Computing the Plane Equation of a Polygon

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The following is a numerically robust method of computing the plane equation of a polygon, described by a list of its vertices in 3D. If the vertices are not exactly on a plane, the result will be a "best fit" plane. This method, due to M. E. Newell, works for arbitrary polygons, and it is preferable to the one that uses only three vertices of the polygon.

Assume that a polygon is given by its vertices $v_1, v_2, \ldots, v_n$, in cyclic order, and each v_i is given by its coordinates in 3D: $v_i = (x_i, y_i, z_i)$. The plane equation, $Ax+By+Cz+D = 0$, is given by the equation

$$(X - P) \bullet N \;=\; 0,$$

where $X = (x, y, z)$, $N = (A, B, C)$ is the normal to the plane, P is a point on the plane, and "$\bullet$" represents the scalar product of two vectors. A, B, C can be computed by:

$$A \;=\; \sum_{i=1}^{n} (y_i - y_{i\oplus 1})(z_i + z_{i\oplus 1}),$$

$$B \;=\; \sum_{i=1}^{n} (z_i - z_{i\oplus 1})(x_i + x_{i\oplus 1}),$$

$$C \;=\; \sum_{i=1}^{n} (x_i - x_{i\oplus 1})(y_i + y_{i\oplus 1}),$$

where "$\oplus$" stands for addition modulo n.

D can be computed as:

$$D \;=\; -P \bullet N,$$

where P is the average of all the vertices (also called the "center of gravity"):

$$P \;=\; \frac{1}{n} \sum_{i=1}^{n} v_i.$$