
  E [[0]] s = 0 E [[1]] s = 1 …
  E [[nd]] s = 10*E[[n]] s + E[[d]] s
  E [[e_1 + e_2]] s = E[[e_1]] s + E[[e_2]] s

Semantics of Imperative Programs

- Syntax

  P ::= x:=e | if B then P else P | P;P | while B do P
- Semantics
  - C : Programs → (State → State)
  - State = Variables → Values

    would be locations → values if we wanted to model aliasing

Every imperative program can be translated into a functional program in a relatively simple, syntax-directed way.

Semantics of Assignment

C[[x:= e]]
 is a function states → states

C[[x:= e]] s = s'
 where s' : variables → values is identical to s except
 s'(x) = E [[e]] s gives the value of e in state s

Semantics of Conditional

C[[if B then P else Q]]
 is a function states → states

C[[if B then P else Q]] s =
 C[[P]] s if E [[B]] s is *true*
 C[[Q]] s if E [[B]] s is *false*

Simplification: assume B cannot diverge or have side effects

Semantics of Iteration

C[[while B do P]]

is a function states → states

C[[while B do P]] = the function f such that

f(s) = s if E [[B]] s is *false*

f(s) = f(C[[P]](s)) if E [[B]] s is *true*

Mathematics of denotational semantics: prove that there is such a function and that it is uniquely determined. “Beyond scope of this course.”

Perspective

- Denotational semantics
  - Assign mathematical meanings to programs in a structured, principled way
  - Imperative programs define mathematical functions
  - Can write semantics using lambda calculus, extended with operators like

    modify : (state × var × value) → state
- Impact
  - Influential theory
  - Applications via

    abstract interpretation, type theory, …

Functional vs Imperative Programs

- Denotational semantics shows
  - Every imperative program can be written as a functional program, using a data structure to represent machine states
- This is a theoretical result
  - I guess “theoretical” means “it’s really true” (?)
- What are the practical implications?
  - Can we use functional programming languages for practical applications?

    Compilers, graphical user interfaces, network routers, ….

What is a *functional* language ?

- “No side effects”
- OK, we have side effects, but we also have higher-order functions…

We will use *pure functional language* to mean
“a language with functions, but without side effects
or other imperative features.”

No-side-effects language test

Within the scope of specific declarations of $x_1,x_2, …, x_n$, all occurrences of an expression e containing only variables $x_1,x_2, …, x_n$, must have the same value.

- Example

```
begin
    integer x=3; integer y=4;
   5*(x+y)-3
   …        ||?  // no new declaration of x or y //
   4*(x+y)+1
end
```

Example languages

- Pure Lisp

  atom, eq, car, cdr, cons, lambda, define
- Impure Lisp: rplaca, rplacd

```
 lambda (x)  (cons
                   (car x)
                   (…  (rplaca (… x …) …) … (car x) … )
))
```

Cannot just evaluate (car x) once

- Common procedural languages are not functional
  - Pascal, C, Ada, C++, Java, Modula, …

    Example functional programs in a couple of slides

Backus' Turing Award

- John Backus was designer of Fortran, BNF, etc.
- Turing Award in 1977
- Turing Award Lecture
  - Functional prog better than imperative programming
  - Easier to reason about functional programs
  - More efficient due to parallelism
  - Algebraic laws
    - Reason about programs
    - Optimizing compilers

Reasoning about programs

- To prove a program correct,
  - must consider everything a program depends on
- In functional programs,
  - dependence on any data structure is *explicit*
- Therefore,
  - easier to reason about functional programs
- Do you believe this?
  - This thesis must be tested in practice
  - Many who prove properties of programs believe this
  - Not many people really prove their code correct

Haskell Quicksort

- Very succinct program

```
qsort [] = []
qsort (x:xs) = qsort elts_lt_x ++ [x]
                            ++ qsort elts_greq_x
    where elts_lt_x = [y | y <- xs, y < x]
          elts_greq_x = [y | y <- xs, y >= x]
```

- This is the whole thing
  - No assignment – just write expression for sorted list
  - No array indices, no pointers, no memory management, …

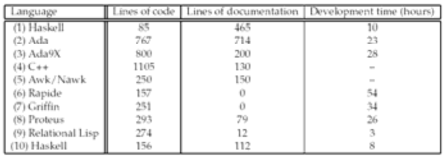
Compare: C quicksort

```
qsort( a, lo, hi ) int a[], hi, lo;
{ int h, l, p, t;
  if (lo < hi) {
     l = lo; h = hi; p = a[hi];
     do {
        while ((l < h) && (a[l] <= p)) l = l+1;
        while ((h > l) && (a[h] >= p)) h = h-1;
        if (l < h) { t = a[l]; a[l] = a[h]; a[h] = t; }
     } while (l < h);
     t = a[l]; a[l] = a[hi]; a[hi] = t;
     qsort( a, lo, l-1 );
     qsort( a, l+1, hi );
  }
}
```

Interesting case study

Hudak and Jones,
Haskell vs Ada vs C++ vs Awk vs ...,
Yale University Tech Report, 1994

- Naval Center programming experiment
  - Separate teams worked on separate languages
  - Surprising differences



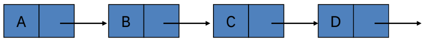
Language	Lines of code	Lines of documentation	Development time (hours)
(1) Haskell	85	465	10
(2) Ada	767	714	23
(3) Ada9X	800	200	28
(4) C++	1105	130	–
(5) Awk/Nawk	250	150	–
(6) Rapide	157	0	54
(7) Griffin	251	0	34
(8) Proteus	293	79	26
(9) Relational Lisp	274	12	3
(10) Haskell	156	112	8

Some programs were incomplete or did not run

  - Many evaluators didn't understand, when shown the code, that the Haskell program was complete. They thought it was a high level partial specification.

Disadvantages of Functional Prog

Functional programs often less efficient. Why?



Change 3rd element of list x to y

(cons (car x) (cons (cadr x) (cons y (cdddr x))))

- Build new cells for first three elements of list

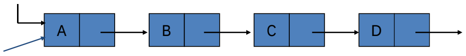
(rplaca (cddr x) y)

- Change contents of third cell of list directly

However, many optimizations are possible

Sample Optimization: Update in Place

Function uses updated list



(lambda (x)

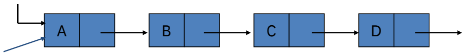
(... (list-update x 3 y) ... (cons 'E (cdr x)) ...)

Can we implement list-update as assignment to cell?

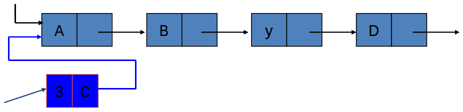
May not improve efficiency if there are multiple pointers to list, but should help if there is only one.

Sample Optimization: Update in Place

Initial list x



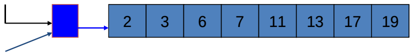
List x after (list-update x 3 y)



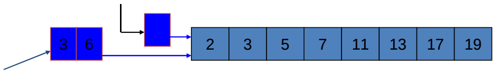
This works better for arrays than lists.

Sample Optimization: Update in Place

Array A



Update(A, 3, 5)



- Approximates efficiency of imperative languages
- Preserves functional semantics (old value persists)

Von Neumann bottleneck

- Von Neumann
  - Mathematician responsible for idea of stored program
- Von Neumann Bottleneck
  - Backus' term for limitation in CPU-memory transfer
- Related to sequentiality of imperative languages
  - Code must be executed in specific order

    function f(x) { if (x<y) then y = x; else x = y; }

    g(f(i), f(j));

Eliminating VN Bottleneck

- No side effects
  - Evaluate subexpressions independently
  - Example
    - function f(x) { return x<y ? 1 : 2; }
    - g(f(i), f(j), f(k), …);
- Does this work in practice? Good idea but …
  - Too much parallelism
  - Little help in allocation of processors to processes
  - …
  - David Shaw promised to build the non-Von …
- Effective, easy concurrency is a *hard* problem