Morphological Image Compression using Skeletons

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What is Mathematical Morphology?

“... a theory for analysis of spatial structures which was initiated by George Matheron and Jean Serra. It is called Morphology since it aims at the analyzing the shape and the forms of the objects. It is Mathematical in the sense that the analysis is based on set theory, topology, lattice, random functions, etc. “

(Serra and Soille, 1994).
Basic Operation: Dilation

- **Dilation**: replacing every pixel with the maximum value of its neighborhood determined by the structure element (SE)

\[ X \oplus B = \{ x+b \mid x \in X, b \in B \} \]

![Diagram showing basic operation of dilation](image)

Original \[\text{ }\] \[\oplus\] \[\text{ }\] SE \[\text{ }\] Dilation
Dilation demonstration

Original Figure

Dilated Image With Circle SE
Dilation example

Dilation (Israel Map, circle SE):
Basic Operation: Erosion

- Erosion: A Dual operation - replacing every pixel with the minimum value of its neighborhood determined by the SE

\[ X \ominus B = (X^c + B)^c \]
Erosion demonstration

Original Figure

Eroded Image With Circle SE
Erosion example

Erosion (Lena, circle SE):
Composed Operations: Opening

- **Opening**: Erosion and then Dilation removes positive peaks narrower than the SE

\[ X \circ B = (X \ B) \oplus B \]
Opening demonstration

Original Figure

Opened Image With Circle SE
Opening example

Opening (Lena, circle SE):

Original B&W Lena

Opened Lena, SE radius = 4
Composed Operations: Closing

- **Closing**: Dilation and then Erosion removes negative peaks narrower than the SE

\[ X \bullet B = (X \oplus B) \quad B \]

![Original SE Closing Diagram](image-url)
Closing demonstration
Closing example

• Closing (Lena, circle SE):
Cleaning “white” noise

Opening & Closing
Cleaning “black” noise

Closing & Opening
Conditioned dilation

\[ \delta^Y (X) = (X \oplus B) \cap Y \]

- Example:
Opening By Reconstruction

\[ \text{Rec}(\text{mask, kernel}) = \lim_{n \to \infty} (\delta^m_{\text{mask}} (\text{kernel}))^n \]

- Open by rec. (dogi):

Original Dogi

Opened Dogi

Opened By Reconstruction Dogi
Granulometry and Ultimate Erosions

\[ U_n(X) = X \ominus nB - \text{Rec}\{X \ominus nB, X \ominus (n+1)B\} \]

- Example:
Geometric Interpretation

Dilation

Erosion

Opening

Closing

Original

SE
The Use for Compression

- The morphological skeleton is a redundant representation of binary images.

- Skeleton point can be eliminated and error free reconstruction can still be obtained.
Morphological Skeleton

• A *Medial Axis* representation of an object:
Structuring Elements and Skeletons

Circle
Square
Rhombus
Line 0
Line 90
Line 45

Skeleton for Square SE
Skeleton for Rhombus SE
Minimal Skeleton

Square SE

Skeleton for Square SE

Minimal Skeleton for Square SE
Choosing SE

• The main problem is to choose the best SE for the object we want to compress
• In many cases, a SE named Boxne found to give best results, with no mathematical explanation
Modified Skeleton

- The **size** of the SE and NOT its shape are to be changed
- Increasing the SE size in successive steps we reduce the amount of information
References


• Elyashiv Kessner and Azriel Sinai, Scattering Measure Of Spraying In Plants, Final project, SIPL 2007.