Finite-state automata

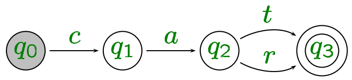
Automata are models of computation: they compute languages.

A finite-state automaton is a five-tuple $\langle Q, q_0, \Sigma, \delta, F\rangle$, where Σ is a finite set of **alphabet** symbols, Q is a finite set of **states**, $q_0 \in Q$ is the **initial state**, $F \subseteq Q$ is a set of **final** (accepting) states and $\delta : Q \times \Sigma \times Q$ is a relation from states and alphabet symbols to states.

Finite-state automata

Example: Finite-state automaton

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{c, a, t, r\}$
- $F = \{q_3\}$
- $\delta = \{\langle q_0, c, q_1\rangle, \langle q_1, a, q_2\rangle, \langle q_2, t, q_3\rangle, \langle q_2, r, q_3\rangle\}$



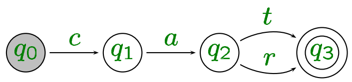
Finite-state automata

The reflexive transitive extension of the transition relation δ is a new relation, $\hat{\delta}$, defined as follows:

- for every state $q \in Q$, $(q, \epsilon, q) \in \hat{\delta}$
- for every string $w \in \Sigma^*$ and letter $a \in \Sigma$, if $(q, w, q') \in \hat{\delta}$ and $(q', a, q'') \in \delta$ then $(q, w \cdot a, q'') \in \hat{\delta}$.

Finite-state automata

Example: Paths
For the finite-state automaton:



$\hat{\delta}$ is the following set of triples:

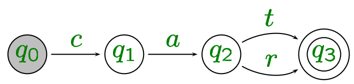
$$\langle q_0, \epsilon, q_0\rangle, \langle q_1, \epsilon, q_1\rangle, \langle q_2, \epsilon, q_2\rangle, \langle q_3, \epsilon, q_3\rangle,$$
$$\langle q_0, c, q_1\rangle, \langle q_1, a, q_2\rangle, \langle q_2, t, q_3\rangle, \langle q_2, r, q_3\rangle,$$
$$\langle q_0, ca, q_2\rangle, \langle q_1, at, q_3\rangle, \langle q_1, ar, q_3\rangle,$$
$$\langle q_0, cat, q_3\rangle, \langle q_0, car, q_3\rangle$$

Finite-state automata

A string w is accepted by the automaton $A = \langle Q, q_0, \Sigma, \delta, F\rangle$ if and only if there exists a state $q_f \in F$ such that $(q_0, w, q_f) \in \hat{\delta}$.

The *language accepted by a finite-state automaton* is the set of all the strings it accepts.

Example: Language
The language of the finite-state automaton:



is $\{$*cat*, *car*$\}$.

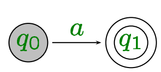
Finite-state automata

Example: Some finite-state automata

$\emptyset$

Finite-state automata

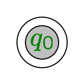
Example: Some finite-state automata



$\{a\}$

Finite-state automata

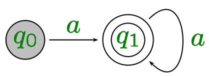
Example: Some finite-state automata



$\{\epsilon\}$

Finite-state automata

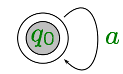
Example: Some finite-state automata



$\{a,\ aa,\ aaa,\ aaaa, \ldots\}$

Finite-state automata

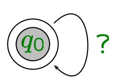
Example: Some finite-state automata



a^*

Finite-state automata

Example: Some finite-state automata



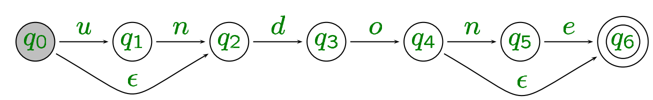
Σ^*

Finite-state automata

An extension: ϵ-moves.

The transition relation δ is extended to: $\delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q$

Example: Automata with ϵ-moves
The language accepted by the following automaton is $\{$*do, undo, done, undone*$\}$:



Finite-state automata

Theorem (Kleene, 1956): The class of languages recognized by finite-state automata is the class of regular languages.

Finite-state automata

Example: Finite-state automata and regular expressions

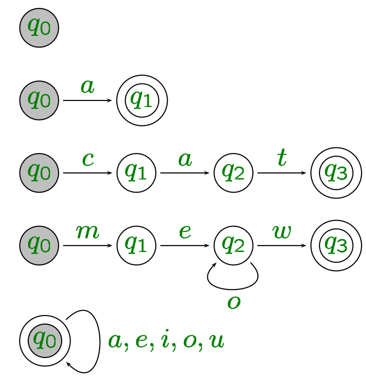
$\emptyset$

a

$((c \cdot a) \cdot t)$

$(((m \cdot e) \cdot (o)^*) \cdot w)$

$((a + (e + (i + (o + u)))))^*$



Operations on finite-state automata

- Concatenation
- Union
- Intersection
- Minimization
- Determinization

Minimization and determinization

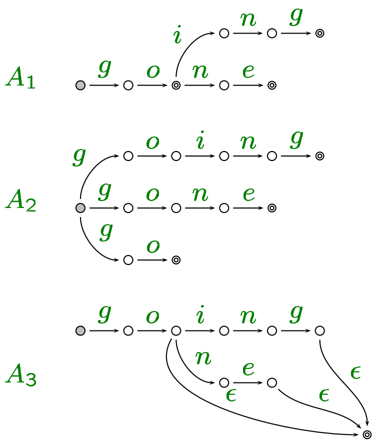
If L is a regular language then there exists a finite-state automaton A accepting L such that the number of states in A is minimal. A is unique up to isomorphism.

A finite-state automaton is **deterministic** if its transition relation is a function.

If L is a regular language then there exists a deterministic, ϵ-free finite-state automaton which accepts it.

Minimization and determinization

Example: Equivalent automata



Applications of finite-state automata in NLP

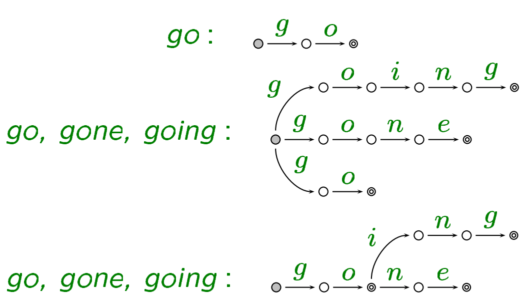
Finite-state automata are efficient computational devices for generating regular languages.

An equivalent view would be to regard them as *recognizing* devices: given some automaton A and a word w, applying the automaton to the word yields an answer to the question: Is w a member of $L(A)$, the language accepted by the automaton?

This reversed view of automata motivates their use for a simple yet necessary application of natural language processing: dictionary lookup.

Applications of finite-state automata in NLP

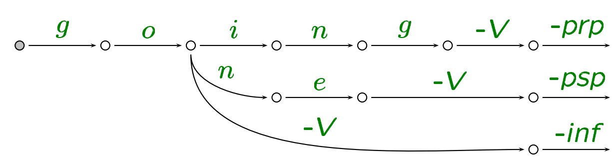
Example: Dictionaries as finite-state automata



Applications of finite-state automata in NLP

Example: Adding morphological information
Add information about part-of-speech, the number of nouns and the tense of verbs:

$$\Sigma = \{a, b, c, \ldots, y, z, \text{-N}, \text{-V}, \text{-sg}, \text{-pl}, \text{-inf}, \text{-prp}, \text{-psp}\}$$



The appeal of regular languages for NLP

- Most phonological and morphological process of natural languages can be straight-forwardly described using the operations that regular languages are closed under.
- The closure properties of regular languages naturally support modular development of finite-state grammars.
- Most algorithms on finite-state automata are linear. In particular, the recognition problem is linear.
- Finite-state automata are reversible: they can be used both for analysis and for generation.

Regular relations

While regular expressions are sufficiently expressive for some natural language applications, it is sometimes useful to define relations over two sets of strings.

Regular relations

Part-of-speech tagging:

I	know	some	new	tricks	
PRON	V	DET	ADJ	N	
said	the	Cat	in	the	Hat
V	DET	N	P	DET	N

Regular relations

Morphological analysis:

I	know	some	new
I-PRON-1-sg	know-V-pres	some-DET-indef	new-ADJ
tricks	said	the	Cat
trick-N-pl	say-V-past	the-DET-def	cat-N-sg
in	the	Hat	
in-P	the-DET-def	hat-N-sg	

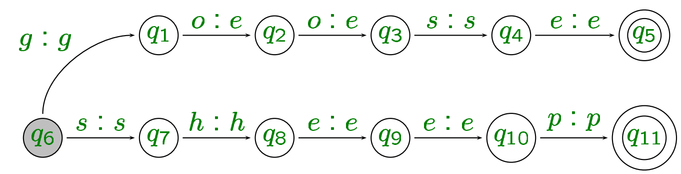
Regular relations

Singular-to-plural mapping:

cat	hat	ox	child	mouse	sheep	goose
cats	hats	oxen	children	mice	sheep	geese

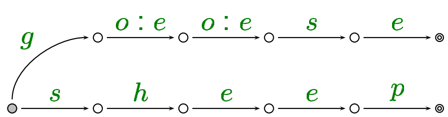
Finite-state transducers

A finite-state transducer is a six-tuple $\langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F\rangle$. Similarly to automata, Q is a finite set of states, $q_0 \in Q$ is the initial state, $F \subseteq Q$ is the set of final (or accepting) states, Σ_1 and Σ_2 are alphabets: finite sets of symbols, not necessarily disjoint (or different). $\delta : Q \times \Sigma_1 \times \Sigma_2 \times Q$ is a relation from states and pairs of alphabet symbols to states.

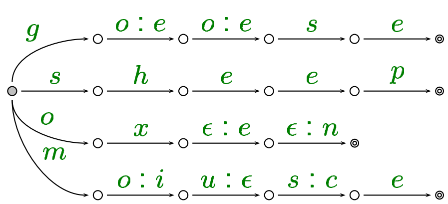


Finite-state transducers

Shorthand notation:



Adding ϵ-moves:



Finite-state transducers

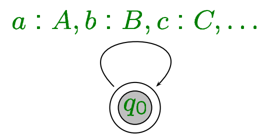
A finite-state transducer defines a set of pairs: a binary relation over $\Sigma_1^* \times \Sigma_2^*$.

The relation is defined analogously to how the language of an automaton is defined: A pair $\langle w_1, w_2\rangle$ is accepted by the transducer $A = \langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F\rangle$ if and only if there exists a state $q_f \in F$ such that $(q_0, w_1, w_2, q_f) \in \hat{\delta}$.

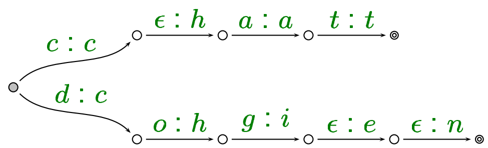
The transduction of a word $w \in \Sigma_1^*$ is defined as $T(w) = \{u \mid (q_0, w, u, q_f) \in \hat{\delta}$ for some $q_f \in F\}$.

Finite-state transducers

Example: The uppercase transducer



Example: English-to-French



Properties of finite-state transducers

Given a transducer $\langle Q, q_0, \Sigma_1, \Sigma_2, \delta, F\rangle$,

- its *underlying automaton* is $\langle Q, q_0, \Sigma_1 \times \Sigma_2, \delta', F\rangle$, where $(q_1, (a, b), q_2) \in \delta'$ iff $(q_1, a, b, q_2) \in \delta$
- its *upper automaton* is $\langle Q, q_0, \Sigma_1, \delta_1, F\rangle$, where $(q_1, a, q_2) \in \delta_1$ iff for some $b \in \Sigma_2$, $(q_1, a, b, q_2) \in \delta$
- its *lower automaton* is $\langle Q, q_0, \Sigma_2, \delta_2, F\rangle$, where $(q_1, b, q_2) \in \delta_2$ iff for some $a \in \Sigma_a$, $(q_1, a, b, q_2) \in \delta$

Properties of finite-state transducers

A transducer T is *functional* if for every $w \in \Sigma_1^*$, $T(w)$ is either empty or a singleton.

Transducers are closed under union: if T_1 and T_2 are transducers, there exists a transducer T such that for every $w \in \Sigma_1^*$, $T(w) = T_1(w) \cup T_2(w)$.

Transducers are closed under inversion: if T is a transducer, there exists a transducer T^{-1} such that for every $w \in \Sigma_1^*$, $T^{-1}(w) = \{u \in \Sigma_2^* \mid w \in T(u)\}$.

The inverse transducer is $\langle Q, q_0, \Sigma_2, \Sigma_1, \delta^{-1}, F\rangle$, where $(q_1, a, b, q_2) \in \delta^{-1}$ iff $(q_1, b, a, q_2) \in \delta$.

Properties of regular relations

Example: Operations on finite-state relations

$R_1 = \{$*tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, pineapple:Ananas, coconut:Koko*$\}$

$R_2 = \{$*grapefruit:pampelmuse, coconut:Kokusnuß*$\}$

$R_1 \cup R_2 = \{$*tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß*$\}$

Properties of finite-state transducers

Transducers are closed under composition: if T_1 is a transduction from Σ_1^* to Σ_2^* and and T_2 is a transduction from Σ_2^* to Σ_3^*, then there exists a transducer T such that for every $w \in \Sigma_1^*$, $T(w) = T_2(T_1(w))$.

The number of states in the composition transducer might be $|Q_1 \times Q_2|$.

Example: Composition of finite-state relations

$R_1 = \{$*tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß*$\}$
$R_2 = \{$*tomate:tomato, ananas:pineapple, pampelmousse:grapefruit, concombre:cucumber, cornichon:cucumber, noix-de-coco:coconut*$\}$
$R_2 \circ R_1 = \{$*tomate:Tomate, ananas:Ananas, pampelmousse:Grapefruit, pampelmousse:Pampelmuse, concombre:Gurke,cornichon:Gurke, noix-de-coco:Koko, noix-de-coco:Kokusnuße*$\}$

Properties of finite-state transducers

Transducers are not closed under intersection.

T_1 T_2

$$T_1(c^n) = \{a^n b^m \mid m \geq 0\}$$
$$T_2(c^n) = \{a^m b^n \mid m \geq 0\} \Rightarrow$$
$$(T_1 \cap T_2)(c^n) = \{a^n b^n\}$$

Transducers with no ϵ-moves are closed under intersection.

Properties of finite-state transducers

- Computationally efficient
- Denote regular relations
- Closed under concatenation, Kleene-star, union
- Not closed under intersection (and hence complementation)
- Closed under composition
- Weights