Combined Attacks for Cryptanalysis of Block Ciphers

Orr Dunkelman

Joint work with Eli Biham and Nathan Keller

August 25, 2005

Topics of the Talk

1. Block Ciphers
2. Cryptanalysis of Block Ciphers
3. Differential Cryptanalysis
4. Linear Cryptanalysis
5. Combined Attacks
6. New Combined Attacks
7. Summary

Block Ciphers

- Basic cryptographic primitive
- Symmetric key primitive
- Can be transformed into stream ciphers, hash functions, and message authentication codes
- Many candidates for “good” block cipher:
  - Data Encryption Standard (DES)
  - Advanced Encryption Standard (AES)
  - IDEA
  - Serpent
  - MARS
  - ...

Block Ciphers (cont.)

- Pair of algorithms: encryption and decryption
- Each accepts blocks of plaintexts in a given size, e.g., 64 or 128 bits
- Encryption defines a keyed permutation of plaintexts into ciphertexts

Due to implementation issues, most block ciphers are iterated:

- Encryption is done in rounds
- Each round applies a relatively "weak" transformation to the data
- The key is used to generate set of subkeys

Cryptanalysis of Block Ciphers

Several basic assumptions:

- The encryption algorithm is known
- Several attack models:
  - Ciphertext only attack
  - Known plaintext attack
  - Chosen plaintext/ciphertext attack
  - Adaptive chosen plaintext (and ciphertext) attack

Generic Attacks

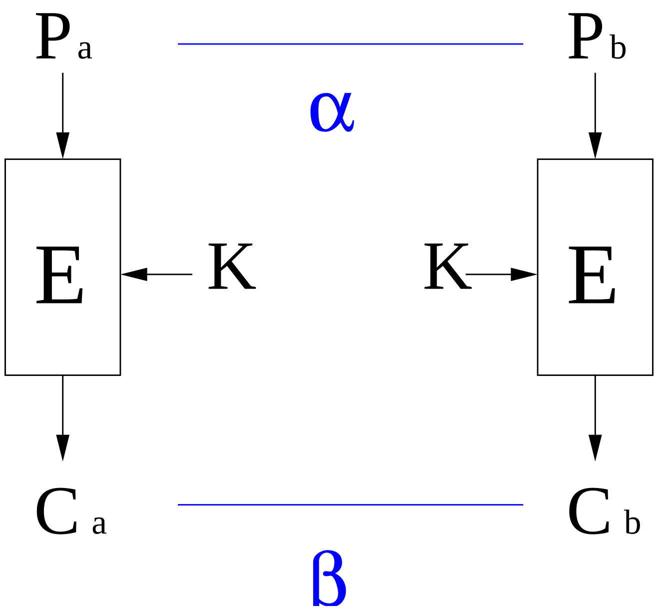
- Exhaustive key search
- Dictionary attack
- Time-memory-data trade-offs

Cryptanalysis Techniques

- Using bad mixture of data with key
- Statistical properties of the cipher
- Structural properties of the cipher
- Algebraic properties of the cipher

Differential Cryptanalysis

- Introduced by Biham and Shamir in 1990
- Studies the development of differences through the encryption function
- A differential $\alpha \rightarrow \beta$ with probability p:



- $O(p^{-1})$ pairs needed to use the differential.

Differential Cryptanalysis (cont.)

Cipher	"Best" Differential		"Best" Differential Attack		
	Rounds	Probability	Rounds	Data	Time
DES	5	$\approx 1/10000$	8	$\approx 20,000$	2^9
	13	$2^{-47.2}$	16	$2^{47.2}$	$2^{35.2}$
FEAL	3	1	4	2	2
	6	2^{12}	8	2^{12}	2^{12}
	n	2^{-2n}	$n+2$	2^{2n}	2^{2n}
AES	1	2^{-6}	3	2^{32}	$O(1)$
	2	2^{-30}	3	2^{32}	$O(1)$
	3	2^{-54}			
	4	$< 2^{-108}$			

Linear Cryptanalysis

- Introduced by Matsui in 1993
- Studies linear approximations of the encryption function
- A linear approximation $\lambda_P \to \lambda_C$ with probability $1/2 \pm q$:

$$\lambda_P \cdot P \oplus \lambda_C \cdot C = \lambda_K \cdot K$$

for $1/2 + q$ of the plaintext-ciphertext pairs (P, C)

- The sign of the bias q is less important
- Requires about $O(q^{-2})$ known plaintexts

Linear Cryptanalysis (cont.)

Cipher	"Best" Approximation		"Best" Linear Attack		
	Rounds	Bias	Rounds	Data	Time
DES	6	$\approx 1/262$	8	$\approx 140,000$	2^{18}
	14	$2^{-20.7}$	16	2^{43}	2^{37}
FEAL	3	$1/2$	4	16	2^{10}
	7	2^{-8}	8	2^{12}	2^{18}
	n	2^{-n-1}	$n+1$	2^{n+3}	2^{n+9}
AES	1	2^{-3}			
	2	2^{-15}			
	3	2^{-27}			
	4	$< 2^{-55}$			

Distinguisher, Key Recovery, and the All the Rest

The statistical properties define *distinguishers*. These distinguishers can distinguish whether a black box is a given cipher or a random permutation.
If there is a distinguisher for r rounds of the cipher, key recovery on $r+i$ rounds is usually:

1. Guess the subkey of i rounds
2. Partially encrypt/decrypt the data the i rounds
3. Run the distinguisher
4. If the distinguisher fails — discard subkey guess
5. If the distinguisher succeeds — right subkey found*

* In several cases attacking r rounds using an r-round distinguisher is possible.

Combined Attacks

Modern ciphers try to avoid good long statistical properties:

- Avalanche criteria
- Wide-trail strategy
- Highly nonlinear functions
- Many rounds

However, good short properties still exist.

Combined Attacks (cont.)

Treats the cipher E as a cascade of two sub-ciphers, i.e., $E = E_1 \circ E_0$.
Use one statistical property for the first sub-cipher, a second one for the second sub-cipher, and stitch them somehow.

- Differential-linear (Langford and Hellman, 1994)
- Miss in the middle (Biryukov, Biham, Shamir, 1999)
- Boomerang attack (Wagner, 1999)

Differential-Linear Attack

- Use differential $\Omega_P \to \Omega_T$ for E_0 (with probability 1)
- Use linear approximation $\lambda_T \to \lambda_C$ for E_1
- Ask for the encryption of the pair $(P_1, P_2 = P_1 \oplus \Omega_P)$
- The intermediate encryption values satisfy $T_1 \oplus T_2 = \Omega_T$
- The value difference between $\lambda_T \cdot T_1$ and $\lambda_T \cdot T_2$ is known (equals to $\lambda_T \cdot \Omega_T$)
- The values $\lambda_C \cdot C_1$ and $\lambda_C \cdot C_2$ are equal with probability $1/2 \pm 2q^2$

Differential-Linear Attack (cont.)

Cipher	Statisitical Property	Rounds	Probability
DES	Differential	3	1/16
		4	1/800
		5	1/10000
		6	1/1000000
	Linear	3	1/26
		4	1/6724
		5	1/70000
		6	1/70000
	Differential-Linear	3	1
		4	1/26
		5	1/26
		6	$\approx 1/173$

Miss-in-the-Middle Attack

- Use (truncated) differential $\alpha \to \beta$ for E_0
- Use (truncated) differential $\delta \to \gamma$ for E_1^{-1}
- If β and γ cannot be the same, e.g., first bit of β is always active, while first bit of γ is zero, then
$$\alpha \not\to \delta$$
- Any key suggesting that $\alpha \to \delta$ is obviously wrong

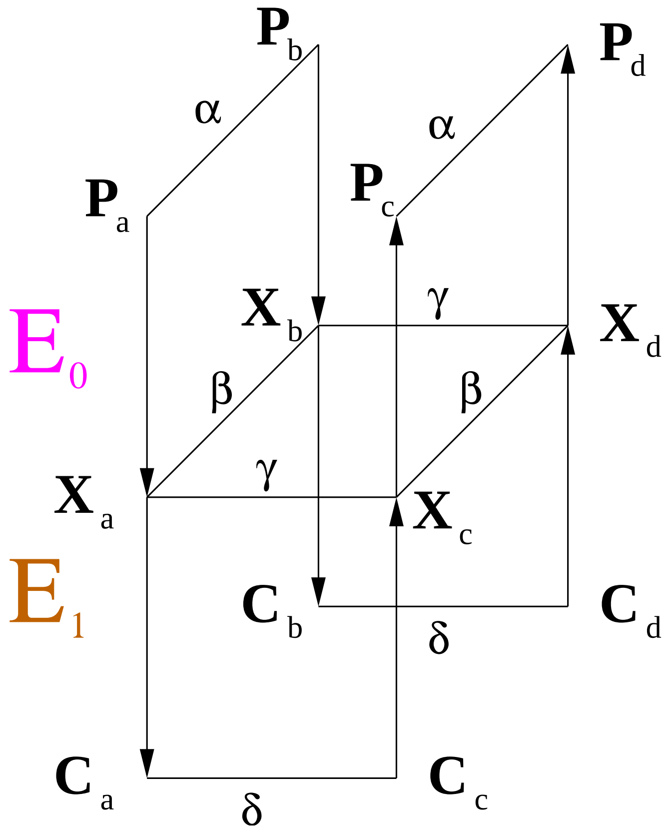
Note: The concept of using impossible events to discard wrong key guesses was used in the attacks on the Enigma.

The Boomerang Attack

The Boomerang Attack

Uses two differentials, $\alpha \longrightarrow \beta$ for E_0, and $\gamma \longrightarrow \delta$ for E_1.

The structure of the boomerang attack:



Probability of obtaining right quartet $p^2 q^2$.

New Combined Attacks

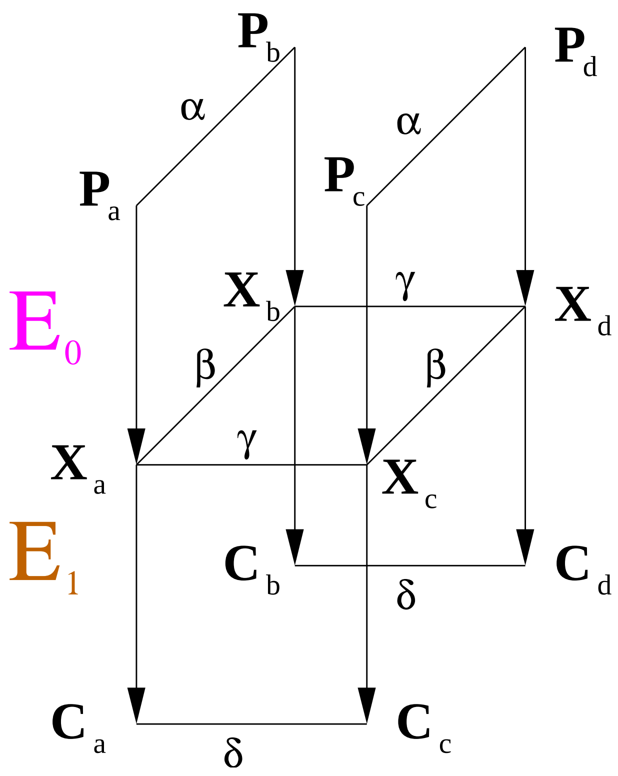
- Amplified Boomerang/Rectangle
- Enhanced Differential-Linear
- Differential-Bilinear
- Higher-Order Differential-Linear
- (Differential-)(Bi)Linear Boomerang

The Amplified Boomerang

Introduced by Kelsey, Kohno, and Schneier in 2000.
Transforms the boomerang attack into a chosen plaintext attack.

- Transformation is based on birthday paradox arguments.
- Right quartet is formed with probability $2^{-(n+1)}pq$.
- New problem — identifying the right quartets.

The Amplified Boomerang (cont.)



Probability of obtaining right quartet $2^{-n-1}p^2q^2$.

Due to the birthday paradox $O(2^{(n+1)/2}/pq)$ pairs suffice for obtaining a right quartet.

The Rectangle Attack

Improves the amplified boomerang attack:

- Better probability for right quartet, using multiple differentials for each sub-cipher (just like in the boomerang attack)
- Better usage of the data (twice the possible quartets for the same data)
- Better algorithms for finding the right quartets

The Rectangle Attack (cont.)

For example, in the rectangle attack on 10-round Serpent:

- Better probability:
  - $p = 2^{-29}$, while $\hat{p} = 2^{-25.4}$.
  - $q = 2^{-47}$, while $\hat{p} = 2^{-35.9}$.

- Better algorithms:
  - The time complexity is reduced from $2^{199.6}$ memory accesses to $2^{173.8}$ memory accesses.

Enhanced Differential-Linear Attack

Allows using differentials with probability less than 1.
Assume that the differential $\Omega_P \to \Omega_T$ has probability p.
The probability that $\lambda_T \cdot T_1 = \lambda_T \cdot T_2$ is

$$\underbrace{p}_{\text{differential holds}} + \underbrace{(1-p) \cdot \frac{1}{2}}_{\text{differential doesn't hold}} = \frac{1}{2} + \frac{p}{2}$$

The probability that $\lambda_C \cdot C_1 = \lambda_C \cdot C_2$ is

$$\frac{1+p}{2} \cdot \left(\frac{1}{2} + 2q^2\right) + \frac{1-p}{2} \cdot \left(\frac{1}{2} - 2q^2\right) = \frac{1}{2} + 2pq^2$$

Can also be used when $\Omega_T \cdot \lambda_T$ is unknown (but fixed).

Differential-Bilinear Attack

Bilinear cryptanalysis is a generalization of linear attack to Feistel block ciphers, presented by Courtois in 2004.
Let the plaintext be $P = (L_0, R_0)$ and the ciphertext $C = (L_n, R_n)$, the bilinear approximation is of the form:

$$L_0[\alpha_0] \cdot R_0[\beta_0] \oplus R_0[\gamma_0] \oplus L_0[\delta_0] \oplus L_n[\alpha_n] \cdot R_n[\beta_n] \oplus R_n[\gamma_n] \oplus L_n[\delta_n] =$$

$$L_0[\epsilon_0] \cdot K[\epsilon_1] \oplus R_0[\zeta_0] \cdot K[\zeta_1] \oplus L_n[\eta_0] \cdot K[\eta_1] \oplus R_n[\theta_0] \cdot K[\theta_1] \oplus K[\iota_1]$$

Best 3-round linear approximation of DES has bias of $1.56 \cdot 2^{-3}$.
Best 3-round bilinear approximation of DES has bias of $1.66 \cdot 2^{-3}$.
For s^5 DES the advantage is much greater (linear bias of 1/8; bilinear bias of 1/4).

Differential-Bilinear Attack (cont.)

The attacker chooses pairs of plaintexts P_1 and $P_2 = P_1 \oplus \Omega_P$.
With probability p the intermediate encryption values T_1 and T_2, have a difference that satisfies the equality

$$T_{1L}[\alpha_0] \cdot T_{1R}[\beta_0] \oplus T_{1L}[\gamma_0] \oplus T_{1R}[\delta_0] = T_{2L}[\alpha_0] \cdot T_{2R}[\beta_0] \oplus T_{1L}[\gamma_0] \oplus T_{2R}[\delta_0],$$

Similarly to the differential-linear case, the pair of ciphertexts C_1 and C_2 satisfies the following equation

$$C_{1L}[\alpha_n] \cdot C_{1L}[\beta_n] \oplus C_{1L}[\gamma_n] \oplus C_{1R}[\delta_n] = C_{2L}[\alpha_n] \cdot C_{2R}[\beta_n] \oplus C_{2L}[\gamma_n] \oplus C_{2R}[\delta_n]$$

with probability $1/2 + 2pq^2$.

Differential-Bilinear Attack (cont.)

There are several differences between differential-linear and differential-bilinear attacks:

- Not any differential can be used in the differential-bilinear attack
- Differential must give the attacker the difference of $L_T[\alpha_0] \cdot R_T[\beta_0]$
- Subkey terms exist in the bilinear approximation

Higher-Order Differential-Linear Attack

Higher-order differentials are differentials of sets of plaintexts.
The common higher-order differential predicts that the XOR value of all ciphertexts is some constant (usually zero).
Our proposed attack can use also SQUARE-like properties (SQUARE, saturation, and Integral attacks).
All we require is that the XOR of some bit(s) over all the intermediate encryption values is fixed.

Higher-Order Differential-Linear Attack (cont.)

Let *Set* be a set of plaintexts $\{P_1, P_2, \ldots, P_m\}$ such that the higher-order differential predicts (with some probability p) the value

$$\oplus_{i=1}^{m} T_i.$$

Let the event I be

$$I = \{\lambda_P \cdot (T_1 \oplus \ldots \oplus T_m) = \lambda_C \cdot (C_1 \oplus \ldots \oplus C_m)\}.$$

Then (under standard independence assumptions) $\Pr[I] = 1/2 + 2^{m-1}q^m$.
This leads to bias of $2^{m-1}pq^m$ in the value of

$$\oplus_{i=1}^{m} \lambda_C \cdot C_i$$

Higher-Order Differential-Linear Attack (cont.)

We can use the higher-order differential-linear attack for:

- Transform any linear approximation with bias 1/2 for r Feistel rounds into an higher-order differential-linear approximation with bias 1/2 for $r+3$ rounds
- When m small, or q very high (very common for weak key classes)
- Increase the linear weak key class of idea from 2^{23} keys into 2^{32} keys.

We note that the higher-order differential-linear attack was developed independently by Shin et al. under the name square-nonlinear attack.

(Differential-)(Bi)Linear Boomerang Attack

Our main observation is that the boomerang process leads to a pair of intermediate encryption values with difference β.
This β difference may be used in (differential-)(bi)linear attack (on the first sub-cipher).

- The cipher is actually decomposed into three sub-ciphers
- If the probability of the first differential p_0, of the second differential p_1, and the (differential-)(bi)linear approximation of E_0 has bias $(2p)q$, the bias for the entire structure is:

$$2p_0\hat{p}_1{}^2(p)q$$

Major Results — The Rectangle Attack

Results of the rectangle attack:

Cipher	Number of Rounds	Complexity: Data	Complexity: Time	Number of Keys
Serpent	10	$2^{126.3}$ CP	$2^{173.8}$	1
AES-192	9	2^{86} RK-CP	2^{125}	256
AES-256	10	$2^{114.9}$ RK-CP	$2^{171.8}$	256
COCONUT98	full†	4 RK-ACPC	1	2
IDEA	6.5	$2^{59.8}$ RK-CP	$2^{88.1}$	4
KASUMI	full	$2^{54.6}$ RK-CP	$2^{91.7}$	4

† – Distinguishing attack, RK – Related-key, CP – Chosen plaintext, ACPC – Adaptive chosen plaintext and ciphertext

Major Results — Enhanced Differential-Linear

Cipher	Number of Rounds	Complexity: Data	Complexity: Time
DES	8	$2^{14.8}$ CP	$2^{14.8}$
	9	$2^{15.8}$ CP	$2^{29.2}$
COCONUT98	full	$2^{27.7}$ CP	$2^{33.7}$
Serpent	11	2^{118} KP	$2^{205.7}$

The Case for KASUMI

- KASUMI is the 3GPP block cipher
- 8-round Feistel; 64-bit block; 128-bit key
- Used to ensure privacy and authenticity of the conversations
- The mode of use are proved to be secure, if the cipher is secure against related-key attacks

Our attack on KASUMI is a related-key rectangle attack, which is a related-key attack...

The Case for KASUMI (cont.)

- Rounds 1–4: differentials with effective probability 2^{-17}
- Rounds 5–7: differentials with effective probability 2^{-2}
- Key recovery on round 8
- Many tricks due to the 128-bit subkey in each round

Questions?

Thank You!