Implementing morphology and phonology

We begin with a simple problem: a lexicon of some natural language is given as a list of words. Suggest a data structure that will provide insertion and retrieval of data. As a first solution, we are looking for time efficiency rather than space efficiency.
The solution: *trie* (word tree).
Access time: $O(|w|)$. Space requirement: $O(\sum_w |w|)$.
A trie can be augmented to store also a morphological dictionary specifying concatenative affixes, especially suffixes. In this case it is better to turn the tree into a graph.
The obtained model is that of *finite-state automata*.

Finite-state technology

Finite-state automata are not only a good model for representing the lexicon, they are also perfectly adequate for representing dictionaries (lexicons+additional information), describing morphological processes that involve concatenation etc.
A natural extension of finite-state automata – finite-state transducers – is a perfect model for most processes known in morphology and phonology, including non-segmental ones.

Formal language theory – definitions

Formal languages are defined with respect to a given *alphabet*, which is a finite set of symbols, each of which is called a *letter*. A finite sequence of letters is called a *string*.

Example: Strings

Let $\Sigma = \{0, 1\}$ be an alphabet. Then all binary numbers are strings over Σ.
If $\Sigma = \{a, b, c, d, \ldots, y, z\}$ is an alphabet then *cat*, *incredulous* and *supercalifragilisticexpialidocious* are strings, as are *tac*, *qqq* and *kjshdflkwjehr*.

Formal language theory – definitions

The *length* of a string w, denoted $|w|$, is the number of letters in w. The unique string of length 0 is called the *empty string* and is denoted ϵ.

If $w_1 = \langle x_1, \ldots, x_n \rangle$ and $w_2 = \langle y_1, \ldots, y_m \rangle$, the *concatenation* of w_1 and w_2, denoted $w_1 \cdot w_2$, is the string $\langle x_1, \ldots, x_n, y_1, \ldots, y_m \rangle$.

$|w_1 \cdot w_2| = |w_1| + |w_2|$.

For every string w, $w \cdot \epsilon = \epsilon \cdot w = w$.

Formal language theory – definitions

Example: Concatenation

Let $\Sigma = \{a, b, c, d, \ldots, y, z\}$ be an alphabet. Then $master \cdot mind = mastermind$, $mind \cdot master = mindmaster$ and $master \cdot master = mastermaster$. Similarly, $learn \cdot s = learns$, $learn \cdot ed = learned$ and $learn \cdot ing = learning$.

Formal language theory – definitions

An *exponent* operator over strings is defined in the following way: for every string w, $w^0 = \epsilon$. Then, for $n > 0$, $w^n = w^{n-1} \cdot w$.

Example: Exponent

If $w = go$, then $w^0 = \epsilon$, $w^1 = w = go$, $w^2 = w^1 \cdot w = w \cdot w = gogo$, $w^3 = gogogo$ and so on.

Formal language theory – definitions

The *reversal* of a string w is denoted w^R and is obtained by writing w in the reverse order. Thus, if $w = \langle x_1, x_2, \ldots, x_n \rangle$, $w^R = \langle x_n, x_{n-1}, \ldots, x_1 \rangle$.

Given a string w, a *substring* of w is a sequence formed by taking contiguous symbols of w in the order in which they occur in w. If $w = \langle x_1, \ldots, x_n \rangle$ then for any i, j such that $1 \leq i \leq j \leq n$, $\langle x_i, \ldots x_j \rangle$ is a substring of w.

Two special cases of substrings are *prefix* and *suffix*: if $w = w_l \cdot w_c \cdot w_r$ then w_l is a prefix of w and w_r is a suffix of w.

Formal language theory – definitions

Example: Substrings

Let $\Sigma = \{a, b, c, d, \ldots, y, z\}$ be an alphabet and w = *indistinguishable* a string over Σ. Then ϵ, *in*, *indis*, *indistinguish* and *indistinguishable* are prefixes of w, while ϵ, *e*, *able*, *distinguishable* and *indistinguishable* are suffixes of w. Substrings that are neither prefixes nor suffixes include *distinguish*, *gui* and *is*.

Formal language theory – definitions

Given an alphabet Σ, the set of all strings over Σ is denoted by Σ^*.
A *formal language* over an alphabet Σ is a subset of Σ^*.

Formal language theory – definitions

Example: Languages

Let $\Sigma = \{a, b, c, \ldots, y, z\}$. The following are formal languages:

- Σ^*;
- the set of strings consisting of consonants only;
- the set of strings consisting of vowels only;
- the set of strings each of which contains at least one vowel and at least one consonant;
- the set of palindromes;

Formal language theory – definitions

Example: Languages

Let $\Sigma = \{a, b, c, \ldots, y, z\}$. The following are formal languages:

- the set of strings whose length is less than 17 letters;
- the set of single-letter strings ($= \Sigma$);
- the set $\{i, you, he, she, it, we, they\}$;
- the set of words occurring in Joyce's Ulysses;
- the empty set;

Note that the first five languages are infinite while the last five are finite.

Formal language theory – definitions

The string operations can be lifted to languages.
If L is a language then the *reversal* of L, denoted L^R, is the language $\{w \mid w^R \in L\}$.
If L_1 and L_2 are languages, then
$L_1 \cdot L_2 = \{w_1 \cdot w_2 \mid w_1 \in L_1 \textit{ and } w_2 \in L_2\}$.

Example: Language operations

$L_1 = \{i, you, he, she, it, we, they\}$, $L_2 = \{smile, sleep\}$.
Then ${L_1}^R = \{i, uoy, eh, ehs, ti, ew, yeht\}$ and $L_1 \cdot L_2 = \{ismile, yousmile, hesmile, shesmile, itsmile, wesmile, theysmile, isleep, yousleep, hesleep, shesleep, itsleep, wesleep, theysleep\}$.

Formal language theory – definitions

If L is a language then $L^0 = \{\epsilon\}$.
Then, for $i > 0$, $L^i = L \cdot L^{i-1}$.

Example: Language exponentiation

Let L be the set of words $\{$*bau, haus, hof, frau*$\}$. Then $L^0 = \{\epsilon\}$, $L^1 = L$ and $L^2 = \{$*baubau, bauhaus, bauhof, baufrau, hausbau, haushaus, haushof, hausfrau, hofbau, hofhaus, hofhof, hoffrau, fraubau, frauhaus, frauhof, fraufrau*$\}$.

Formal language theory – definitions

The *Kleene closure* of L and is denoted L^* and is defined as $\bigcup_{i=0}^{\infty} L^i$.
$L^+ = \bigcup_{i=1}^{\infty} L^i$.

Example: Kleene closure

Let $L = \{dog, cat\}$. Observe that $L^0 = \{\epsilon\}$, $L^1 = \{dog, cat\}$, $L^2 = \{catcat, catdog, dogcat, dogdog\}$, etc. Thus L^* contains, among its infinite set of strings, the strings ϵ, *cat*, *dog*, *catcat*, *catdog*, *dogcat*, *dogdog*, *catcatcat*, *catdogcat*, *dogcatcat*, *dogdogcat*, etc.
The notation for Σ^* should now become clear: it is simply a special case of L^*, where $L = \Sigma$.

Regular expressions

Regular expressions are a formalism for defining (formal) languages. Their "syntax" is formally defined and is relatively simple. Their "semantics" is sets of strings: the denotation of a regular expression is a set of strings in some formal language.

Regular expressions

Regular expressions are defined recursively as follows:

- $\emptyset$ is a regular expression
- ϵ is a regular expression
- if $a \in \Sigma$ is a letter then a is a regular expression
- if r_1 and r_2 are regular expressions then so are $(r_1 + r_2)$ and $(r_1 \cdot r_2)$
- if r is a regular expression then so is $(r)^*$
- nothing else is a regular expression over Σ.

Regular expressions

Example: Regular expressions

Let Σ be the alphabet $\{a, b, c, \ldots, y, z\}$. Some regular expressions over this alphabet are:

- $\emptyset$
- a
- $((c \cdot a) \cdot t)$
- $(((m \cdot e) \cdot (o)^*) \cdot w)$
- $(a + (e + (i + (o + u))))$
- $((a + (e + (i + (o + u)))))^*$

Regular expressions

For every regular expression r its denotation, $[\![r]\!]$, is a set of strings defined as follows:

- $[\![\emptyset]\!] = \emptyset$
- $[\![\epsilon]\!] = \{\epsilon\}$
- if $a \in \Sigma$ is a letter then $[\![a]\!] = \{a\}$
- if r_1 and r_2 are regular expressions whose denotations are $[\![r_1]\!]$ and $[\![r_2]\!]$, respectively, then $[\![(r_1 + r_2)]\!] = [\![r_1]\!] \cup [\![r_2]\!]$, $[\![(r_1 \cdot r_2)]\!] = [\![r_1]\!] \cdot [\![r_2]\!]$ and $[\![(r_1)^*]\!] = [\![r_1]\!]^*$

Regular expressions

Example: Regular expressions and their denotations

$\emptyset$	$\emptyset$
a	$\{a\}$
$((c \cdot a) \cdot t)$	$\{c \cdot a \cdot t\}$
$(((m \cdot e) \cdot (o)^*) \cdot w)$	$\{mew,\ meow,\ meoow,\ meooow,\ \ldots\}$
$(a + (e + (i + (o + u))))$	$\{a, e, i, o, u\}$
$((a + (e + (i + (o + u)))))^*$	*all strings of 0 or more vowels*

Regular expressions

Example: Regular expressions

Given the alphabet of all English letters, $\Sigma = \{a, b, c, \ldots, y, z\}$, the language Σ^* is denoted by the regular expression Σ^*.

The set of all strings which contain a vowel is denoted by $\Sigma^* \cdot (a + e + i + o + u) \cdot \Sigma^*$.

The set of all strings that begin in "*un*" is denoted by $(un)\Sigma^*$.

The set of strings that end in either "*tion*" or "*sion*" is denoted by $\Sigma^* \cdot (s + t) \cdot (ion)$.

Note that all these languages are infinite.

Regular languages

A language is **regular** if it is the denotation of some regular expression.
Not all formal languages are regular.

Properties of regular languages

Closure properties:
A class of languages $\mathcal{L}$ is said to be closed under some operation '$\bullet$' if and only if whenever two languages L_1, L_2 are in the class ($L_1, L_2 \in \mathcal{L}$), also the result of performing the operation on the two languages is in this class: $L_1 \bullet L_2 \in \mathcal{L}$.

Properties of regular languages

Regular languages are closed under:

- Union
- Intersection
- Complementation
- Difference
- Concatenation
- Kleene-star
- Substitution and homomorphism

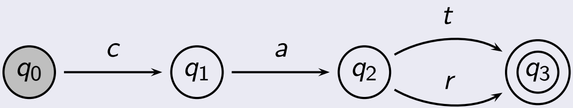
Finite-state automata

Automata are models of computation: they compute languages.
A finite-state automaton is a five-tuple $\langle Q, q_0, \Sigma, \delta, F \rangle$, where Σ is a finite set of **alphabet** symbols, Q is a finite set of **states**, $q_0 \in Q$ is the **initial state**, $F \subseteq Q$ is a set of **final** (accepting) states and $\delta : Q \times \Sigma \times Q$ is a relation from states and alphabet symbols to states.

Finite-state automata

Example: Finite-state automaton

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{c, a, t, r\}$
- $F = \{q_3\}$
- $\delta = \{\langle q_0, c, q_1\rangle, \langle q_1, a, q_2\rangle, \langle q_2, t, q_3\rangle, \langle q_2, r, q_3\rangle\}$



Finite-state automata

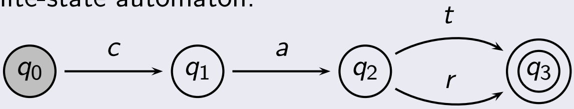
The reflexive transitive extension of the transition relation δ is a new relation, $\hat{\delta}$, defined as follows:

- for every state $q \in Q$, $(q, \epsilon, q) \in \hat{\delta}$
- for every string $w \in \Sigma^*$ and letter $a \in \Sigma$, if $(q, w, q') \in \hat{\delta}$ and $(q', a, q'') \in \delta$ then $(q, w \cdot a, q'') \in \hat{\delta}$.

Finite-state automata

Example: Paths

For the finite-state automaton:



$\hat{\delta}$ is the following set of triples:

$$\begin{gathered}\langle q_0, \epsilon, q_0\rangle, \langle q_1, \epsilon, q_1\rangle, \langle q_2, \epsilon, q_2\rangle, \langle q_3, \epsilon, q_3\rangle, \\ \langle q_0, c, q_1\rangle, \langle q_1, a, q_2\rangle, \langle q_2, t, q_3\rangle, \langle q_2, r, q_3\rangle, \\ \langle q_0, ca, q_2\rangle, \langle q_1, at, q_3\rangle, \langle q_1, ar, q_3\rangle, \\ \langle q_0, cat, q_3\rangle, \langle q_0, car, q_3\rangle\end{gathered}$$

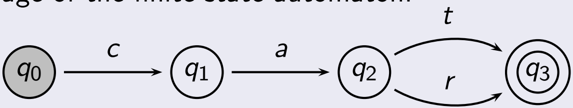
Finite-state automata

A string w is accepted by the automaton $A = \langle Q, q_0, \Sigma, \delta, F\rangle$ if and only if there exists a state $q_f \in F$ such that $(q_0, w, q_f) \in \hat{\hat{\delta}}$.

The *language accepted by a finite-state automaton* is the set of all the strings it accepts.

Example: Language

The language of the finite-state automaton:



is $\{cat,\ car\}$.

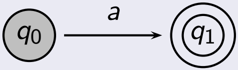
Finite-state automata

Example: Some finite-state automata

$\emptyset$

Finite-state automata

Example: Some finite-state automata



$\{a\}$

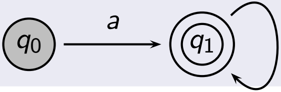
Finite-state automata

Example: Some finite-state automata

$\{\epsilon\}$

Finite-state automata

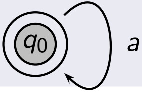
Example: Some finite-state automata



$a\ \{a,\ aa,\ aaa,\ aaaa, \ldots\}$

Finite-state automata

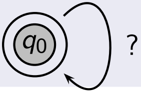
Example: Some finite-state automata



a^*

Finite-state automata

Example: Some finite-state automata



Σ^*

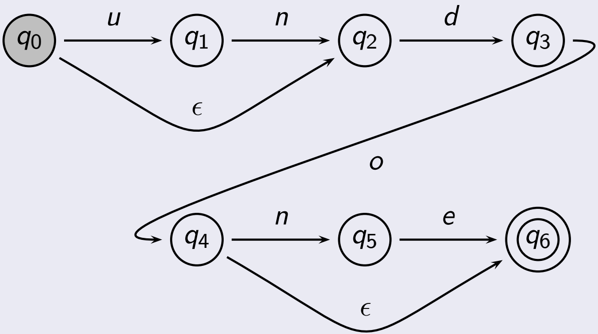
Finite-state automata

An extension: ϵ-moves.

The transition relation δ is extended to: $\delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q$

Example: Automata with ϵ-moves

The language accepted by the following automaton is $\{$*do, undo, done, undone*$\}$:

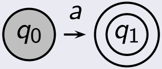
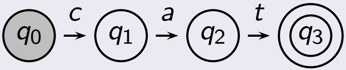


Finite-state automata

Theorem (Kleene, 1956): The class of languages recognized by finite-state automata is the class of regular languages.

Finite-state automata

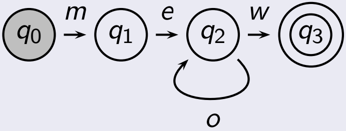
Example: Finite-state automata and regular expressions

$\emptyset$	
a	
$((c \cdot a) \cdot t)$	

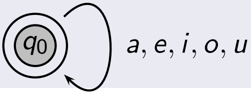
Finite-state automata

Example: Finite-state automata and regular expressions

$(((m \cdot e) \cdot (o)^*) \cdot w)$



$((a + (e + (i + (o + u)))))^*$



Operations on finite-state automata

- Concatenation
- Union
- Intersection
- Minimization
- Determinization

Minimization and determinization

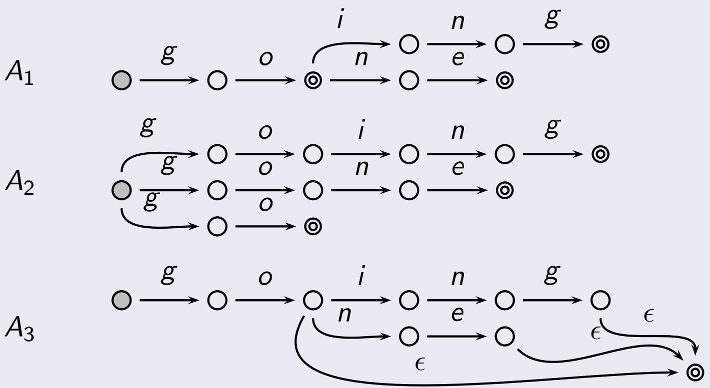
If L is a regular language then there exists a finite-state automaton A accepting L such that the number of states in A is minimal. A is unique up to isomorphism.

A finite-state automaton is **deterministic** if its transition relation is a function.

If L is a regular language then there exists a deterministic, ϵ-free finite-state automaton which accepts it.

Minimization and determinization

Example: Equivalent automata



Applications of finite-state automata in NLP

Finite-state automata are efficient computational devices for generating regular languages.

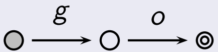
An equivalent view would be to regard them as *recognizing* devices: given some automaton A and a word w, applying the automaton to the word yields an answer to the question: Is w a member of $L(A)$, the language accepted by the automaton?

This reversed view of automata motivates their use for a simple yet necessary application of natural language processing: dictionary lookup.

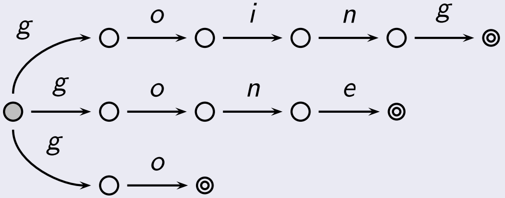
Applications of finite-state automata in NLP

Example: Dictionaries as finite-state automata

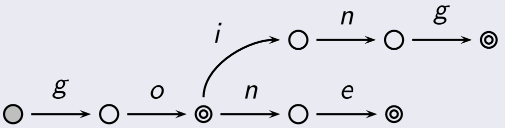
go :



go, gone, going :



go, gone, going :

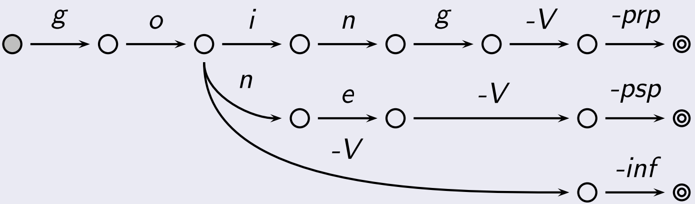


Applications of finite-state automata in NLP

Example: Adding morphological information

Add information about part-of-speech, the number of nouns and the tense of verbs:

$$\Sigma = \{a,\ b,\ c,\ \ldots,\ y,\ z,\ \text{-}N,\ \text{-}V,\ \text{-}sg,\ \text{-}pl,\ \text{-}inf,\ \text{-}prp,\ \text{-}psp\}$$



The appeal of regular languages for NLP

- Most phonological and morphological process of natural languages can be straight-forwardly described using the operations that regular languages are closed under.
- The closure properties of regular languages naturally support modular development of finite-state grammars.
- Most algorithms on finite-state automata are linear. In particular, the recognition problem is linear.
- Finite-state automata are reversible: they can be used both for analysis and for generation.

Regular relations

While regular expressions are sufficiently expressive for some natural language applications, it is sometimes useful to define relations over two sets of strings.

Regular relations

Part-of-speech tagging:

I	know	some	new	tricks	
PRON	V	DET	ADJ	N	
said	the	Cat	in	the	Hat
V	DET	N	P	DET	N

Regular relations

Morphological analysis:

I	know	some	new
I-PRON-1-sg	know-V-pres	some-DET-indef	new-ADJ
tricks	said	the	Cat
trick-N-pl	say-V-past	the-DET-def	cat-N-sg
in	the	Hat	
in-P	the-DET-def	hat-N-sg	

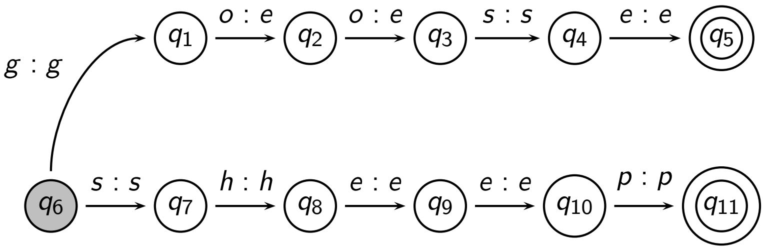
Regular relations

Singular-to-plural mapping:

cat	hat	ox	child	mouse	sheep	goose
cats	hats	oxen	children	mice	sheep	geese

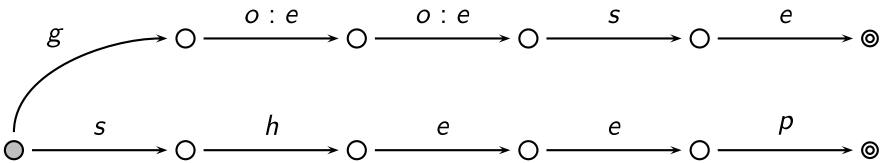
Finite-state transducers

A finite-state transducer is a six-tuple $\langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F \rangle$. Similarly to automata, Q is a finite set of states, $q_0 \in Q$ is the initial state, $F \subseteq Q$ is the set of final (or accepting) states, Σ_1 and Σ_2 are alphabets: finite sets of symbols, not necessarily disjoint (or different). $\delta : Q \times \Sigma_1 \times \Sigma_2 \times Q$ is a relation from states and pairs of alphabet symbols to states.

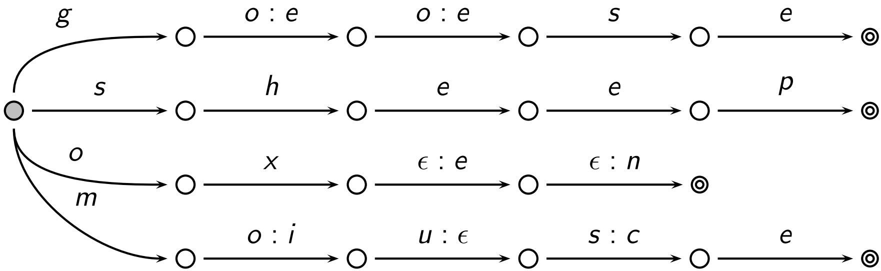


Finite-state transducers

Shorthand notation:



Adding ϵ-moves:



Finite-state transducers

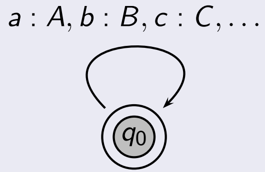
A finite-state transducer defines a set of pairs: a binary relation over $\Sigma_1^* \times \Sigma_2^*$.

The relation is defined analogously to how the language of an automaton is defined: A pair $\langle w_1, w_2 \rangle$ is accepted by the transducer $A = \langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F \rangle$ if and only if there exists a state $q_f \in F$ such that $(q_0, w_1, w_2, q_f) \in \hat{\delta}$.

The transduction of a word $w \in \Sigma_1^*$ is defined as $T(w) = \{u \mid (q_0, w, u, q_f) \in \hat{\delta} \text{ for some } q_f \in F\}$.

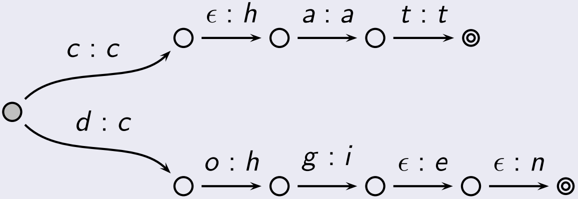
Finite-state transducers

Example: The uppercase transducer



Finite-state transducers

Example: English-to-French



Properties of finite-state transducers

Given a transducer $\langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F\rangle$,

- its *underlying automaton* is $\langle Q, q_0, \Sigma_1 \times \Sigma_2, \delta', F\rangle$, where $(q_1, (a, b), q_2) \in \delta'$ iff $(q_1, a, b, q_2) \in \delta$
- its *upper automaton* is $\langle Q, q_0, \Sigma_1, \delta_1, F\rangle$, where $(q_1, a, q_2) \in \delta_1$ iff for some $b \in \Sigma_2$, $(q_1, a, b, q_2) \in \delta$
- its *lower automaton* is $\langle Q, q_0, \Sigma_2, \delta_2, F\rangle$, where $(q_1, b, q_2) \in \delta_2$ iff for some $a \in \Sigma_a$, $(q_1, a, b, q_2) \in \delta$

Properties of finite-state transducers

- A transducer T is *functional* if for every $w \in \Sigma_1^*$, $T(w)$ is either empty or a singleton.
- Transducers are closed under union: if T_1 and T_2 are transducers, there exists a transducer T such that for every $w \in \Sigma_1^*$, $T(w) = T_1(w) \cup T_2(w)$.
- Transducers are closed under inversion: if T is a transducer, there exists a transducer T^{-1} such that for every $w \in \Sigma_1^*$, $T^{-1}(w) = \{u \in \Sigma_2^* \mid w \in T(u)\}$.
- The inverse transducer is $\langle Q, q_0, \Sigma_2, \Sigma_1, \delta^{-1}, F\rangle$, where $(q_1, a, b, q_2) \in \delta^{-1}$ iff $(q_1, b, a, q_2) \in \delta$.

Properties of regular relations

Example: Operations on finite-state relations

$$R_1 = \{\textit{tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, pineapple:Ananas, coconut:Koko}\}$$

$$R_2 = \{\textit{grapefruit:pampelmuse, coconut:Kokusnuß}\}$$

$$R_1 \cup R_2 = \{\textit{tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß}\}$$

Properties of finite-state transducers

Transducers are closed under composition: if T_1 is a transduction from Σ_1^* to Σ_2^* and and T_2 is a transduction from Σ_2^* to Σ_3^*, then there exists a transducer T such that for every $w \in \Sigma_1^*$, $T(w) = T_2(T_1(w))$.
The number of states in the composition transducer might be $|Q_1 \times Q_2|$.

Example: Composition of finite-state relations

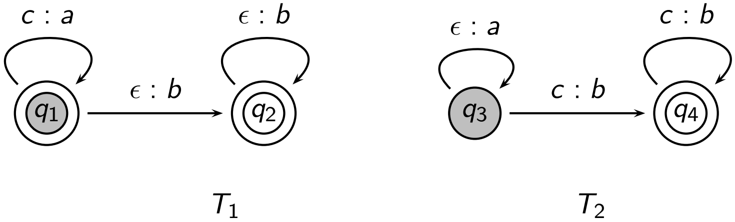
$R_1 = \{$*tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß*$\}$

$R_2 = \{$*tomate:tomato, ananas:pineapple, pampelmousse:grapefruit, concombre:cucumber, cornichon:cucumber, noix-de-coco:coconut*$\}$

$R_2 \circ R_1 = \{$*tomate:Tomate, ananas:Ananas, pampelmousse:Grapefruit, pampelmousse:Pampelmuse, concombre:Gurke,cornichon:Gurke, noix-de-coco:Koko, noix-de-coco:Kokusnuße*$\}$

Properties of finite-state transducers

Transducers are not closed under intersection.



$$T_1(c^n) = \{a^n b^m \mid m \geq 0\}$$
$$T_2(c^n) = \{a^m b^n \mid m \geq 0\} \Rightarrow$$
$$(T_1 \cap T_2)(c^n) = \{a^n b^n\}$$

Transducers with no ϵ-moves are closed under intersection.

Properties of finite-state transducers

- Computationally efficient
- Denote regular relations
- Closed under concatenation, Kleene-star, union
- Not closed under intersection (and hence complementation)
- Closed under composition
- Weights