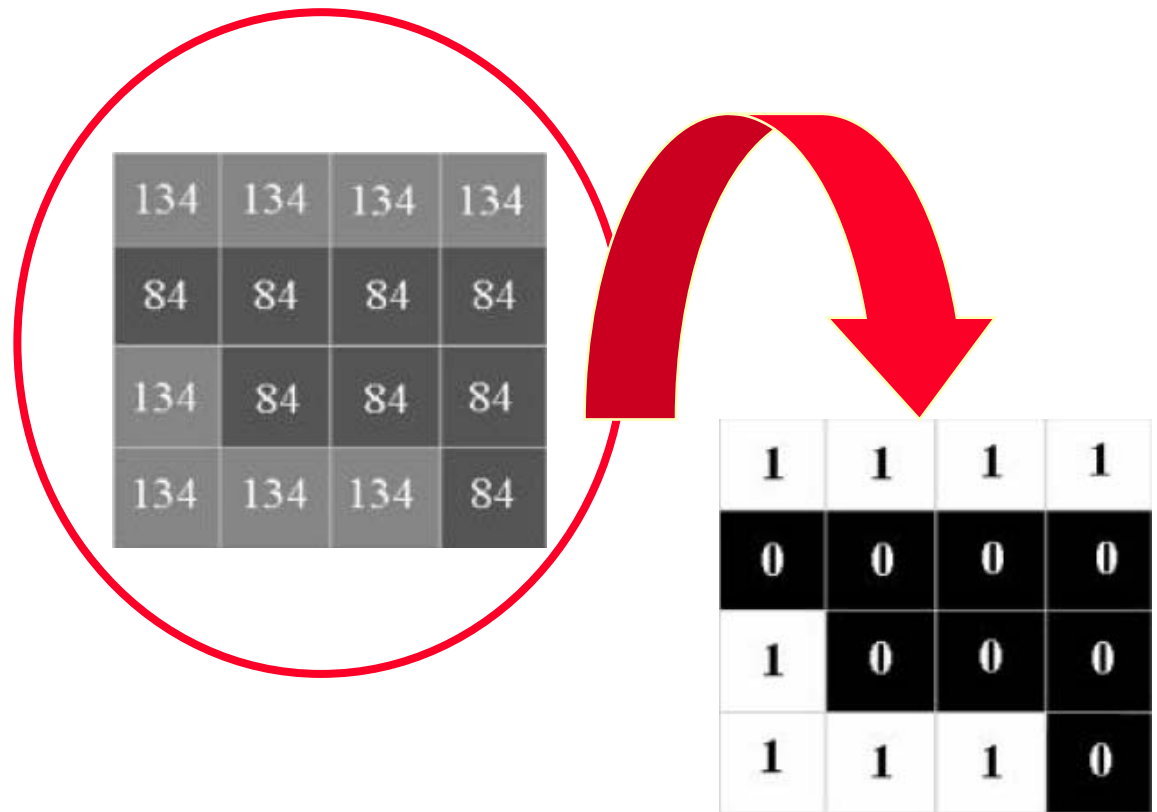


Block Truncation Coding (BTC)



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Basic Idea

- Originally: Preserve the first two sample moments of a small block ($n \times n$)

Average:

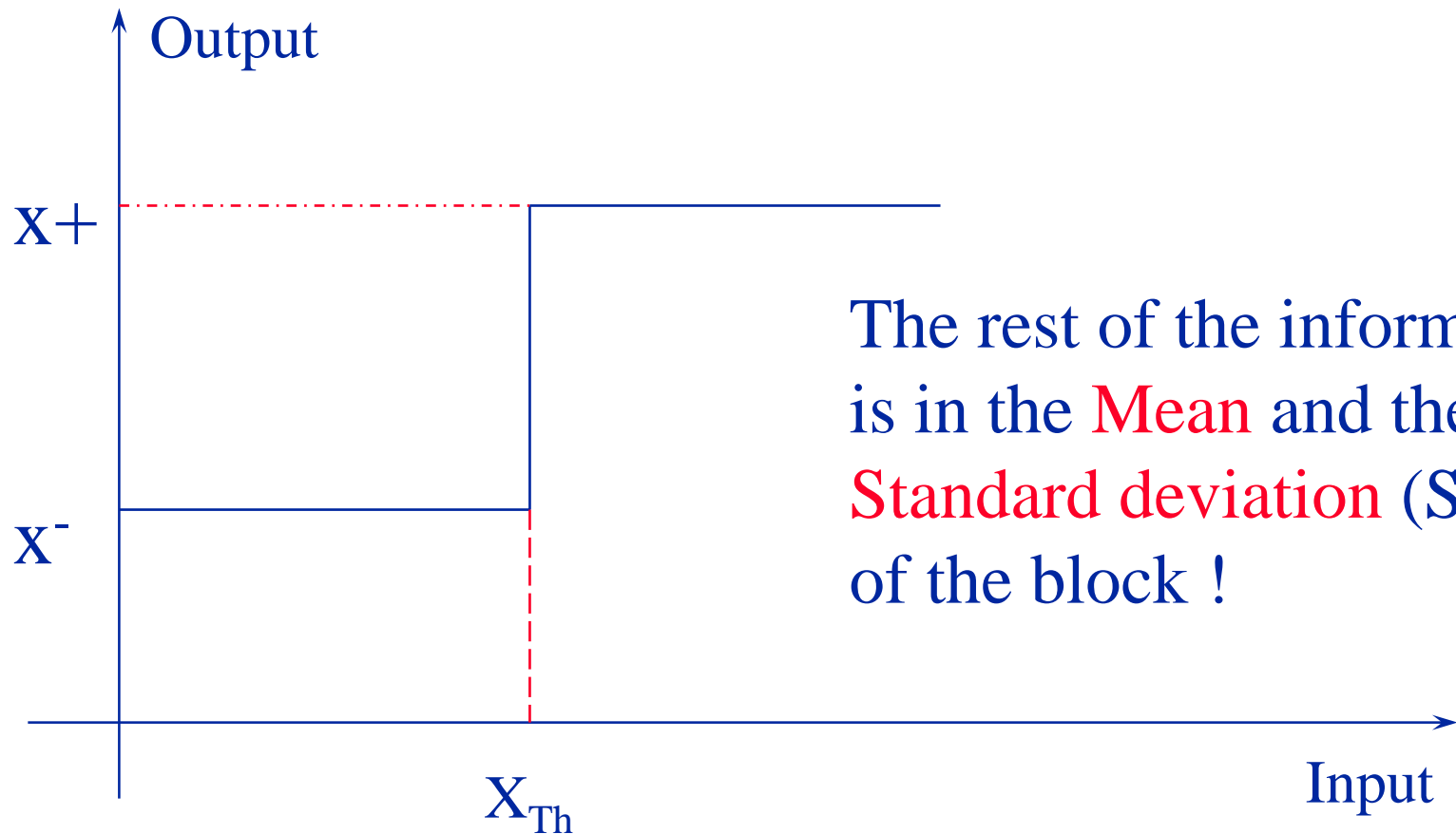
$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Standard Deviation:

$$\sigma = \sqrt{\overline{x^2} - \bar{x}^2}$$

Define a 1-bit (2-level quantizer: x^+ and x^-)
with x_{Th} , such that preserve the two moments

Two Level (binary) Quantizer



The rest of the information is in the **Mean** and the **Standard deviation (SD)** of the block !

Advantages of BTC

- Small **complexity** (faster than TC).
- Preserving **edges**.
- Each **block** can be compressed separately according to its variance.
- Fixed and **Adaptive bit-allocation** optional.

BTC Encoding

- Assume a 512x512 image with 256 gray levels.
- The **threshold** will be the mean value (X_{ave}).
- For each block we transmit bit-level matrix, X_{sd} and X_{ave} .
- The levels X^+ and X^- can be determined by setting up the expressions that **equate (preserve) the moments** before and after quantization.

Levels Selection

$$n^2 \bar{x} = n^- x^- - n^+ x^+$$

$$n^2 \bar{x}^2 = n^- (x^-)^2 - n^+ (x^+)^2$$

Where n^+ and n^- are the number of pixels above and below the threshold (mean)

$$x^- = \bar{x} - \sigma \sqrt{\frac{n^+}{n^-}} \quad x^+ = \bar{x} + \sigma \sqrt{\frac{n^-}{n^+}}$$

Levels Selection (Cont'd)

- Output levels are biased symmetrically around the **mean level**
- Both positive and negative biases are proportional to the SD
- Levels are rounded to the number of allowed bits, so **moment preservation** is not exact

BTC Example

$$x = \begin{bmatrix} 136 & 27 & 144 & 216 \\ 172 & 83 & 43 & 219 \\ 200 & 254 & 1 & 128 \\ 64 & 32 & 96 & 25 \end{bmatrix}$$

Mean=115

SD=77.93

Thresholding

$X^+ = 193$ $X^- = 37$ $n^+ = n^- = 8$

$$x' = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

BTC Decoding

- From the moment preservation principle:

$$x_{rec} = \begin{bmatrix} 193 & 37 & 193 & 193 \\ 193 & 37 & 37 & 193 \\ 193 & 193 & 37 & 193 \\ 37 & 37 & 37 & 37 \end{bmatrix}$$

For N-level reconstruction we use a
Max-Lloyd Quantization !

Compression Ratio

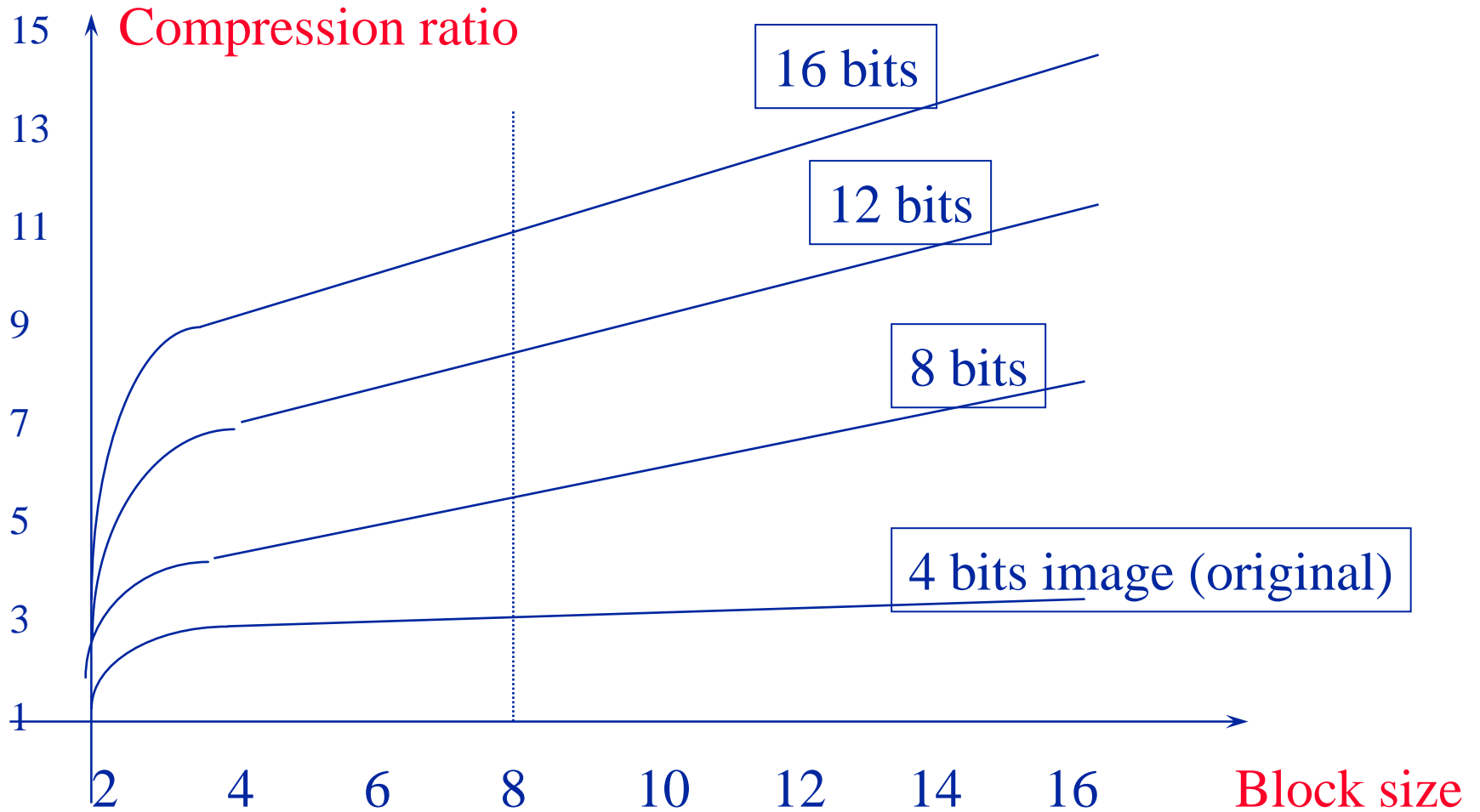
- The higher the **block size** - the higher the compression ratio
- For L bits-per-pixel we have n^2L bits describing a $n \times n$ image
- Assuming that the **mean and SD** are defined also with b bits, we get a total number of $(n^2 + 2b)$ bits in the output:

$$R_{(n)} = \frac{bn^2}{n^2 + 2b} = \frac{8 * 4 * 4}{4 * 4 + 16} = 4$$

Increasing Compression

- Assigning only 6 bits to the mean and 4 bits to the SD: $R=4.923$ (1.625 bpp)
- Another option: assigning 10 bits together to the mean and SD, while the exact number of bits for the mean depends on the SD.

Compression ratio Vs. Block size



Block Size=8, Bitrate=0.935bpp,
Side information=0.25bpp, SNR=30dB



And in color



Original Image



Encoded at 1.89 bpp

Source: Handbook of Image and Video Processing,
Block Truncation Coding (BTC), Edward J. Delp, Martha Saenz, and Paul Salama

Other BTC techniques

- Error criteria: Minimum MSE.
- Error criteria: Minimum MAE.
- Save 3rd order moment.