Part I

Administration and Course Overview
Section 1

Administration
Important Details

1. There will be a final exam.
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2. Course website: Can be reached from Ronen’s homepage.
Course Prerequisites

1. Computational Models
2. Probability theory.
Course Material

1. Books:
   1.1 Oded Goldreich. Foundations of Cryptography.
   1.2 Jonathan Katz and Yehuda Lindell. An Introduction to Modern Cryptography.

2. Lecture notes
   2.1 Ran Canetti www.cs.tau.ac.il/~canetti/f08.html
   2.2 Yehuda Lindell
       u.cs.biu.ac.il/~lindell/89-856/main-89-856.html
   2.3 Luca Trevisan www.cs.berkeley.edu/~daw/cs276/
   2.4 Salil Vadhan people.seas.harvard.edu/~salil/cs120/
   2.5 Benny Applebaum and Iftach Haitner http://moodle.tau.ac.il/2016/course/view.php?id=368416201
Section 2

Course Topics
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Basic primitives in cryptography (i.e., one-way functions, pseudorandom generators and zero-knowledge proofs).

- Focus on *formal* definitions and *rigorous* proofs.
- The goal is not studying some list, but to understand cryptography.
- Start with “what is security?”
- Only then do we ask how to achieve it.
- Start from the bottom and work our way up.
Part II

Foundation of Cryptography
Some stories and motivation

- Encryption (symmetric and public-key).
- Coin tossing over the phone (impossible information theoretically, but possible against poly-time adversaries).
Section 3

Cryptography and Computational Hardness
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1. What is Cryptography?
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2. Hardness assumptions, why do we need them?
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Cryptography and Computational Hardness

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3. Does \( P \neq NP \) suffice?

\( \mathcal{NP} \): all (languages) \( L \subseteq \{0, 1\}^* \) for which there exists a polynomial-time algorithm \( V \) and (a polynomial) \( p \in \text{poly} \) such that the following hold:

3.1 \( V(x, w) = 0 \) for any \( x \not\in L \) and \( w \in \{0, 1\}^* \)

3.2 For any \( x \in L \), \( \exists w \in \{0, 1\}^* \) with \( |w| \leq p(|x|) \) and \( V(x, w) = 1 \)

\( P \neq NP \): i.e., \( \exists L \in \mathcal{NP} \), such that for any polynomial-time algorithm \( A \), \( \exists x \in \{0, 1\}^* \) with \( A(x) \neq 1 \) \( L(x) \)

4. Problems: hard on the average. No known solution

5. One-way functions: an efficiently computable function that no efficient algorithm can invert.
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**polynomial-time algorithms:** an algorithm $A$ runs in polynomial-time, if $\exists p \in \text{poly}$ such that the running time of $A(x)$ is bounded by $p(|x|)$ for any $x \in \{0, 1\}^*$

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Part III

Notation
Notation I

- For $t \in \mathbb{N}$, let $[t] := \{1, \ldots, t\}$.
- Given a string $x \in \{0, 1\}^*$ and $0 \leq i < j \leq |x|$, let $x_{i,\ldots,j}$ stands for the substring induced by taking the $i, \ldots, j$ bit of $x$ (i.e., $x[i] \ldots x[j]$).
- Given a function $f$ defined over a set $U$, and a set $S \subseteq U$, let $f(S) := \{f(x) : x \in S\}$, and for $y \in f(U)$ let $f^{-1}(y) := \{x \in U : f(x) = y\}$.
- $\text{poly}$ stands for the set of all polynomials.
- The worst-case running-time of a polynomial-time algorithm on input $x$, is bounded by $p(|x|)$ for some $p \in \text{poly}$.
- A function is polynomial-time computable, if there exists a polynomial-time algorithm to compute it.
- $\text{PPT}$ stands for probabilistic polynomial-time algorithms.
- A function $\mu : \mathbb{N} \mapsto [0, 1]$ is negligible, denoted $\mu(n) = \text{neg}(n)$, if for any $p \in \text{poly}$ there exists $n' \in \mathbb{N}$ with $\mu(n) \leq 1/p(n)$ for any $n > n'$.
The support of a distribution $P$ over a finite set $\mathcal{U}$, denoted $\text{Supp}(P)$, is defined as $\{u \in \mathcal{U} : P(u) > 0\}$.

Given a distribution $P$ and an event $E$ with $\Pr_P[E] > 0$, we let $(P \mid E)$ denote the conditional distribution $P$ given $E$ (i.e., $(P \mid E)(x) = \frac{P(x) \wedge E}{\Pr_P[E]}$).

For $t \in \mathbb{N}$, let $U_t$ denote a random variable uniformly distributed over $\{0, 1\}^t$.

Given a random variable $X$, we let $x \leftarrow X$ denote that $x$ is distributed according to $X$ (e.g., $\Pr_{x \leftarrow X}[x = 7]$).

Given a final set $S$, we let $x \leftarrow S$ denote that $x$ is uniformly distributed in $S$.

We use the convention that when a random variable appears twice in the same expression, it refers to a single instance of this random variable. For instance, $\Pr[X = X] = 1$ (regardless of the definition of $X$).
Given distribution $P$ over $\mathcal{U}$ and $t \in \mathbb{N}$, we let $P^t$ over $\mathcal{U}^t$ be defined by $D^t(x_1, \ldots, x_t) = \prod_{i \in [t]} D(x_i)$.

Similarly, given a random variable $X$, we let $X^t$ denote the random variable induced by $t$ independent samples from $X$. 

Distribution and random variables II