

עֵיבוֹד שְׁפוֹת טְבַעִיּוֹת

שׁוּלֵי וִינְטֵנֶר

Parsing

Recognition: Given a (context-free) grammar G and a string of words w , determine whether $w \in L(G)$.

Parsing: If $w \in L(G)$, produce the (tree) structure that is assigned by G to w .

Parsing

General requirements for a parsing algorithm:

- **Generality:** the algorithm must be applicable to *any* grammar
- **Completeness:** the algorithm must produce *all* the results in case of ambiguity
- **Efficiency**
- **Flexibility:** a good algorithm can be easily modified

Parsing

Parameters that define different parsing algorithm:

Orientation: Top-down vs. bottom-up vs. mixed

Direction: Left-to-right vs. right-to-left vs. mixed (e.g., island-driven)

Handling multiple choice: Dynamic programming vs. parallel processing vs. backtracking

Search: Breadth-first or Depth-first

A bottom-up recognition algorithm

Assumptions:

- The grammar is given in Chomsky Normal Form: each rule is either of the form $A \rightarrow BC$ (where A, B, C are non-terminals) or of the form $A \rightarrow a$ (where a is a terminal).
- The string to recognize is $w = w_1 \dots w_n$.
- A set of indices $\{0, 1, \dots, n\}$ is defined to point to positions between the input string's words:
0 the 1 cat 2 in 3 the 4 hat 5

An example grammar

$D \rightarrow the$ $NP \rightarrow D N$

$N \rightarrow cat$ $PP \rightarrow P NP$

$N \rightarrow hat$ $NP \rightarrow NP PP$

$P \rightarrow in$

Example sentences:

the cat in the hat

the cat in the hat in the hat

The CYK algorithm

Bottom-up, chart-based recognition algorithm for grammars in CNF

To recognize a string of length n , uses a *chart*: a n -dimensional matrix of size $n \times (n + 1)$

Invariant: a non-terminal A is stored in the $[i, j]$ entry of the chart iff $A \Rightarrow w_{i+1} \dots w_j$

Consequently, the chart is triangular. A word w is recognized iff the start symbol S is in the $[0, n]$ entry of the chart

The idea: build all constituents up to the i -th position before constructing the $i + 1$ position; build smaller constituents before constructing larger ones

The CYK algorithm

```
for j := 1 to n do
  for all rules  $A \rightarrow w_j$  do
    chart[j-1, j] := chart[j-1, j]  $\cup$  {A}
  for i := j-2 downto 0 do
    for k := i+1 to j-1 do
      for all  $B \in \text{chart}[i, k]$  do
        for all  $C \in \text{chart}[k, j]$  do
          for all rules  $A \rightarrow B C$  do
            chart[i, j] := chart[i, j]  $\cup$  {A}
  if  $S \in \text{chart}[0, n]$  then accept else reject
```


The CYK algorithm

Extensions:

- Parsing in addition to recognition
- Support for ϵ -rules
- General context-free grammars (not just CYK)

Parsing schemata

To provide a unified framework for discussing various parsing algorithms we use *parsing schemata*, which are generalized schemes for describing the principles behind specific parsing algorithms. This is a generalization of the *parsing as deduction* paradigm.

A parsing schema consists of four components:

- a set of items
- a set of axioms
- a set of deduction rules
- a set of goal items

Parsing schema: CYK

Given a grammar $G = \langle \Sigma, V, S, P \rangle$ and a string $w = w_1 \dots w_n$:

Items: $[i, A, j]$ for $A \in V$ and $0 \leq i, j \leq n$
(state that $A \xRightarrow{*} w_{i+1} \dots w_j$)

Axioms: $[i, A, i + 1]$ when $A \rightarrow w_{i+1} \in P$

Goals: $[0, S, n]$

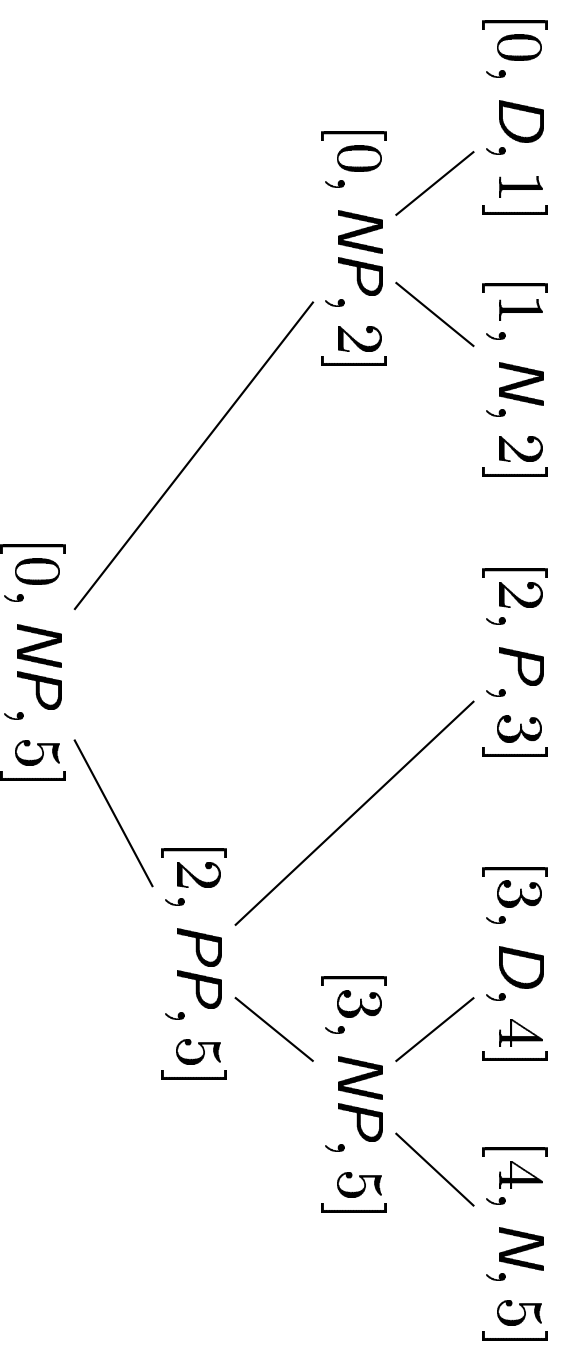
Inference rules:

$$\begin{array}{ccc} [i, B, j] & [j, C, k] & \\ \hline [i, A, k] & & A \rightarrow B C \end{array}$$

CYK parsing schema: deduction example

$D \rightarrow the$ $NP \rightarrow D N$
 $N \rightarrow cat$ $PP \rightarrow P NP$
 $N \rightarrow hat$ $NP \rightarrow NP PP$
 $P \rightarrow in$

0 the 1 cat 2 in 3 the 4 hat 5



Parsing: bottom-up schema (Shift–Reduce)

Items: $[\alpha\bullet, j]$ (state that $\alpha w_{j+1} \dots w_n \xRightarrow{*} w_1 \dots w_n$)

Axioms: $[\bullet, 0]$

Goals: $[S\bullet, n]$

Inference rules:

$[\alpha\bullet, j]$

Shift

$[\alpha w_{j+1}\bullet, j + 1]$

$[\alpha\gamma\bullet, j]$

Reduce

$B \rightarrow \gamma$

$[\alpha B\bullet, j]$

Bottom-up deduction: example

Parsing: top-down schema

Item form: $[\bullet\beta, j]$ (state that $S \Rightarrow^* w_1 \dots w_j \beta$)

Axioms: $[\bullet S, 0]$

Goals: $[\bullet, n]$

Inference rules:

Scan
$$\frac{[\bullet w_{j+1} \beta, j]}{[\bullet \beta, j + 1]}$$

$[\bullet B \beta, j]$

Predict
$$\frac{B \rightarrow \gamma}{[\bullet \gamma \beta, j]}$$

Top-down deduction: example

Input: 0 the 1 cat 2 in 3 the 4 hat 5

| | |
|----------------|---------------------------|
| [•NP, 0] | <i>axiom</i> |
| [•NP PP, 0] | <i>predict NP → NP PP</i> |
| [•D N PP, 0] | <i>predict NP → D N</i> |
| [•the N PP, 0] | <i>predict D → the</i> |
| [•N PP, 1] | <i>scan</i> |
| [•cat PP, 1] | <i>predict N → cat</i> |
| [•PP, 2] | <i>scan</i> |
| [•P NP, 2] | <i>predict PP → P NP</i> |
| [•in NP, 2] | <i>predict P → in</i> |
| [•NP, 3] | <i>scan</i> |
| [•D N, 3] | <i>predict NP → D N</i> |
| [•the N, 3] | <i>predict D → the</i> |
| [•N, 4] | <i>scan</i> |
| [•hat, 4] | <i>predict N → hat</i> |
| [•, 5] | <i>scan</i> |

Top-down parsing: algorithm

```
Parse( $\beta, j$ ) ::  
  if  $\beta = w_{j+1} \cdot \beta'$  then return parse( $\beta', j + 1$ )  
  else if  $\beta = B \cdot \beta'$  then  
    for every rule  $B \rightarrow \gamma \in P$   
      if Parse( $\gamma \cdot \beta', j$ ) then return true  
  return false  
  
if Parse( $S, 0$ ) then accept else reject
```

Top-down vs. Bottom-up parsing

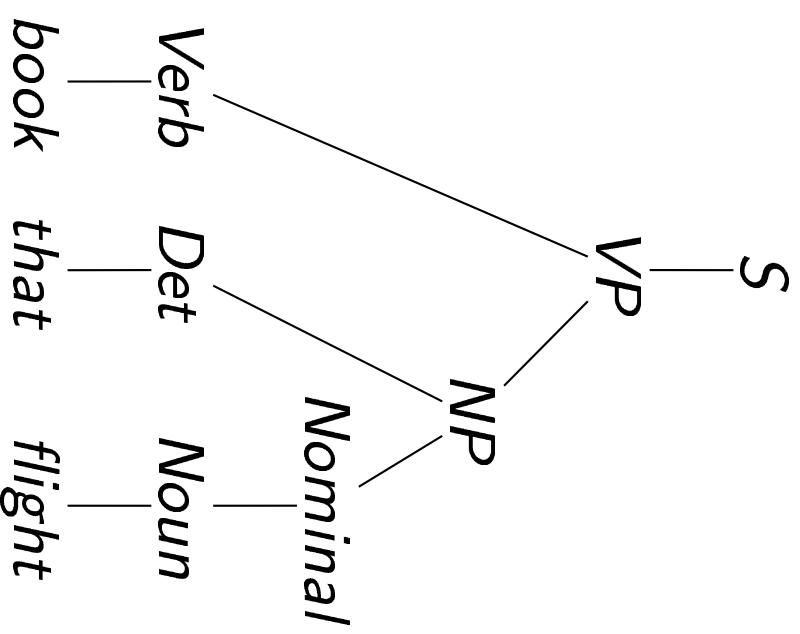
Two inherent constraints:

1. The root of the tree is S
2. The yield of the tree is the input word

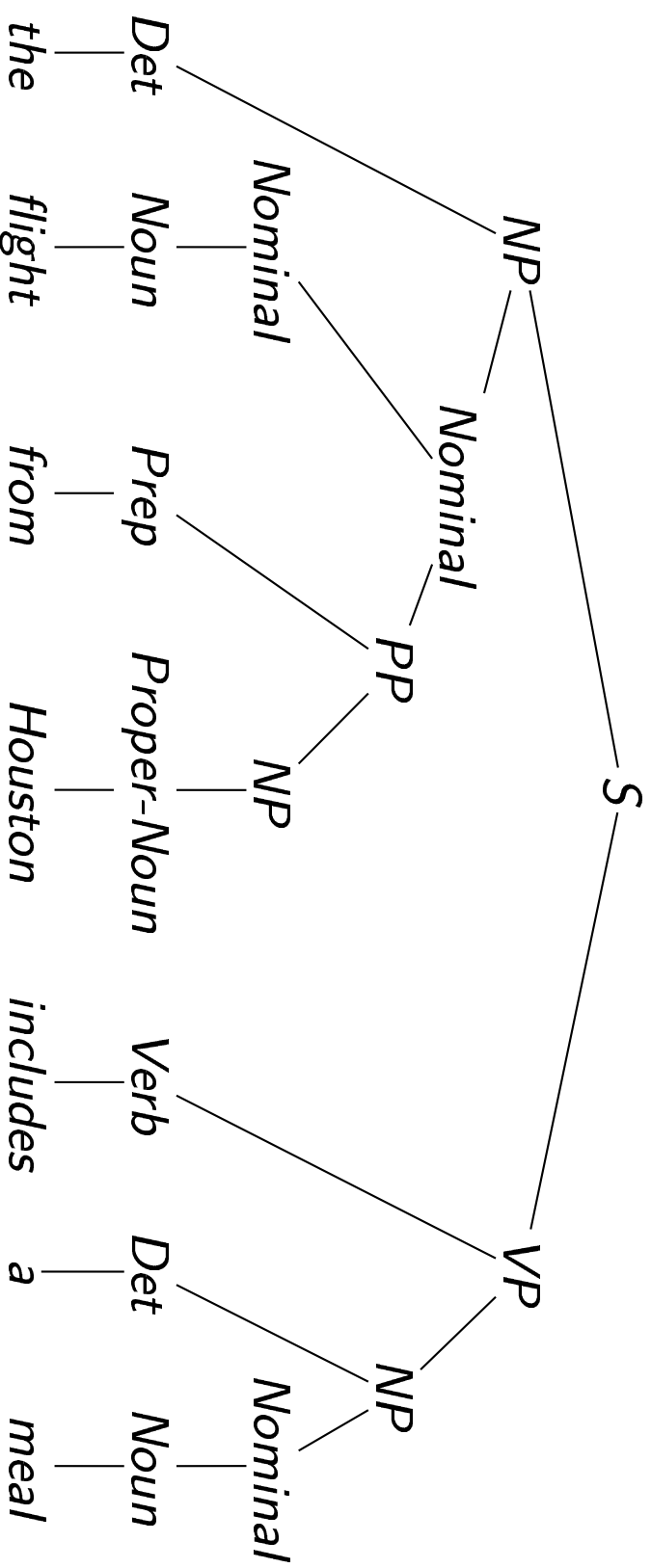
An example grammar

| | |
|------------------------------------|--|
| $S \rightarrow NP VP$ | $Det \rightarrow that this a$ |
| $S \rightarrow Aux NP VP$ | $Noun \rightarrow book flight meal$ |
| $S \rightarrow VP$ | $Verb \rightarrow book include includes$ |
| $VP \rightarrow Verb$ | $Prep \rightarrow from to on$ |
| $VP \rightarrow Verb NP$ | $Proper-Noun \rightarrow Houston TWA$ |
| $NP \rightarrow Det Nominal$ | $Aux \rightarrow does$ |
| $NP \rightarrow Proper-Noun$ | |
| $Nominal \rightarrow Noun$ | |
| $Nominal \rightarrow Noun Nominal$ | |
| $Nominal \rightarrow Nominal PP$ | |
| $PP \rightarrow Prep NP$ | |

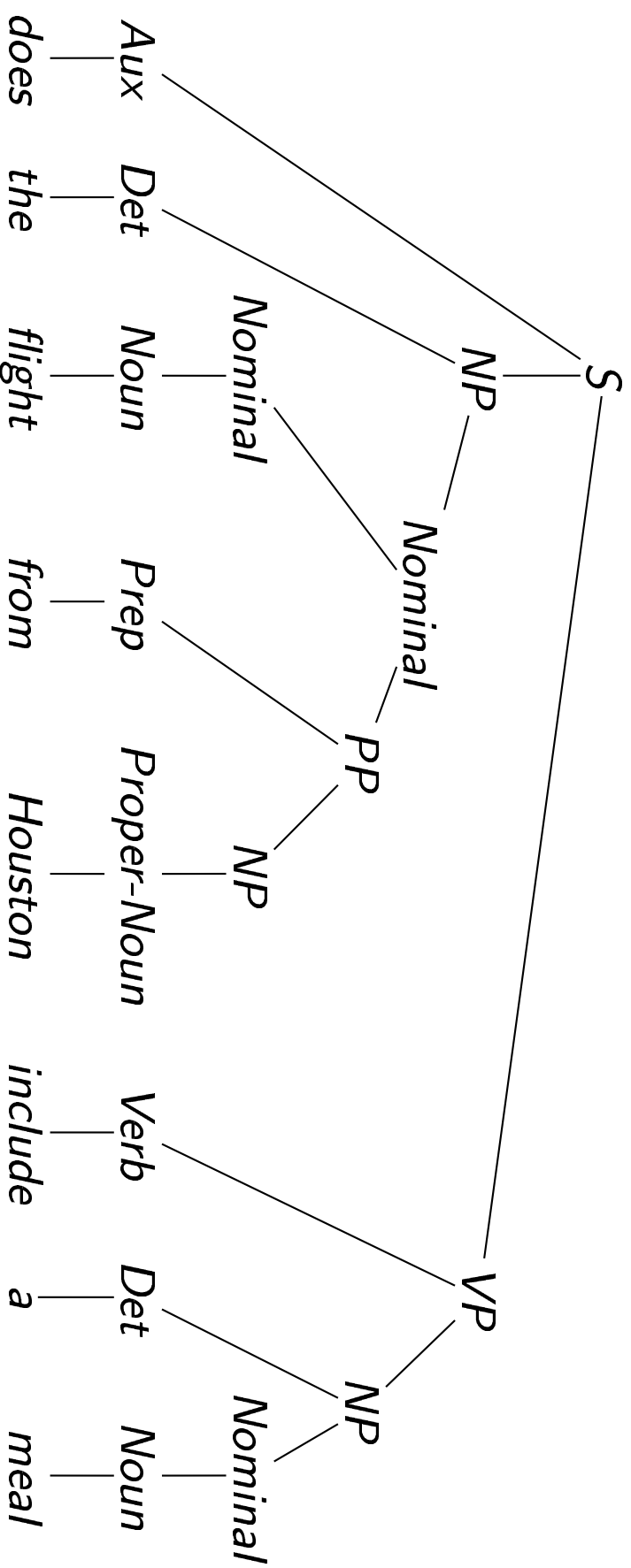
An example derivation tree



An example derivation tree



An example derivation tree



Top-down vs. Bottom-up parsing

When expanding the top-down search space, which local trees are created?

Top-down vs. Bottom-up parsing

To reduce “blind” search, add bottom-up filtering.

Observation: when trying to Parse(β, j), where $\beta = B\gamma$, the parser succeeds only if $B \Rightarrow^* w_{j+1}\beta$.

Definition: A word w is a **left-corner** of a non-terminal B iff $B \Rightarrow^* w\beta$ for some β .

Top-down parsing with bottom-up filtering

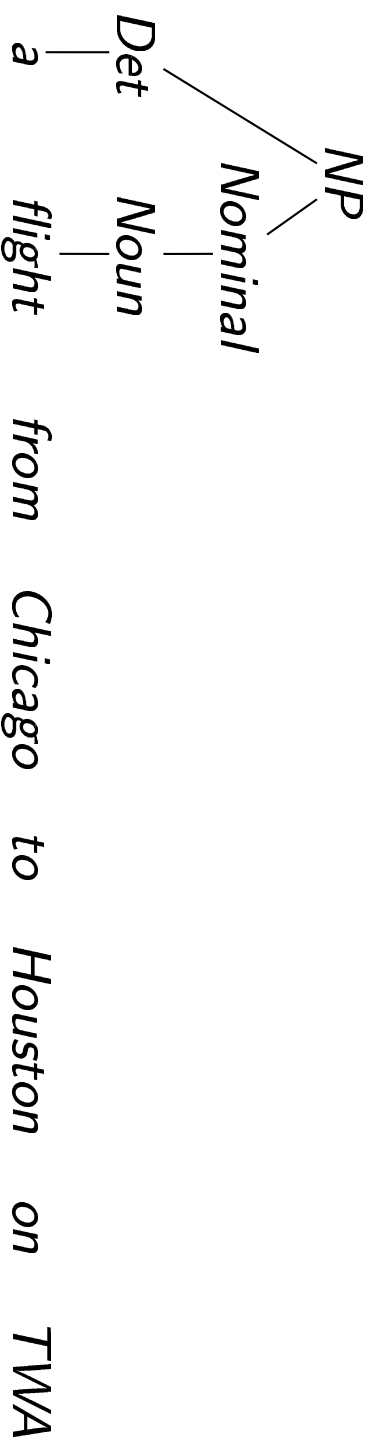
```
Parse( $\beta, j$ ) ::  
  if  $\beta = w_{j+1} \cdot \beta'$  then return parse( $\beta', j + 1$ )  
  else if  $\beta = B \cdot \beta'$  then  
    if  $w_{j+1}$  is a left-corner of  $B$  then  
      for every rule  $B \rightarrow \gamma \in P$   
        if Parse( $\gamma \cdot \beta', j$ ) then return true  
    return false  
  if Parse( $S, 0$ ) then accept else reject
```

Top-down vs. Bottom-up parsing

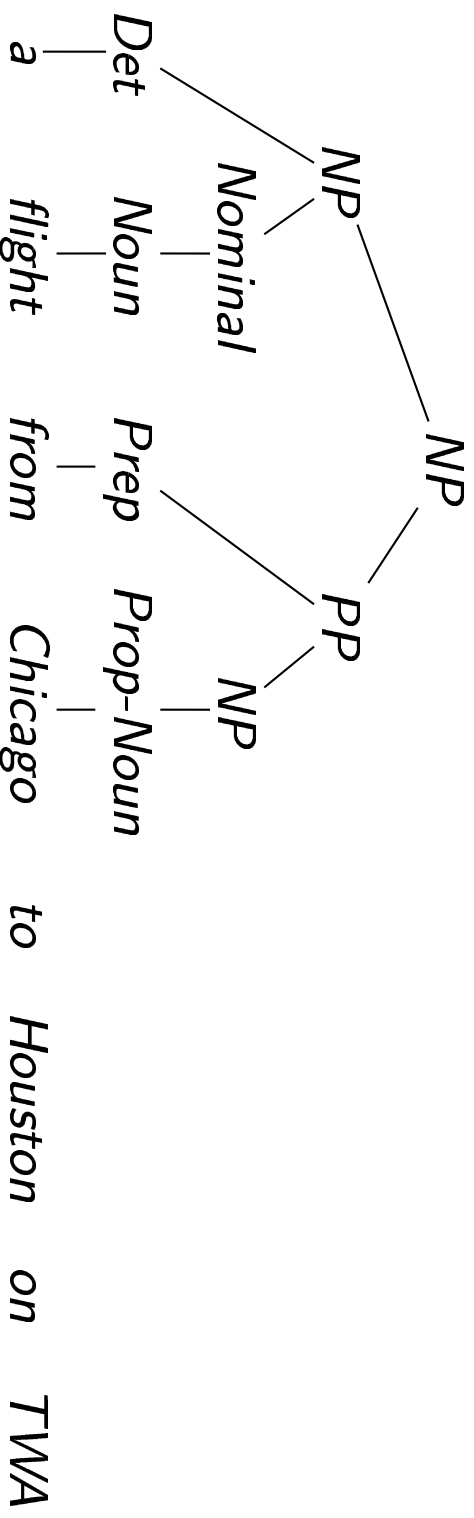
Even with bottom-up filtering, top-down parsing suffers from the following problems:

- Left recursive rules can cause non-termination:
 $NP \rightarrow NP PP.$
- Even when parsing terminates, it might take exponentially many steps.
- Constituents are computed over and over again

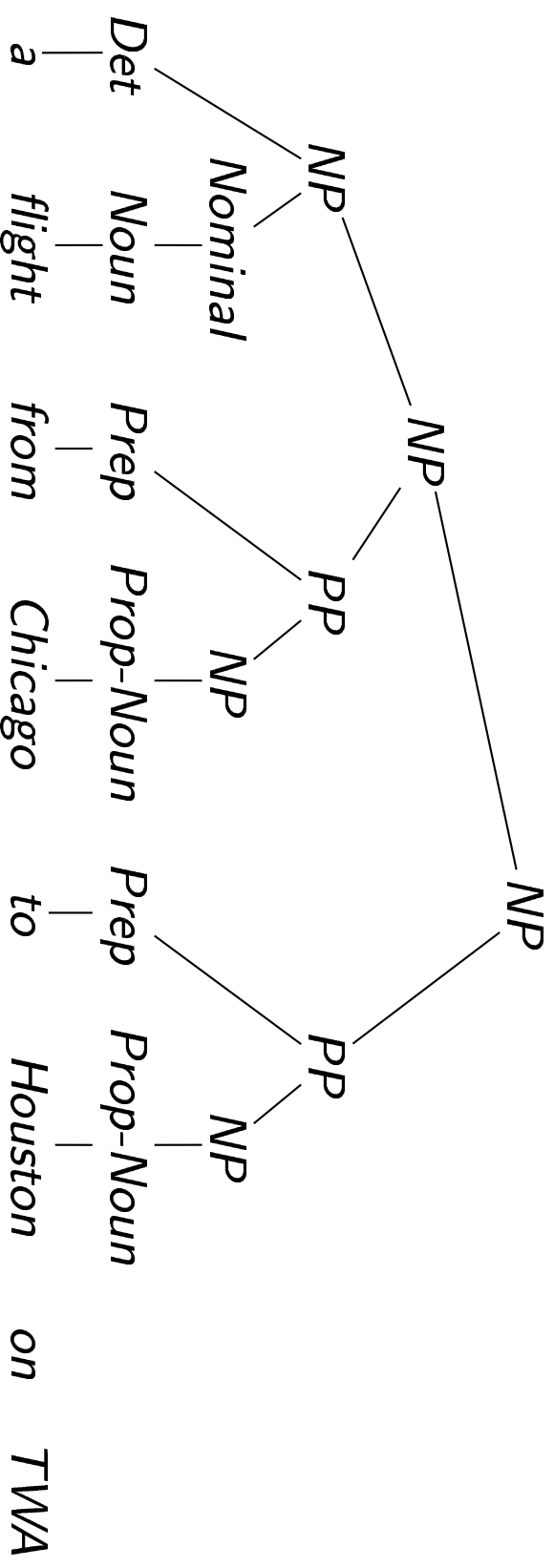
Top-down parsing: repeated generation of sub-trees



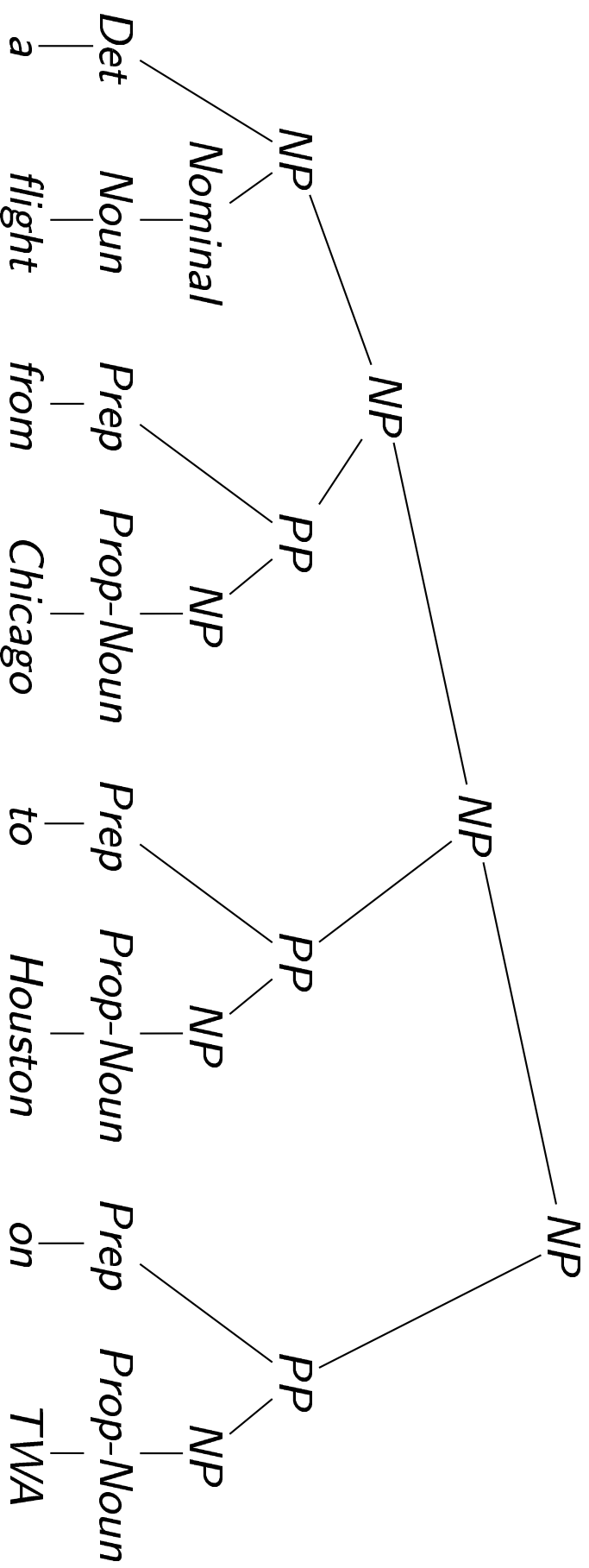
Top-down parsing: repeated generation of sub-trees



Top-down parsing: repeated generation of sub-trees



Top-down parsing: repeated generation of sub-trees



Top-down parsing: repeated generation of sub-trees

Reduplication:

| Constituent | # |
|--|---|
| <i>a flight</i> | 4 |
| <i>from Chicago</i> | 3 |
| <i>to Houston</i> | 2 |
| <i>on TWA</i> | 1 |
| <i>a flight from Chicago</i> | 3 |
| <i>a flight from Chicago to Houston</i> | 2 |
| <i>a flight from Chicago to Houston on TWA</i> | 1 |

Top-down vs. Bottom-up parsing

When expanding the bottom-up search space, which local trees are created?

Top-down vs. Bottom-up parsing

Bottom-up parsing suffers from the following problems:

- All possible analyses of every substring are generated, even when they can never lead to an S , or can never combine with their neighbors
- ϵ -rules can cause performance degradation
- Reduplication of effort

Earley's parsing algorithm

- Dynamic programming: partial results are stored in a chart
- Combines top-down predictions with bottom-up scanning
- No reduplication of computation
- Left-recursion is correctly handled
- ϵ -rules are handled correctly
- Worst-case complexity: $O(n^3)$

Earley's parsing algorithm

Basic concepts:

Dotted rules: if $A \rightarrow \alpha\beta$ is a grammar rule then $A \rightarrow \alpha \bullet \beta$ is a dotted rule.

Edges: if $A \rightarrow \alpha \bullet \beta$ is a dotted rule and i, j are indices into the input string then $[i, A \rightarrow \alpha \bullet \beta, j]$ is an edge. An edge is **passive** (or **complete**) if $\beta = \epsilon$, **active** otherwise.

Actions: The algorithm performs three operations: *scan*, *predict* and *complete*.

Earley's parsing algorithm

scan: read an input word and add a corresponding complete edge to the chart.

predict: when an active edge is added to the chart, predict all possible edges that can follow it

complete: when a complete edge is added to the chart, combine it with appropriate active edges

Earley's parsing algorithm

rightsisters: given an active edge $A \rightarrow \alpha \bullet B\beta$, return all dotted rules $B \rightarrow \bullet \gamma$

leftsisters: given a complete edge $B \rightarrow \gamma \bullet$, return all dotted edges $A \rightarrow \alpha \bullet B\beta$

combination:

$$[i, A \rightarrow \alpha \bullet B\beta, k] * [k, B \rightarrow \gamma \bullet, j] = [i, A \rightarrow \alpha B \bullet \beta, j]$$

Parsing: Earley deduction

Item form: $[i, A \rightarrow \alpha \bullet \beta, j]$ (state that $S \xRightarrow{*} w_1 \dots w_i A \gamma$,
and also that $\alpha \xRightarrow{*} w_{i+1} \dots w_j$)

Axioms: $[0, S' \rightarrow \bullet S, 0]$

Goals: $[0, S' \rightarrow S \bullet, n]$

Parsing: Earley deduction

Inference rules:

$$[i, A \rightarrow \alpha \bullet w_{j+1} \beta, j]$$

Scan

$$[i, A \rightarrow \alpha w_{j+1} \bullet \beta, j + 1]$$

$$[i, A \rightarrow \alpha \bullet B \beta, j]$$

Predict

$$B \rightarrow \gamma$$

$$[j, B \rightarrow \bullet \gamma, j]$$

$$[i, A \rightarrow \alpha \bullet B \beta, k] \quad [k, B \rightarrow \gamma \bullet, j]$$

Complete

$$[i, A \rightarrow \alpha B \bullet \beta, j]$$

Earley's parsing algorithm

```
Parse ::  
  enteredge([0, s' → • s, 0])  
  for j := 1 to n do  
    for every rule  $A \rightarrow w_j$  do  
      enteredge([j-1, A → w_j •, j])  
  
if  $S' \rightarrow S \bullet \in C[0, n]$  then accept else reject
```

Earley's parsing algorithm

```
enteredge(i, edge, j) ::  
  if edge  $\notin$  C[i, j] then /* occurs check */  
    C[i, j] := C[i, j]  $\cup$  {edge}  
  if edge is active then /* predict */  
    for edge'  $\in$  rightsisters(edge) do  
      enteredge([j, edge', j])  
  if edge is passive then /* complete */  
    for edge'  $\in$  leftsisters(edge) do  
      for k such that edge'  $\in$  C[k, i] do  
        enteredge([k, edge', j])
```