Home Assignment 5

1. A context-free grammar $G = \langle V, \Sigma, P, S \rangle$ is **lexical** iff for every production $A \to \alpha \in P$, either $\alpha \in V^*$ or $\alpha \in \Sigma$. Prove that every CFG is weakly equivalent to a lexical CFG. Suggest an algorithm for transforming a general CFG to a lexical one and prove that the languages of the two grammars coincide.

2. Which of the following feature structures subsumes the other?

$$A = \left[\text{F}: \boxed{3}\left[\text{F}: \boxed{3}\right]\right], \qquad B = \boxed{2}\left[\text{F}: \boxed{2}\right]$$

3. Let:

$$A = \left[\text{F}: a\right]$$
$$B = \left[\text{G}: \left[\text{F}: a\right]\right]$$
$$C = \begin{bmatrix}\text{F}: & a \\ \text{G}: & \left[\text{F}: a\right]\end{bmatrix}$$
$$D = \begin{bmatrix}\text{F}: & a \\ \text{G}: & \boxed{1}\left[\text{F}: a\right]\end{bmatrix}$$
$$E = \begin{bmatrix}\text{F}: & \boxed{1}\left[\text{F}: a\right] \\ \text{G}: & \boxed{1}\end{bmatrix}$$
$$F = \begin{bmatrix}\text{F}: & \boxed{1} \\ \text{G}: & \boxed{1}\end{bmatrix}$$

Which of the following holds?

(a) $A \sqcup B = C$
(b) $A \sqcup C = D$
(c) $A \sqcup F = C$
(d) $A \sqcup F = E$
(e) $B \sqcup F = E$
(f) $C \sqcup D = D$
(g) $C \sqcup D = E$
(h) $D \sqcup D = E$
(i) $E \sqcup F = E$

4. Following is a CFG generating Hebrew noun phrases. Augment it with feature structures to enforce agreement on definiteness between the noun and the adjectives:

kaddur gadol hitgalgel
ha-kaddur ha-gadol hitgalgel
*kaddur ha-gadol hitgalgel
*ha-kaddur gadol hitgalgel

$NP \to NP\ ADJP$
$NP \to N \mid D\ N$
$ADJP \to N \mid ADJ\ N$
$N \to kaddur \mid tappux$
$ADJ \to gadol \mid 'adom$
$D \to ha\text{-}$

5. Extend G_2, the unification grammar for E_0, such that transitive verbs with a sentential object are accounted for, too. The grammar must generate also the following sentences:

   Rachel thinks that the sheep sleep

   Rachel knows that Jacob loves her

   Laban knows that Rachel thinks that Jacob loves her

6. Extend G_2 such that simple possessive phrases accounted for, too. Extend the class of pronouns to include the genitive ones: *my, your, his, her, its, our, their*. The grammar must generate also the following sentences:

   Rachel loves her sheep

   Her sheep love her

7. Show that every CFG is weakly equivalent to a unification grammar; that is, for every CFG G there exists a unification grammar G' such that $L(G) = L(G')$.

8. Show a unification grammar G such that $L(G)$ is the set of all natural numbers, represented as strings of decimal digits. In addition, make sure that the first form of the derivation is associated with a feature structure which is a list representation of the input string. For example, if the number "1 2 3" is derivable, the feature structure associated with the start symbol is the following feature structure:

$$\begin{bmatrix} \text{FIRST}: & 1 & \\ \text{REST}: & \begin{bmatrix} \text{FIRST}: & 2 & \\ \text{REST}: & \begin{bmatrix} \text{FIRST}: & 3 \\ \text{REST}: & elist \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

9. Design a unification grammar for the (formal) language $L = \{a^n b^m c^n d^m \mid \leq n \leq m\}$.

Good Luck!