Morphological Image Compression using Skeletons

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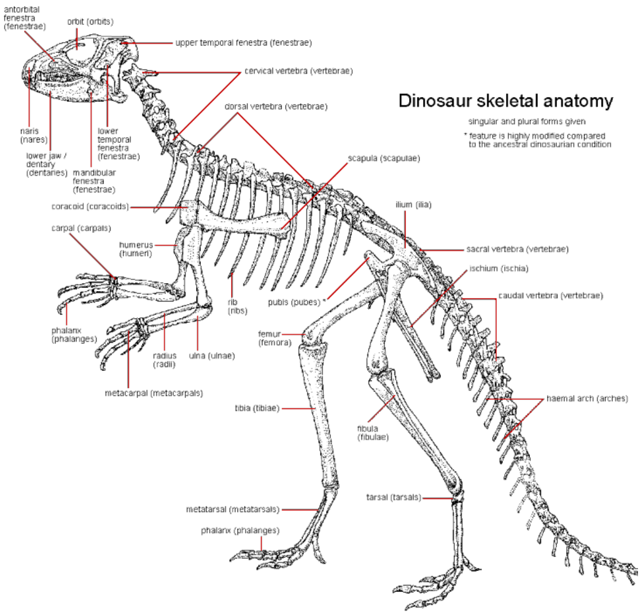


What is Mathematical Morphology ?

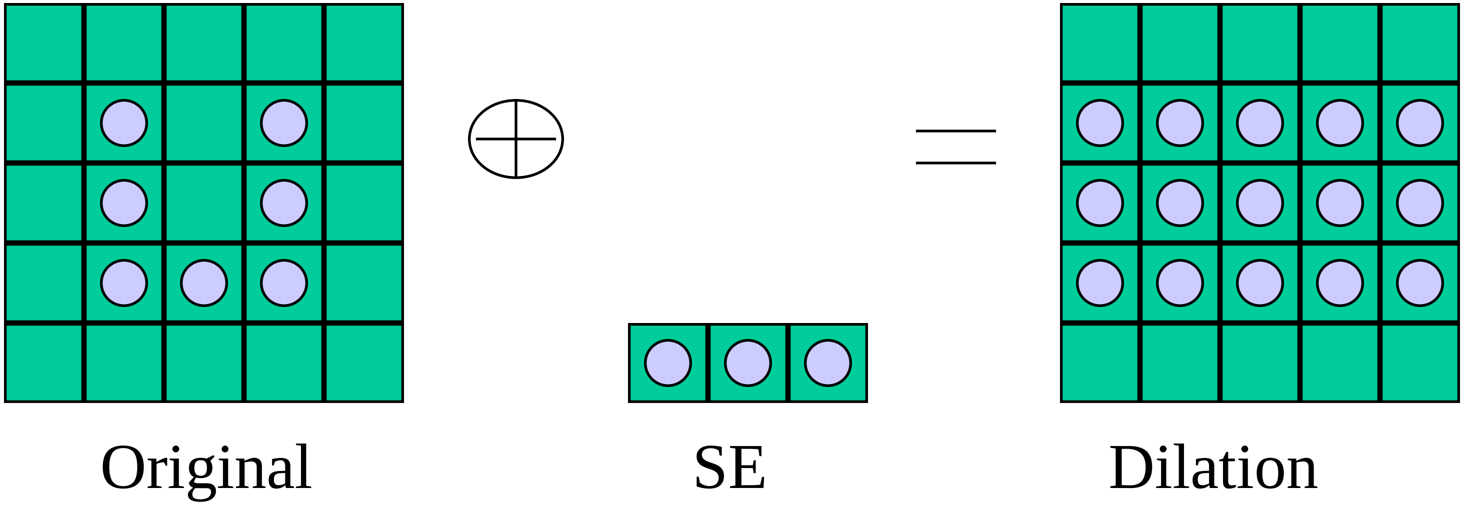
“... a theory for analysis of spatial structures which was initiated by George Matheron and Jean Serra. It is called Morphology since it aims at the analyzing the shape and the forms of the objects. It is Mathematical in the sense that the analysis is based on set theory, topology, lattice, random functions, etc. “

(Serra and Soille, 1994).

Basic Operation: Dilation

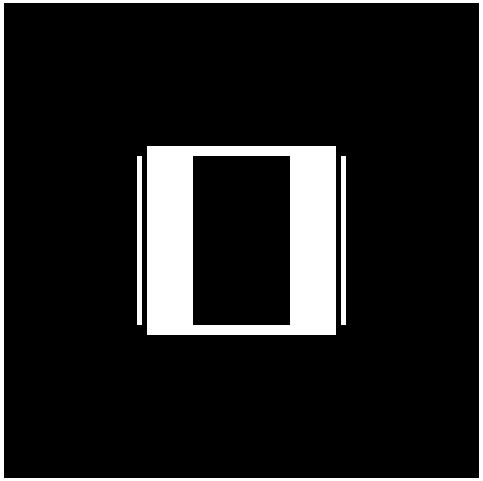
- Dilation: replacing every pixel with the maximum value of its neighborhood determined by the structure element (SE)

$$X \oplus B = \{x + b \mid x \in X, b \in B\}$$

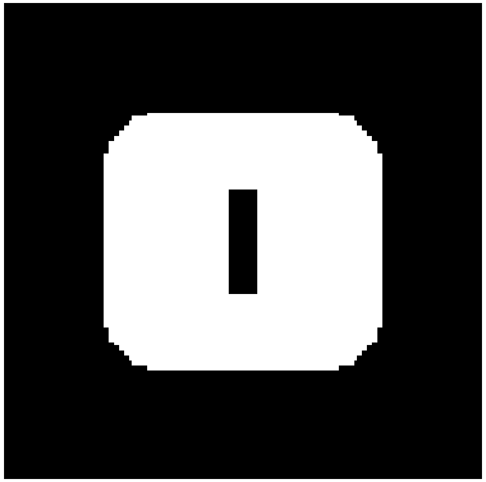


Dilation demonstration

Original Figure

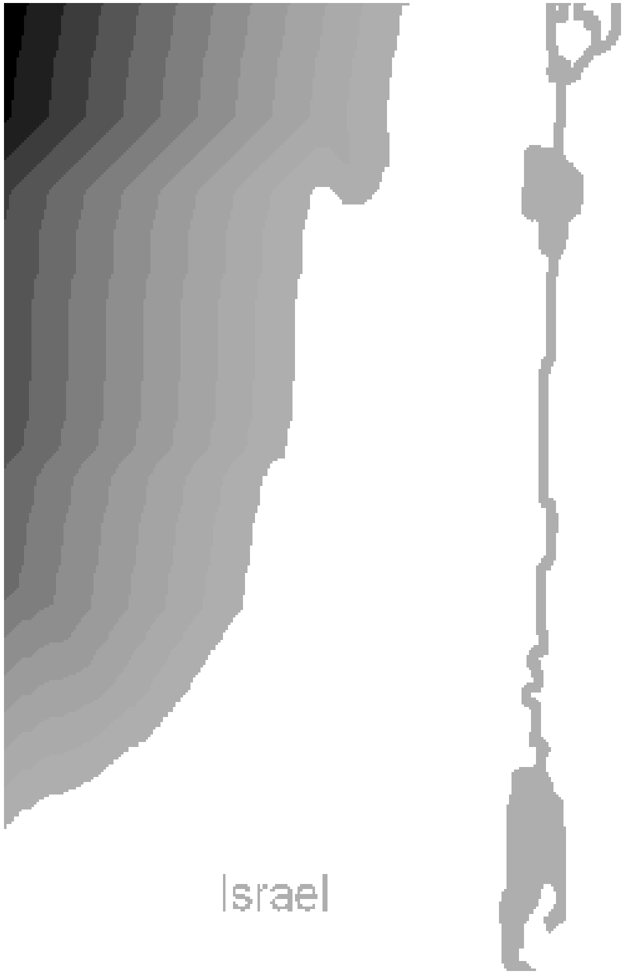


Dilated Image With Circle SE



Dilation example

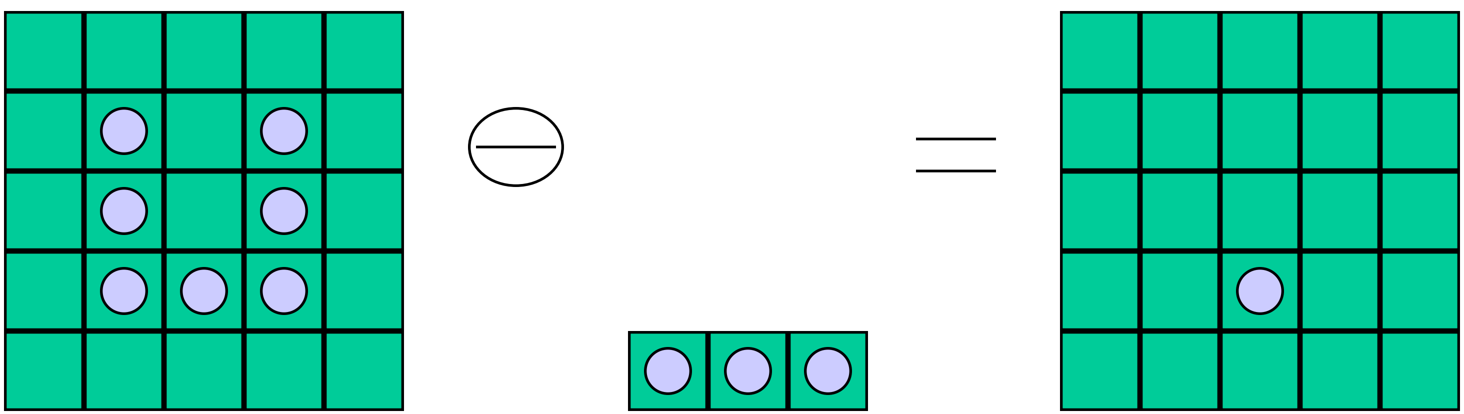
Dilation (Israel Map, circle SE):



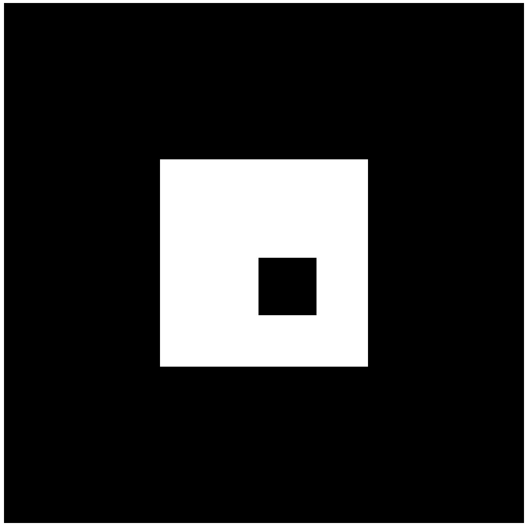
Basic Operation: Erosion

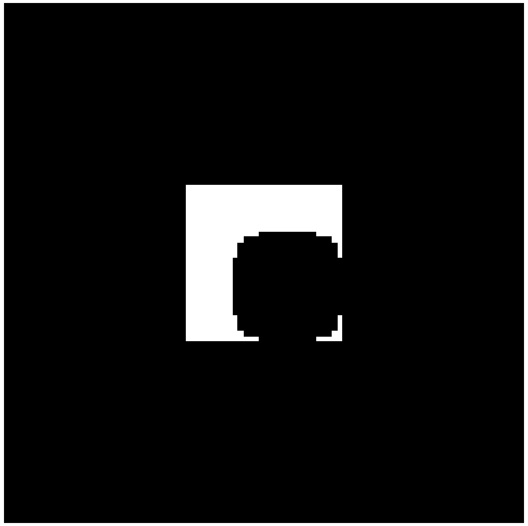
- Erosion: A Dual operation - replacing every pixel with the minimum value of its neighborhood determined by the SE

$$X \quad B = \left(X^{C} + B\right)^{C}$$



Erosion demonstration





Erosion example

Erosion (Lena, circle SE):

Original B&W Lena

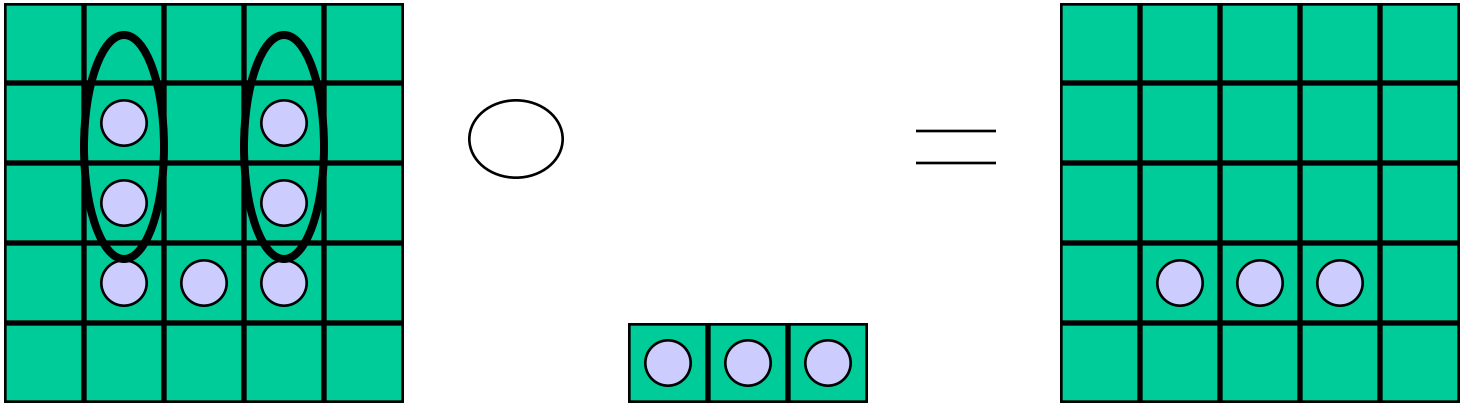
Eroded Lena, SE radius = 4

Composed Operations: Opening

- Opening: Erosion and than Dilation

removes positive peaks narrower than the SE

$$X \circ B = (X \quad B) \oplus B$$



Original

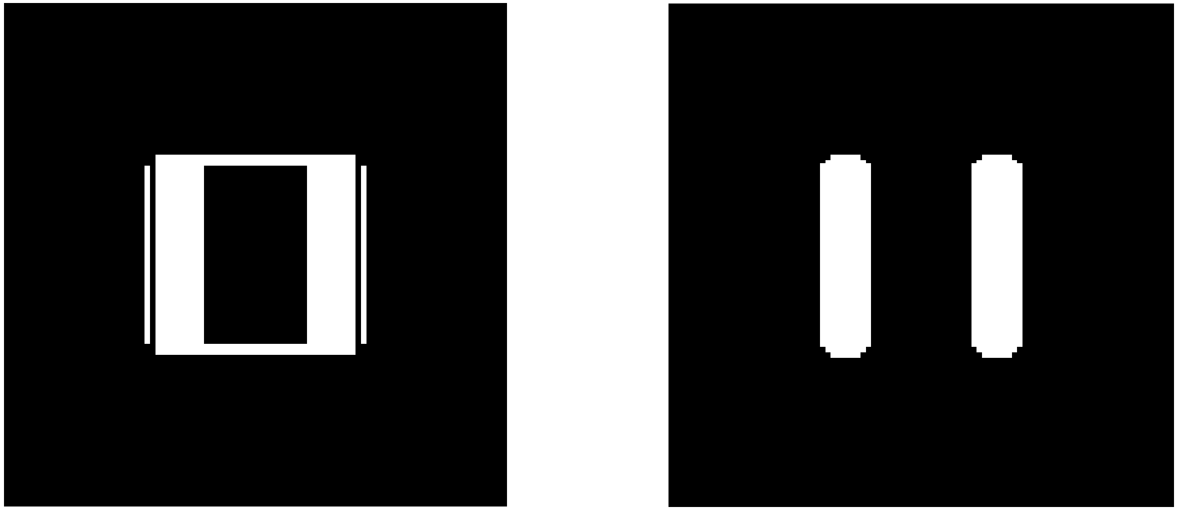
SE

Opening

Opening demonstration

Original Figure

Opened Image With Circle SE



Opening example

Opening (Lena, circle SE):

Original B&W Lena

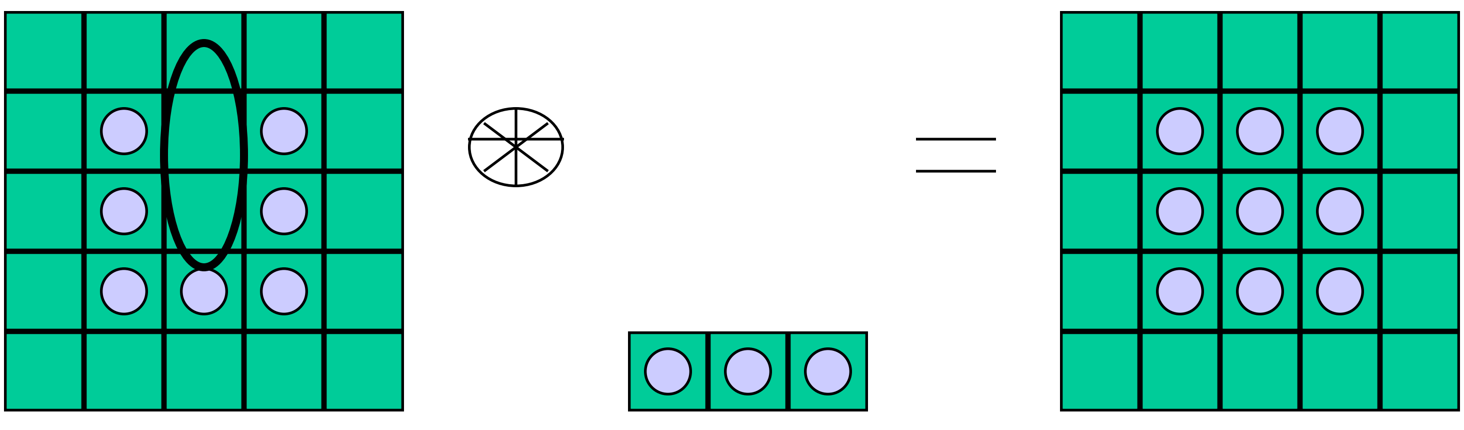
Opened Lena, SE radius = 4

Composed Operations: Closing

- Closing: Dilation and than Erosion

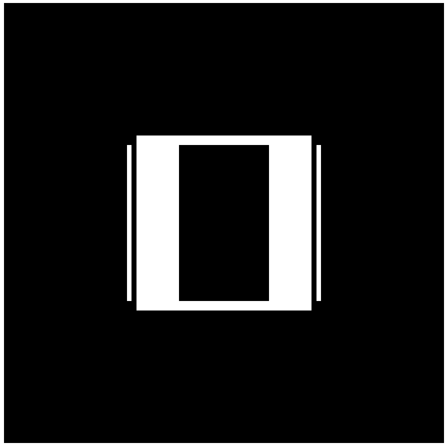
removes negative peaks <u>narrower</u> than the SE

$$X \bullet B = (X \oplus B) \quad B$$

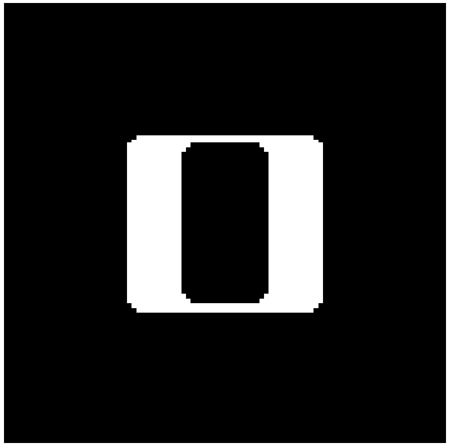


Closing demonstration

Original Figure



Closed Image With Circle SE



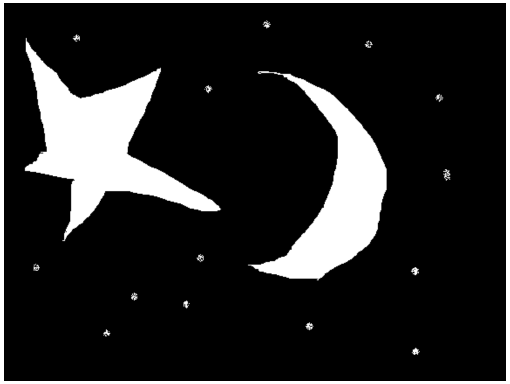
Closing example

- Closing (Lena, circle SE):

Original B&W Lena

Closed Lena, SE radius = 4

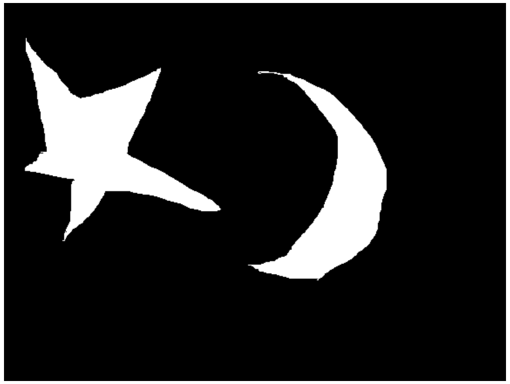
Cleaning “white” noise



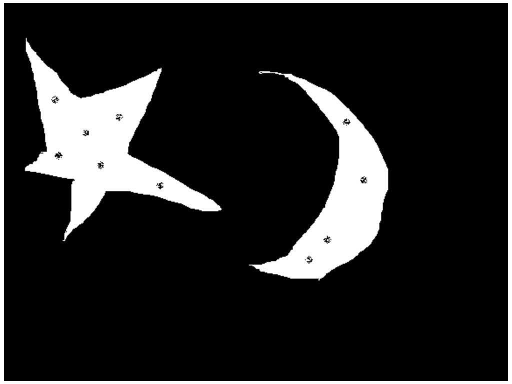
Opening

&

Closing



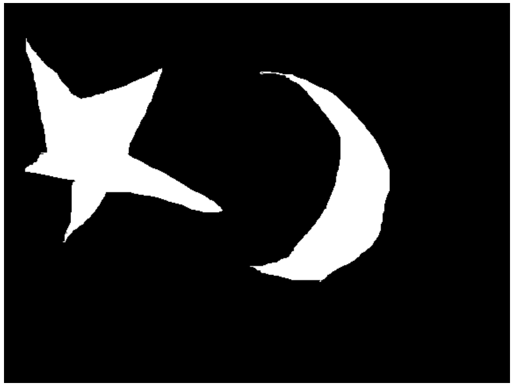
Cleaning “black” noise



Closing

&

Opening

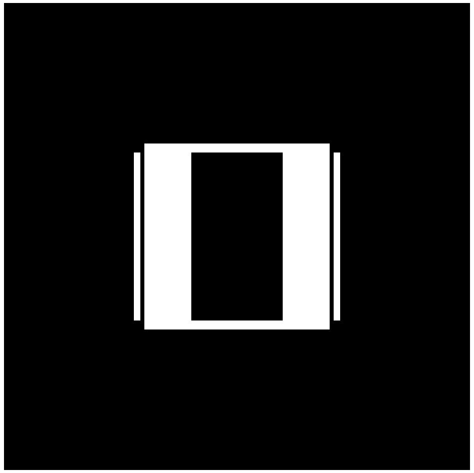


Conditioned dilation

$$\delta^Y(X) = (X \oplus B) \cap Y$$

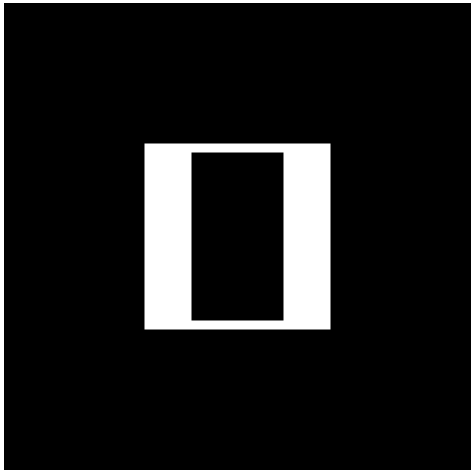
- Example:

Original Figure



Opened Figure By Circle SE

Conditioned Dilated Figure



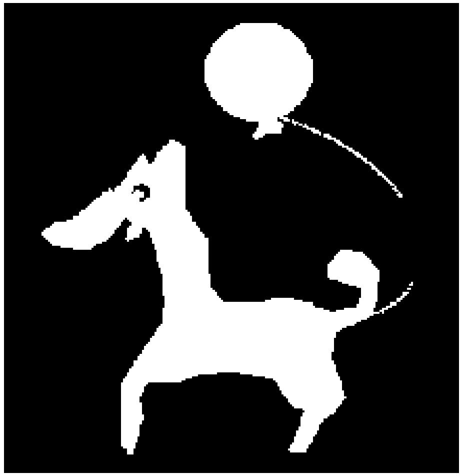
Opening By Reconstruction

$$Rec(mask, kernel) = \lim_{n \to \infty}(\delta^{mask}(kernel))^n$$

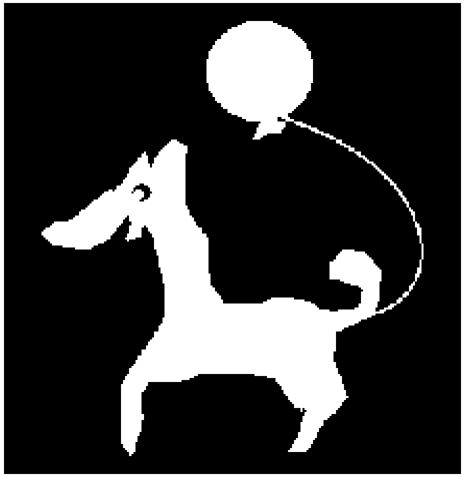
- Open by rec. (dogi):

Original Dogi

Opened Dogi



Opened By Reconstruction Dogi



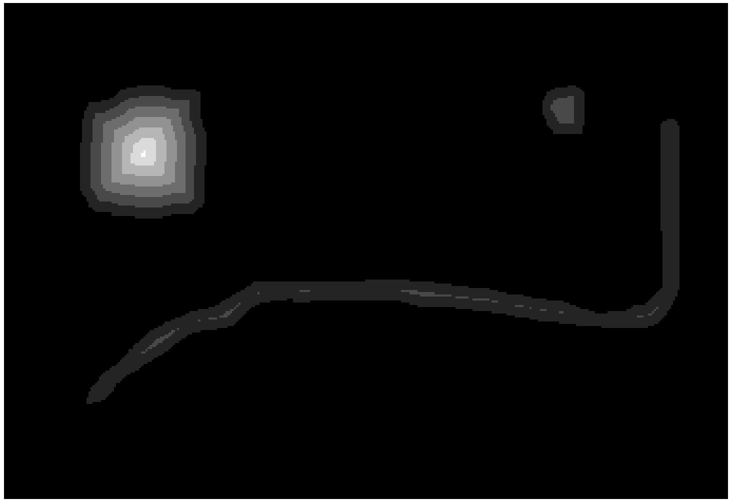
Granulometry and Ultimate Erosions

$$U_n(X) = X \ominus nB - Rec\left\{X \ominus nB, X \ominus (n+1)B\right\}$$

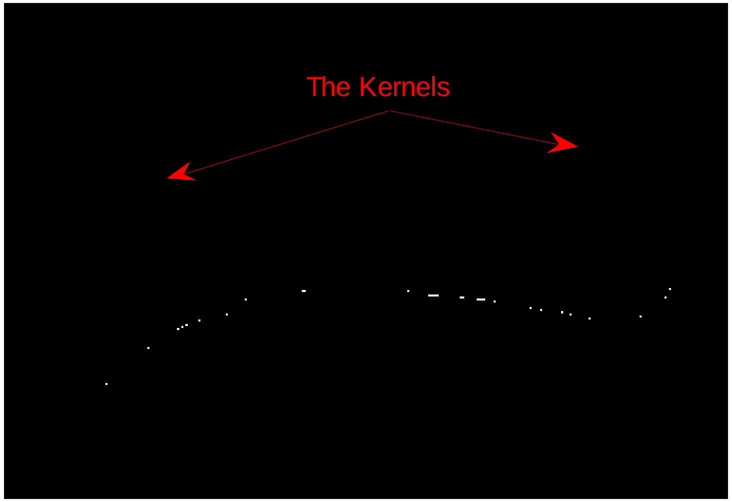
- Example:

The Original Example

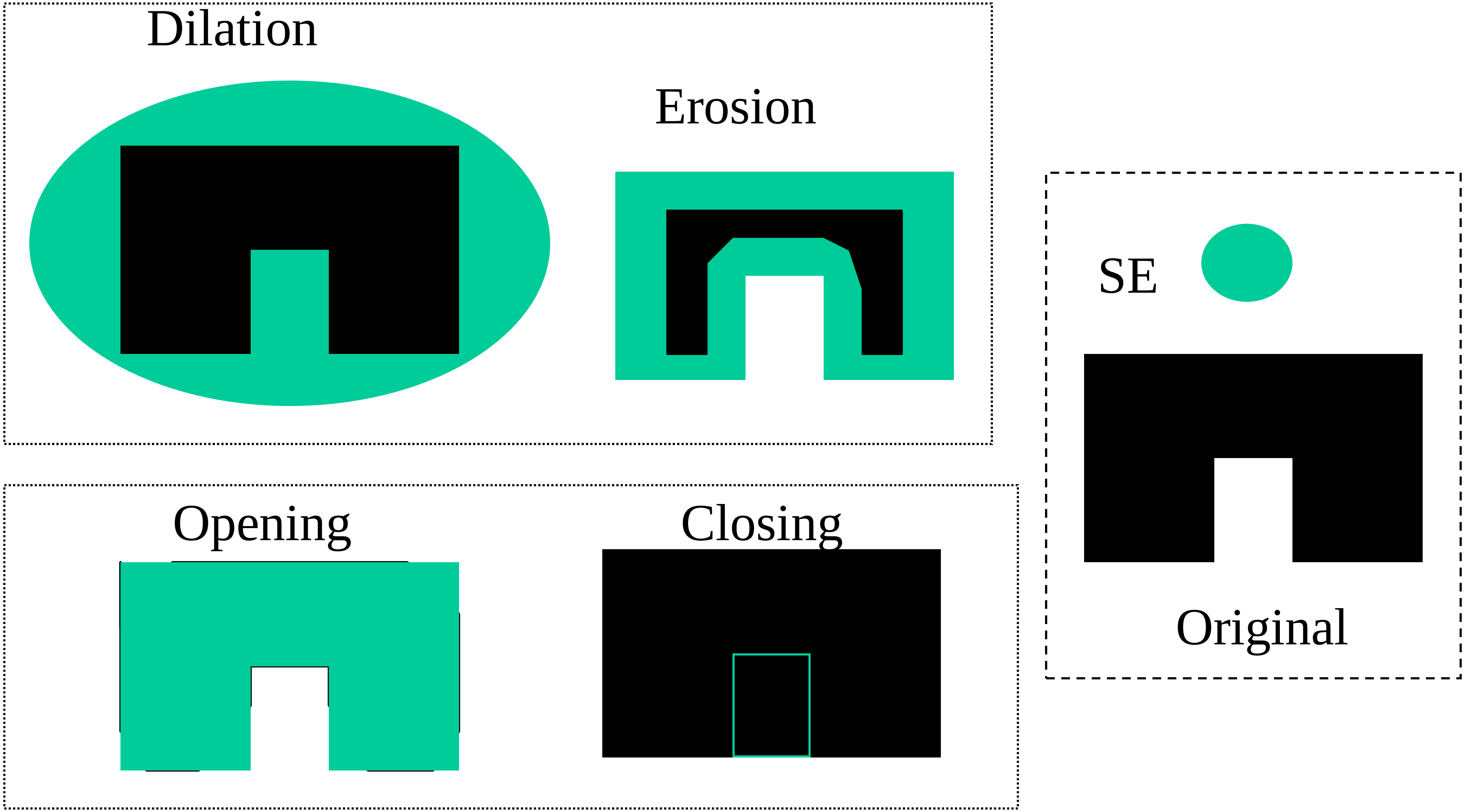
The Eroded Example



The Kernels By Ultimate Erosions



Geometric Interpretation

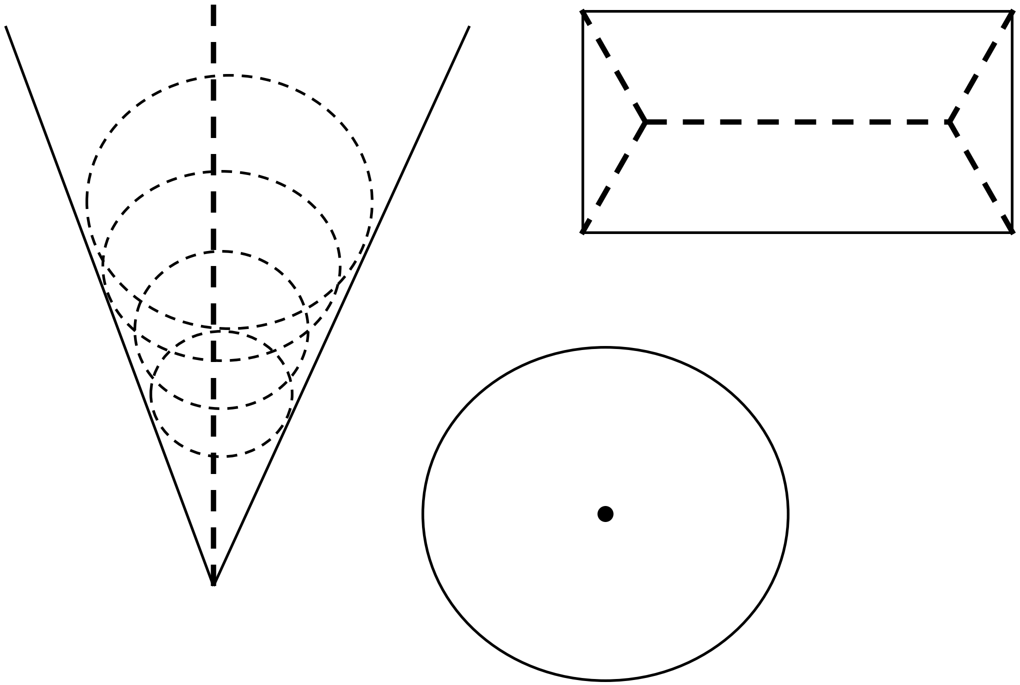


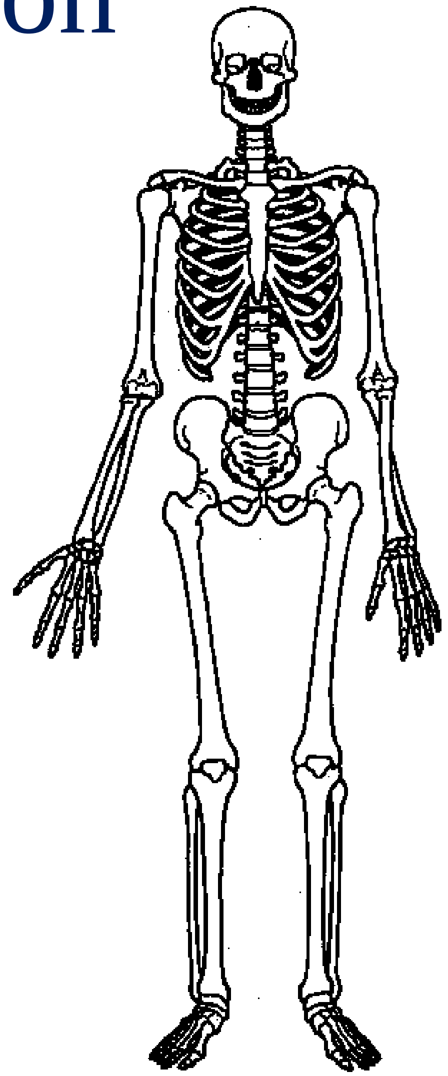
The Use for Compression

- The morphological skeleton is a redundant representation of binary images.
- Skeleton point can be eliminated and error free reconstruction can still be obtained.

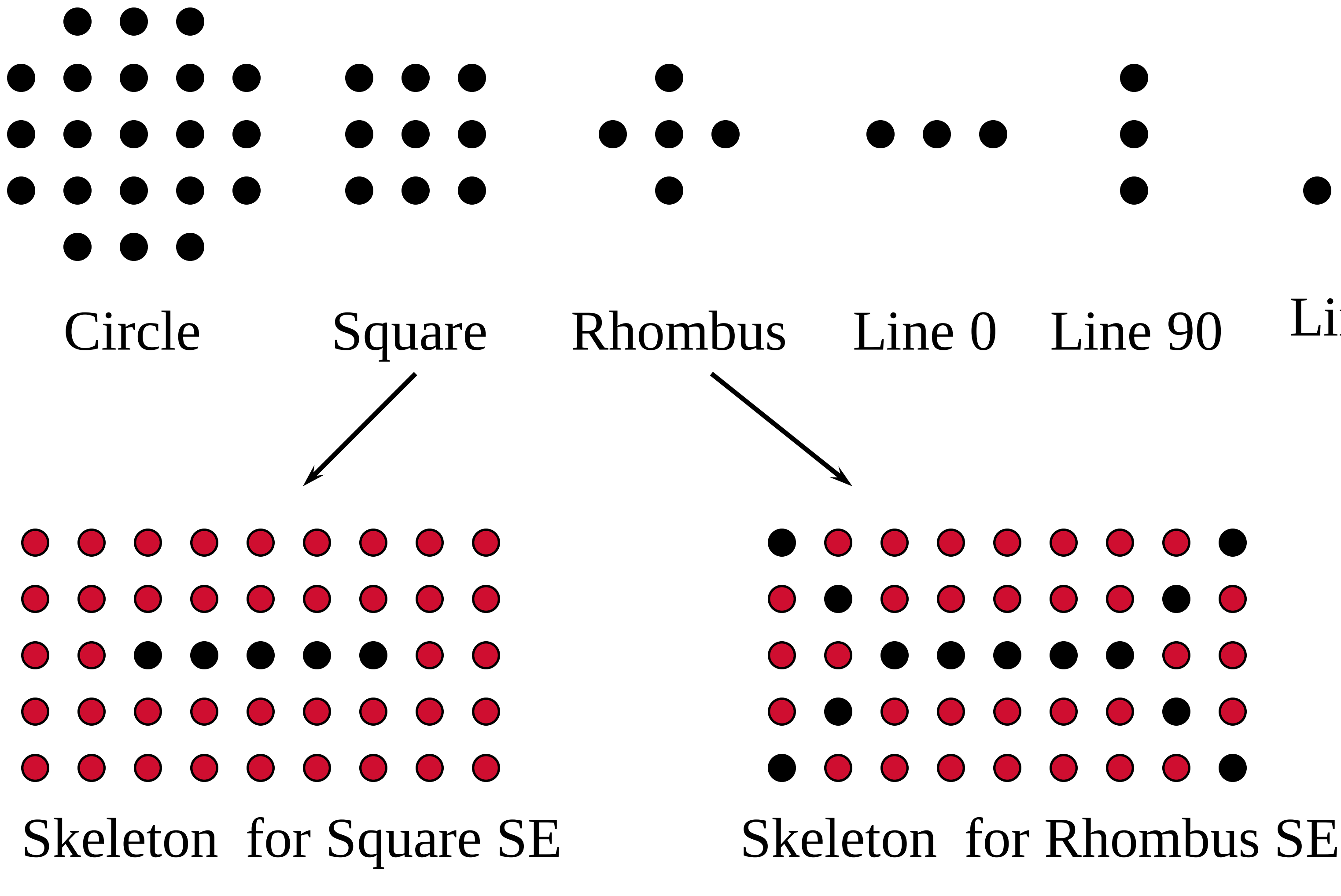
Morphological Skeleton

- A *Medial Axis* representation of an object:

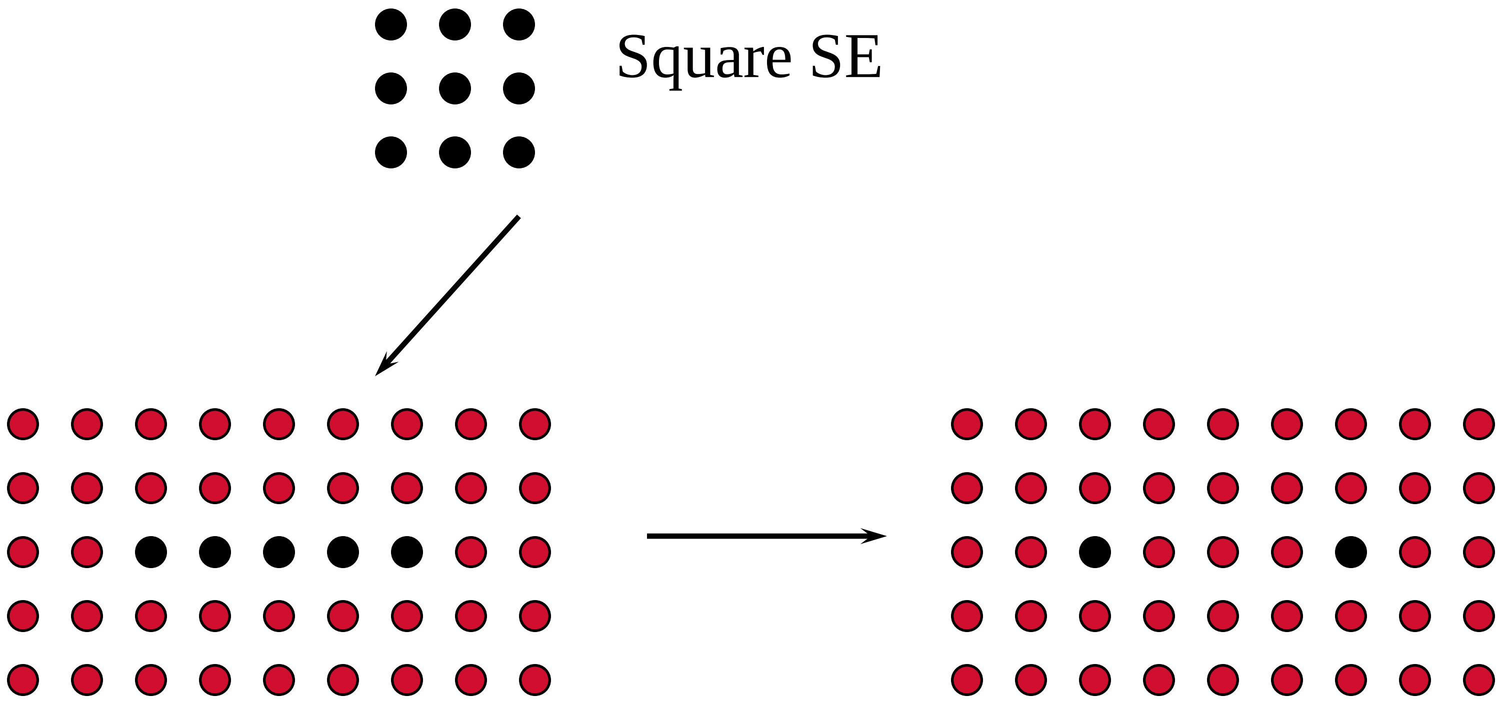




Structuring Elements and Skeletons



Minimal Skeleton

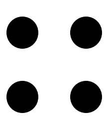


Skeleton for Square SE

Minimal Skeleton for Square SE

Choosing SE

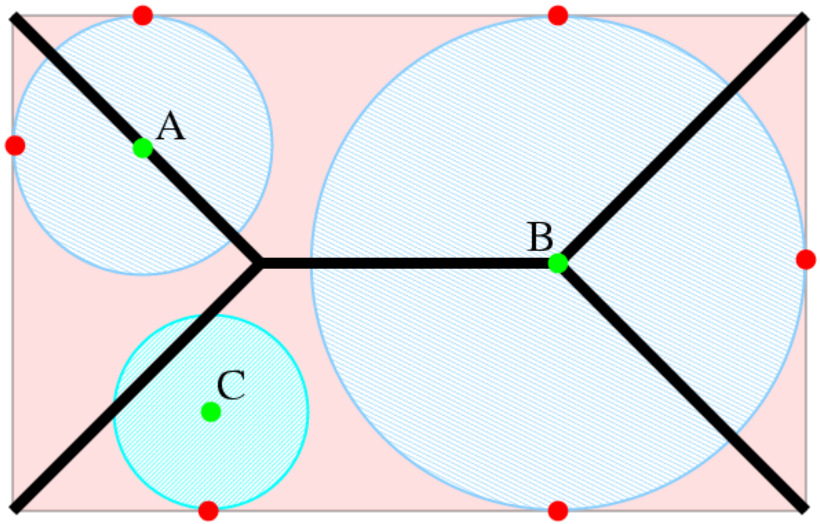
- The main problem is to choose the best SE for the object we want to compress
- In many cases, a SE named Boxne found to give best results, with no mathematical explanation



Boxne

Modified Skeleton

- The size of the SE and NOT its shape are to be changed
- Increasing the SE size in successive steps we reduce the amount of information



References

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- Keshet (Kresch), Renato , Ph.D. Thesis, Morphological Image Representation for Coding Applications, 1995.
- Elyashiv Kessner and Azriel Sinai , Scattering Measure Of Spraying In Plants, Final project, SIPL 2007.