9th International Workshop on Parallel Matrix Algorithms and Applications
Bordeaux, France, July 6-8, 2016

Scaling preconditioners for parallel CG-accelerated projection techniques

Dan Gordon*†[1] and Rachel Gordon[2]

[1]Dept. of Computer Science – University of Haifa, Haifa 34988, Israel
[2]Dept. of Aerospace Engineering – The Technion, Haifa 32000, Israel

Abstract

We consider the problem of solving strongly indefinite linear systems that arise from a variety of domains, such as circuit simulation, CFD, acoustic and thermal problems, and many others. The matrices of such systems often have a highly irregular structure with an extremely large variance in both the size and the number of elements in the rows and columns. Such problems gave rise to sophisticated preconditioning techniques, which often aim to place the largest element on the diagonal. Another problem with huge matrices is that the ILU-type preconditioners are inherently sequential and their parallel versions are not as useful in all cases.

We examine the runtime behavior of several CG variants, such as CGNR, CGNE, CGMN (and its block-parallel version CARP-CG), combined with various row and column scalings. We also introduce a new variant: CG acceleration of a method originally introduced in the context of image reconstruction from projections [Censor, Gordon & Gordon, PARCO 2001]. This method is called "CARP1" in the CARP paper [Gordon & Gordon, SISC 2005], and also "DROP" in [Censor et al., SISC 2008]. CARP1 is just CARP with blocks consisting of a single equation, but its CG acceleration, called CARP1-CG has not been studied before. CARP1 incorporates a mechanism which takes account of the number of elements in every column.

CARP-CG has been used successfully on many problematic linear systems arising from partial differential equations (PDEs), such as convection-dominated problems, the Helmholtz equation at high frequencies, problems with discontinuous coefficients, and problems with large off-diagonal elements. More recently, CARP-CG was found to be useful for solving difficult linear systems arising from the solution of eigenvalue problems, and for solving the elastic wave equation in the frequency domain. Even though CARP-CG is essentially a domain decomposition (DD) method, its application to large 3D wave problems does not exhibit the problem of artificial reflections at subdomain boundaries; a problem common to other DD methods.

The efficacy of CARP-CG on such problems is due to the a fact that it is a CG acceleration of CARP, which is a block-parallel version of KACZ - the Kaczmarz algorithm. KACZ is SOR on the system $AA^Ty=b, x=A^Ty$, so it inherently normalizes the equations. A key reason for the robustness of KACZ is that after the normalization of A, all elements

*Speaker
†Corresponding author: gordon@cs.haifa.ac.il

on the diagonal of AA^T are 1, while the off-diagonal elements are < 1. The robustness of KACZ carries over to its parallelization and acceration in CARP-CG.

We present several simple schemes for the acceleration and improved accuracy of the above algorithms. These schemes are various combinations of row and column scalings. The row scaling consists of dividing each equation by the L_2-norm of its coefficients, which we call equation normalization (EN). EN has also been shown to be useful for GMRES and Bi-CGSTAB on problems with discontinuous coefficients [Gordon & Gordon, JCAM 2010].

We also use two types of column scalings: the first one, called column normalization (CN), consists of dividing each column by the L_2-norm of the column elements. The second type of column scaling is sparsity-oriented: each column is divided by the number of its nonzeros; this scaling is called CS. These scalings can also be combined, for example, EN-CN-CS.

The results of this study show that the above CG methods can benefit very significantly from these scalings and their combination. For a given number of iterations, relative residual results can be improved on some problems by several orders of magnitude. Equivalently, relative residual goals can be reached in a significantly shorter time.

Keywords: Circuit problems, CG acceleration, CARP_CG, row scaling, column scaling, sparsity_oriented scaling.