Home Assignment 5

1. A context-free grammar $G = \langle V, \Sigma, P, S \rangle$ is lexical iff for every production $A \to \alpha \in P$, either $\alpha \in V^*$ or $\alpha \in \Sigma$. Prove that every CFG is weakly equivalent to a lexical CFG. Suggest an algorithm for transforming a general CFG to a lexical one and prove that the languages of the two grammars coincide.

2. Which of the following feature structures subsumes the other?
   \[ A = \begin{bmatrix} F : & 3 \end{bmatrix} \]
   \[ B = \begin{bmatrix} F : & 2 \end{bmatrix} \]

3. Let:
   \begin{align*}
   A &= \begin{bmatrix} F : & a \end{bmatrix} \\
   B &= \begin{bmatrix} G : & [F : & a] \end{bmatrix} \\
   C &= \begin{bmatrix} F : & a \\
   G : & [F : & a] \end{bmatrix} \\
   D &= \begin{bmatrix} F : & a \\
   G : & [F : & a] \end{bmatrix} \\
   E &= \begin{bmatrix} F : & [F : & a] \\
   G : & 1 \end{bmatrix} \\
   F &= \begin{bmatrix} F : & 1 \\
   G : & 1 \end{bmatrix}
   \end{align*}

Which of the following holds?
(a) $A \cup B = C$
(b) $A \cup C = D$
(c) $A \cup F = C$
(d) $A \cup F = E$
(e) $B \cup F = E$
(f) $C \cup D = D$
(g) $C \cup D = E$
(h) $D \cup D = E$
(i) $E \cup F = E$

4. Following is a CFG generating Hebrew noun phrases. Augment it with feature structures to enforce agreement on definiteness between the noun and the adjectives:

   kaddur gadol hitgalgel
   ha-kaddur ha-gadol hitgalgel
   *kaddur ha-gadol hitgalgel
   *ha-kaddur gadol hitgalgel

   \begin{align*}
   NP &\to NP \ ADJP \\
   NP &\to N \mid D \ N \\
   ADJP &\to N \mid ADJ \ N \\
   N &\to kaddur \mid tappux \\
   ADJ &\to gadol \mid 'adom \\
   D &\to ha-
   \end{align*}
5. Extend $G_2$, the unification grammar for $E_0$, such that transitive verbs with a sentential object are accounted for, too. The grammar must generate also the following sentences:

Rachel thinks that the sheep sleep
Rachel knows that Jacob loves her
Laban knows that Rachel thinks that Jacob loves her

6. Extend $G_2$ such that simple possessive phrases accounted for, too. Extend the class of pronouns to include the genitive ones: my, your, his, her, its, our, their. The grammar must generate also the following sentences:

Rachel loves her sheep
Her sheep love her

7. Show that every CFG is weakly equivalent to a unification grammar; that is, for every CFG $G$ there exists a unification grammar $G'$ such that $L(G) = L(G')$.

8. Show a unification grammar $G$ such that $L(G)$ is the set of all natural numbers, represented as strings of decimal digits. In addition, make sure that the first form of the derivation is associated with a feature structure which is a list representation of the input string. For example, if the number “1 2 3” is derivable, the feature structure associated with the start symbol is the following feature structure:

$$
\begin{align*}
\text{FIRST} & : 1 \\
\text{REST} & : \begin{cases}
\text{FIRST} & : 2 \\
\text{REST} & : \begin{cases}
\text{FIRST} & : 3 \\
\text{REST} & : \text{elist}
\end{cases}
\end{cases}
\end{align*}
$$

9. Design a unification grammar for the (formal) language $L = \{a^nb^mc^n d^m \mid n \leq m\}$.

**Good Luck!**