# Corner cutting with trapezoidal augmentation for area-preserving smoothing of polygons and polylines

ONLINE SUPPLEMENT

Dan Gordon[1]

The purpose of this note is to provide the user with some implemention details of the trapezoidal construction of CCA2. It is assumed that the reader is familiar with the relevant paper.

Fig. 1 illustrates the trapezoidal construction on an edge $AB$ of some polygon. For simplicity, it is assumed that $A$ coincides with the origin of an $(X,Y)$ coordinate system and that $B$ lies on the positive $X$-axis; other cases are obtained by a simple translation and rotation transformation. We denote by $\beta$ the angle made by the other edge adjacent to $A$ with the $X$-axis, so the line equation of the adjacent edge is $y = (\tan\beta)x$.

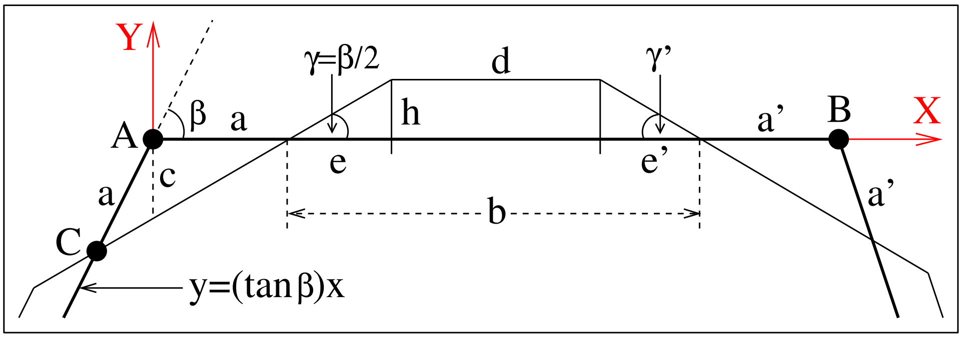


Figure 1: Illustration of the trapezoidal construction of CCA2.

The first step is to determine the cut lengths at $A$ and $B$. This is done by evaluating the internal angle at each vertex and using the relevant $\alpha$-profile. For example, the angle at $A$ is $\pi - \beta$. Denote by $a$ and $a'$ the cut lengths at $A$ and $B$, respectively. Since the cut triangle is isosceles, the angle between the $X$-axis and the cutting line at $A$ is $\gamma = \beta/2$. Hence, the equation of the cutting line is $y = (\tan\gamma)x - c$, where $c$ is the distance between $A$ and the point at which the cutting line intersects the $Y$-axis. As can be seen from the figure, $c = a\tan\gamma$.

We now know the coordinates of the three vertices of the cut triangle at $A$: they are $(0,0),(a,0)$, and the intersection of the cutting line with the other edge, denoted in the figure by $C$. Alternately, the coordinates of $C$ can be found as follows: let $\ell$ be the length of the adjacent edge, $(-x_1,-y_1)$ the coordinates of adjacent vertex, and we denote the coordinates of $C$ by $(-x_2,-y_2)$. Note that $x_i, y_i > 0$. Now, by similarity of triangles, we have:

$$\frac{x_2}{x_1} = \frac{y_2}{y_1} = \frac{a}{\ell}.$$

[1]Dept. of Computer Science University of Haifa Haifa 31905, Israel. Email: gordon@cs.haifa.ac.il

We now calculate the area of the cut triangle at $A$ from its coordinates. Similarly, we calculate the area of the cut triangle at $B$. We denote by $\mathbf{S}$ half the sum of the two cut areas, and we need to construct the trapezoid so that its area will be $\mathbf{S}$. This is done as follows.

From the previous calculations, we know the position and length $b$ of the trapezoid base. We need to calculate the height $h$ of the trapezoid. Let $d$ denote the length of the top of the trapezoid; $d$ is also not known. Denote by $e, e'$ the bases of the two triangles, and $\gamma, \gamma'$ the base angles of the trapezoid, as shown in the figure. We have the following equations:

$$d + e + e' = b, \;\; e = \frac{h}{\tan\gamma}, \;\; e' = \frac{h}{\tan\gamma'},$$

from which we get

$$d + h \underbrace{\left( \frac{1}{\tan\gamma} + \frac{1}{\tan\gamma'} \right)}_{\text{denote: } t} = b \implies d + ht = b \implies h = \frac{b-d}{t}.$$

The area of the trapezoid should be equal to $\mathbf{S}$, so

$$\frac{d+b}{2} h = \mathbf{S}.$$

Substituting for $h$, we get $(d+b)(b-d) = 2\mathbf{S}t \implies b^2 - d^2 = 2\mathbf{S}t$. Therefore,

$$d = \sqrt{b^2 - 2\mathbf{S}t},$$

and this gives us the required value of $h = (b-d)/t$. The coordinates of the vertices at the top of the trapezoid are $(a+e, h)$ and $(a+e+d, h)$.

**Note:** if we get $b^2 - 2\mathbf{S}t \leq 0$, i.e., $d$ is zero or imaginary, then a proper trapezoid cannot be constructed, and it is necessary to decrease one or both of the cut areas.