Implementing morphology and phonology

We begin with a simple problem: a lexicon of some natural language is given as a list of words. Suggest a data structure that will provide insertion and retrieval of data. As a first solution, we are looking for time efficiency rather than space efficiency.

The solution: *trie* (word tree).

Access time: $O(|w|)$. Space requirement: $O(\sum_w |w|)$.

A trie can be augmented to store also a morphological dictionary specifying concatenative affixes, especially suffixes. In this case it is better to turn the tree into a graph.

The obtained model is that of *finite-state automata*.

Finite-state technology

Finite-state automata are not only a good model for representing the lexicon, they are also perfectly adequate for representing dictionaries (lexicons+additional information), describing morphological processes that involve concatenation etc.

A natural extension of finite-state automata – finite-state transducers – is a perfect model for most processes known in morphology and phonology, including non-segmental ones.

Formal language theory – definitions

Formal languages are defined with respect to a given *alphabet*, which is a finite set of symbols, each of which is called a *letter*.

A finite sequence of letters is called a *string*.

> Example: Strings
> Let $\Sigma = \{0, 1\}$ be an alphabet. Then all binary numbers are strings over Σ.
>
> If $\Sigma = \{a, b, c, d, \ldots, y, z\}$ is an alphabet then *cat*, *incredulous* and *supercalifragilisticexpialidocious* are strings, as are *tac*, *qqq* and *kjshdflkwjehr*.

Formal language theory – definitions

The *length* of a string w, denoted $|w|$, is the number of letters in w. The unique string of length 0 is called the *empty string* and is denoted ϵ.

If $w_1 = \langle x_1, \ldots, x_n \rangle$ and $w_2 = \langle y_1, \ldots, y_m \rangle$, the *concatenation* of w_1 and w_2, denoted $w_1 \cdot w_2$, is the string $\langle x_1, \ldots, x_n, y_1, \ldots, y_m \rangle$. $|w_1 \cdot w_2| = |w_1| + |w_2|$.

For every string w, $w \cdot \epsilon = \epsilon \cdot w = w$.

Formal language theory – definitions

Example: Concatenation
Let $\Sigma = \{a, b, c, d, \ldots, y, z\}$ be an alphabet. Then *master* $\cdot$ *mind* $=$ *mastermind*, *mind* $\cdot$ *master* $=$ *mindmaster* and *master* $\cdot$ *master* $=$ *mastermaster*. Similarly, *learn* $\cdot$ *s* $=$ *learns*, *learn* $\cdot$ *ed* $=$ *learned* and *learn* $\cdot$ *ing* $=$ *learning*.

Formal language theory – definitions

An *exponent* operator over strings is defined in the following way: for every string w, $w^0 = \epsilon$. Then, for $n > 0$, $w^n = w^{n-1} \cdot w$.

Example: Exponent
If $w =$ *go*, then $w^0 = \epsilon$, $w^1 = w =$ *go*, $w^2 = w^1 \cdot w = w \cdot w =$ *gogo*, $w^3 =$ *gogogo* and so on.

Formal language theory – definitions

The *reversal* of a string w is denoted w^R and is obtained by writing w in the reverse order. Thus, if $w = \langle x_1, x_2, \ldots, x_n \rangle$, $w^R = \langle x_n, x_{n-1}, \ldots, x_1 \rangle$.

Given a string w, a *substring* of w is a sequence formed by taking contiguous symbols of w in the order in which they occur in w. If $w = \langle x_1, \ldots, x_n \rangle$ then for any i, j such that $1 \leq i \leq j \leq n$, $\langle x_i, \ldots x_j \rangle$ is a substring of w.

Two special cases of substrings are *prefix* and *suffix*: if $w = w_l \cdot w_c \cdot w_r$ then w_l is a prefix of w and w_r is a suffix of w.

Formal language theory – definitions

Example: Substrings
Let $\Sigma = \{a, b, c, d, \ldots, y, z\}$ be an alphabet and $w =$ *indistinguishable* a string over Σ. Then ϵ, *in*, *indis*, *indistinguish* and *indistinguishable* are prefixes of w, while ϵ, *e*, *able*, *distinguishable* and *indistinguishable* are suffixes of w. Substrings that are neither prefixes nor suffixes include *distinguish*, *gui* and *is*.

Formal language theory – definitions

Given an alphabet Σ, the set of all strings over Σ is denoted by Σ^*.

A *formal language* over an alphabet Σ is a subset of Σ^*.

Formal language theory – definitions

Example: Languages
Let $\Sigma = \{a, b, c, \ldots, y, z\}$. Then Σ^* is the set of all strings over the Latin alphabet. Any subset of this set is a language. In particular, the following are formal languages:

Formal language theory – definitions

- Σ^*;
- the set of strings consisting of consonants only;
- the set of strings consisting of vowels only;
- the set of strings each of which contains at least one vowel and at least one consonant;
- the set of palindromes;
- the set of strings whose length is less than 17 letters;
- the set of single-letter strings;
- the set $\{$*i, you, he, she, it, we, they*$\}$;
- the set of words occurring in Joyce's Ulysses;
- the empty set;

Note that the first five languages are infinite while the last five are finite.

Formal language theory – definitions

The string operations can be lifted to languages.

If L is a language then the *reversal* of L, denoted L^R, is the language $\{w \mid w^R \in L\}$.

If L_1 and L_2 are languages, then
$L_1 \cdot L_2 = \{w_1 \cdot w_2 \mid w_1 \in L_1 \text{ and } w_2 \in L_2\}$.

Example: Language operations
$L_1 = \{$*i, you, he, she, it, we, they*$\}$, $L_2 = \{$*smile, sleep*$\}$.

Then ${L_1}^R = \{$*i, uoy, eh, ehs, ti, ew, yeht*$\}$ and $L_1 \cdot L_2 = \{$*ismile, yousmile, hesmile, shesmile, itsmile, wesmile, theysmile, isleep, yousleep, hesleep, shesleep, itsleep, wesleep, theysleep*$\}$.

Formal language theory – definitions

If L is a language then $L^0 = \{\epsilon\}$.
Then, for $i > 0$, $L^i = L \cdot L^{i-1}$.

Example: Language exponentiation
Let L be the set of words {*bau, haus, hof, frau*}. Then $L^0 = \{\epsilon\}$, $L^1 = L$ and $L^2 =$ {*baubau, bauhaus, bauhof, baufrau, hausbau, haushaus, haushof, hausfrau, hofbau, hofhaus, hofhof, hoffrau, fraubau, frauhaus, frauhof, fraufrau*}.

Formal language theory – definitions

The *Kleene closure* of L and is denoted L^* and is defined as $\bigcup_{i=0}^{\infty} L^i$.
$L^+ = \bigcup_{i=1}^{\infty} L^i$.

Example: Kleene closure
Let $L =$ {*dog, cat*}. Observe that $L^0 = \{\epsilon\}$, $L^1 =$ {*dog, cat*}, $L^2 =$ {*catcat, catdog, dogcat, dogdog*}, etc. Thus L^* contains, among its infinite set of strings, the strings ϵ, *cat*, *dog*, *catcat*, *catdog*, *dogcat*, *dogdog*, *catcatcat*, *catdogcat*, *dogcatcat*, *dogdogcat*, etc.

The notation for Σ^* should now become clear: it is simply a special case of L^*, where $L = \Sigma$.

Regular expressions

Regular expressions are a formalism for defining (formal) languages. Their "syntax" is formally defined and is relatively simple. Their "semantics" is sets of strings: the denotation of a regular expression is a set of strings in some formal language.

Regular expressions

Regular expressions are defined recursively as follows:

- $\emptyset$ is a regular expression
- ϵ is a regular expression
- if $a \in \Sigma$ is a letter then a is a regular expression
- if r_1 and r_2 are regular expressions then so are $(r_1 + r_2)$ and $(r_1 \cdot r_2)$
- if r is a regular expression then so is $(r)^*$
- nothing else is a regular expression over Σ.

Regular expressions

Example: Regular expressions
Let Σ be the alphabet {*a*, *b*, *c*, ..., *y*, *z*}. Some regular expressions over this alphabet are:

- $\emptyset$
- a
- $((c \cdot a) \cdot t)$
- $(((m \cdot e) \cdot (o)^*) \cdot w)$
- $(a + (e + (i + (o + u))))$
- $((a + (e + (i + (o + u)))))^*$

Regular expressions

For every regular expression r its denotation, $[\![r]\!]$, is a set of strings defined as follows:

- $[\![\emptyset]\!] = \emptyset$
- $[\![\epsilon]\!] = \{\epsilon\}$
- if $a \in \Sigma$ is a letter then $[\![a]\!] = \{a\}$
- if r_1 and r_2 are regular expressions whose denotations are $[\![r_1]\!]$ and $[\![r_2]\!]$, respectively, then $[\![(r_1 + r_2)]\!] = [\![r_1]\!] \cup [\![r_2]\!]$, $[\![(r_1 \cdot r_2)]\!] = [\![r_1]\!] \cdot [\![r_2]\!]$ and $[\![(r_1)^*]\!] = [\![r_1]\!]^*$

Regular expressions

Example: Regular expressions and their denotations

$\emptyset$	$\emptyset$
a	$\{a\}$
$((c \cdot a) \cdot t)$	$\{c \cdot a \cdot t\}$
$(((m \cdot e) \cdot (o)^*) \cdot w)$	{*mew, meow, meoow, meooow, meoo*
$(a + (e + (i + (o + u))))$	$\{a, e, i, o, u\}$
$((a + (e + (i + (o + u)))))^*$	*all strings of 0 or more vowels*

Regular expressions

Example: Regular expressions
Given the alphabet of all English letters, $\Sigma = \{a, b, c, \ldots, y, z\}$, the language Σ^* is denoted by the regular expression Σ^*.

The set of all strings which contain a vowel is denoted by $\Sigma^* \cdot (a + e + i + o + u) \cdot \Sigma^*$.

The set of all strings that begin in "*un*" is denoted by $(un)\Sigma^*$.

The set of strings that end in either "*tion*" or "*sion*" is denoted by $\Sigma^* \cdot (s + t) \cdot (ion)$.

Note that all these languages are infinite.

Properties of regular languages

Closure properties:

A class of languages $\mathcal{L}$ is said to be closed under some operation '$\bullet$' if and only if whenever two languages L_1, L_2 are in the class ($L_1, L_2 \in \mathcal{L}$), also the result of performing the operation on the two languages is in this class: $L_1 \bullet L_2 \in \mathcal{L}$.

Properties of regular languages

Regular languages are closed under:

- Union
- Intersection
- Complementation
- Difference
- Concatenation
- Kleene-star
- Substitution and homomorphism