Block Truncation Coding (BTC)

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Update: Jan. 2006
Basic Idea

• Originally: Preserve the first two sample moments of a small block \((n \times n)\)

  Average:

  \[
  \bar{x} = \frac{1}{n^2} \sum_{i=1}^{n} x_i
  \]

  Standard Deviation:

  \[
  \sigma = (x^2 - \bar{x}^2)
  \]

Define a 1-bit (2-level quantizer: \(x^+\) and \(x^-\)) with \(x_{Th}\), such that preserve the two moments
Two Level (binary) Quantizer

The rest of the information is in the Mean and the Standard deviation (SD) of the block!
Advantages of BTC

- Small **complexity** (faster than TC).
- Preserving **edges**.
- Each **block** can be compressed separately according to its variance.
- Fixed and **Adaptive bit-allocation** optional.
BTC Encoding

• Assume a 512x512 image with 256 gray levels.
• The **threshold** will be the mean value ($x_{ave}$).
• For each block we transmit bit-level matrix, $x_{sd}$ and $x_{ave}$.
• The levels $X^+$ and $X^-$ can be determined by setting up the expressions that **equate** (preserve) the **moments** before and after quantization.
Levels Selection

\[ n^2 \bar{x} = n^- x^- - n^+ x^+ \]

\[ n^2 \bar{x}^2 = n^- (x^-)^2 - n^+ (x^+)^2 \]

Where \( n^+ \) and \( n^- \) are the number of pixels above and below the threshold (mean)

\[ x^- = \bar{x} - \sigma \sqrt{\frac{n^+}{n^-}} \quad x^+ = \bar{x} + \sigma \sqrt{\frac{n^-}{n^+}} \]
Levels Selection (Cont’d)

• Output levels are biased symmetrically around the mean level

• Both positive and negative biases are proportional to the SD

• Levels are rounded to the number of allowed bits, so moment preservation is not exact
BTC Example

Mean=115  SD=77.93

Thresholding

\[ x = \begin{bmatrix} 136 & 27 & 144 & 216 \\ 172 & 83 & 43 & 219 \\ 200 & 254 & 1 & 128 \\ 64 & 32 & 96 & 25 \end{bmatrix} \]

\[ x' = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ X^+ = 193 \quad X^- = 37 \quad n^+ = n^- = 8 \]
BTC Decoding

• From the moment preservation principle:

\[
X_{rec} = \begin{bmatrix}
193 & 37 & 193 & 193 \\
193 & 37 & 37 & 193 \\
193 & 193 & 37 & 193 \\
37 & 37 & 37 & 37
\end{bmatrix}
\]

For N-level reconstruction we use a Max-Lloyd Quantization!
Compression Ratio

• The higher the block size - the higher the compression ratio

• For $L$ bits-per-pixel we have $n^2L$ bits describing a $nxn$ image

• Assuming that the mean and SD are defined also with $b$ bits, we get a total number of $(n^2 + 2b)$ bits in the output:

$$R(n) = \frac{bn^2}{n^2 + 2b} = \frac{8 \times 4 \times 4}{4 \times 4 + 16} = 4$$
Increasing Compression

• Assigning only 6 bits to the mean and 4 bits to the SD: $R=4.923$ (1.625 bpp)

• Another option: assigning 10 bits together to the mean and SD, while the exact number of bits for the mean depends on the SD.
Compression ratio Vs. Block size

- 4 bits image (original)
- 8 bits
- 12 bits
- 16 bits

Compression ratio

Block size
Block Size=8, Bitrate=0.935bpp, Side information=0.25bpp, SNR=30dB
And in color

Original Image

Encoded at 1.89 bpp

Source: Handbook of Image and Video Processing, Block Truncation Coding (BTC), Edward J. Delp, Martha Saenz, and Paul Salama
Other BTC techniques

- Error criteria: Minimum MSE.
- Error criteria: Minimum MAE.
- Save 3rd order moment.