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Computational Aspects in Image Analysis of Symmetry and of its Perception

Hagit Hel-Or
Dept of Math and Computer Science
Bar-Ilan University
Ramat-Gan, 52900 Israel
e-mail: hagit@cs.biu.ac.il

David Avnir
Department of Organic Chemistry
The Hebrew University of Jerusalem
Jerusalem, 91904 Israel
david@chem.ch.huji.ac.il

Abstract

Symmetry as a characteristic of shape and form has been widely studied both in the artistic and esthetic aspect on one hand and in the mathematical and computational aspect on the other. Symmetry is typically viewed as a discrete feature: an object is either symmetric or non-symmetric. However visual perception and natural behavior and phenomena treat symmetry as a continuous feature, relating to statements such as "one object is **more** symmetric than another" or "an object is **more** mirror symmetric than rotational symmetric". With this notion in mind, we view symmetry as a continuous feature and define a Continuous Symmetry Measure (CSM) to quantify the "amount" of symmetry of different shapes and the "amount" of different symmetries of a single shape. The characteristics of the CSM provides the ability to analyze the symmetries of images, shapes and 3D objects while taking into account the hierarchical and continuous nature of these symmetries and allowing for noise in the data such as occlusion and fuzzy data.

1 Introduction

Symmetry is one of the basic features of shape and form. It has been widely studied from various aspects ranging from artistic to mathematical. However the classical view of symmetry as a binary feature - either an object is symmetric or it is not - is inadequate to describe the symmetries found in the natural world. It is inconsistent with visual perception and natural behavior which treat symmetry as a continuous feature, relating to statements such as "one object is **more** symmetric than another" or "an object is **more** mirror symmetric than rotational symmetric". With this notion in mind, we view symmetry as a continuous feature. Accordingly, a Continuous Symmetry Measure (CSM) has been defined [11, 3] which quantifies the "amount" of symmetry of different shapes and the "amount" of different symmetries of a single shape.

2 The Continuous Symmetry Measure (CSM)

We define the **Continuous Symmetry Measure** (CSM) as the minimum 'effort' required to transform a given shape into a symmetric shape. This 'effort' is measured by the mean of the squared distances each point is moved from its location in the original shape to its location in the symmetric shape. Note that no a priori symmetric reference shape is assumed.

A shape P is represented by a sequence of n points $\{P_i\}_{i=0}^{n-1}$. We define a distance between every two shapes P and Q:

$$d(P,Q) = d(\{P_i\},\{Q_i\}) = \frac{1}{n}\sum_{i=1}^{n} \|P_i - Q_i\|^2$$

We define the **Symmetry Transform** $\hat{P}$ of P as the symmetric shape closest to P in terms of the distance d. The **Continuous Symmetry Measure** of P denoted $S(P)$ is defined as the distance to the closest symmetric shape:

$$S(P) = d(P, \hat{P})$$

The CSM of a shape $P = \{P_i\}_{i=0}^{n-1}$ is evaluated by finding the symmetry transform $\hat{P}$ of P and computing: $S(P) = \frac{1}{n}\Sigma_{i=0}^{n-1}\|P_i - \hat{P}_i\|^2$. This definition of the CSM implicitly implies invariance to rotation and translation. Normalization of the original shape prior to the transformation additionally allows invariance to scale.

A geometrical algorithm was developed to find the Symmetry Transform and the CSM of a shape (see [3, 11, 6] for details).

The general definition of the CSM enables evaluation of a given shape for different types of symmetries (mirror-symmetries, rotational symmetries etc). Moreover, this generalization allows comparisons between the different symmetry types, and allows expressions such as “a shape is more mirror-symmetric than C_2-symmetric”.

An additional feature of the CSM is that we obtain the symmetric shape which is ‘closest’ to the given one, enabling visual evaluation of the CSM.

The CSM approach to measuring symmetry allows the hierarchical nature of symmetry to be expressed and quantified, as will be discussed below. Additionally, the CSM method can deal with noisy and occluded data, also discussed below.

3 Measuring the CSM of Shapes, Images and 3D Objects

The versatility of the CSM method has induced its use in various fields such as Chemistry [9, 5], Psychology [4], Archaeology [2] and more. Underlying these studies is the ability of the CSM to evaluate continuous symmetry in shapes, images and objects. Previous approaches to evaluating symmetry in shapes, images and objects which mainly rely on the binary concept of symmetry are reviewed in [11, 3].

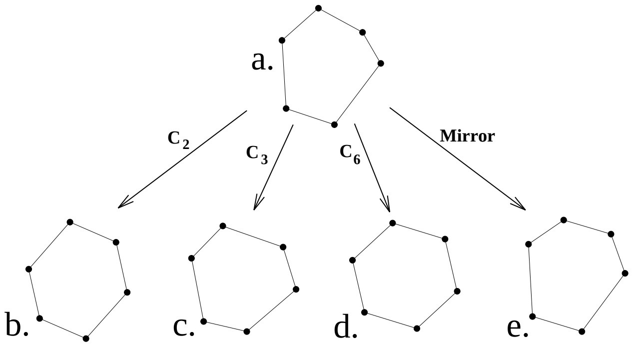

Figure 1: Symmetry Transforms of a 2D polygon.

a) 2D polygon and its Symmetry Transform with respect to b) C_2-symmetry (SD = 1.87). c) C_3-symmetry (SD = 1.64). d) C_6-symmetry (SD = 2.53). e) Mirror-symmetry (SD = 0.66).

3.1 CSM of a Set of Points

Given a set of points, possibly with connectivities between these points, the CSM is calculated using the geometrical algorithm previously developed [11, 3]. Given a shape such as a polygon, the vertices can be used as the input to the algorithms. Figure 1 shows an example of a shape and its Symmetry Transforms and CSM values with respect to several types of symmetries.

3.2 CSM of a Continuous Shape

Measuring the CSM of a general shape, requires the pre-process of representing the shape by a collection of points. Several methods have been suggested to sample the continuous contour of general shapes in order to obtain a collection of points as input to the CSM algorithm [8, 11]. Figure 2 shows an example of a general shape whose contour has been sampled, a collection of representative points obtained and the Symmetry Transforms and CSM values were calculated with respect to several types of symmetries.

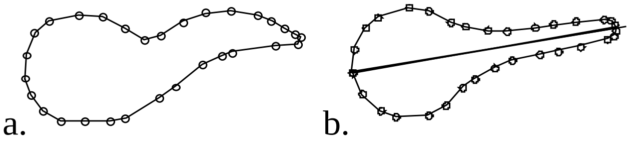

Figure 2: Symmetry Transform of a general 2D shape.
a) The contour of the shape is sampled. b) The Symmetry Transform of the shape is found.

3.3 CSM of Gray-scale Images

Two approaches have been used in applying the CSM to gray-scale images. One approach segments areas of interest from the image and regards their contour as 2D shapes (see Figure 8 for example).
Another approach in dealing with images, lets pixel values denote elevation, and considers an image as a 3D object on which 3D symmetries can be measured. Figure 3 shows a range image and a gray-scale image for which the 3D mirror symmetry transform was computed, the 3D reflection plane was found and the 3D object rotated to a frontal vertical view.

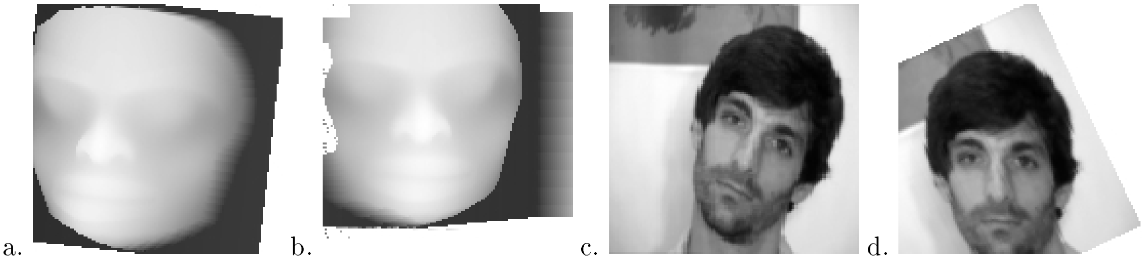

Figure 3: Applying CSM with respect to 3D mirror-symmetry to find orientation of a 3D object.
a) Original range image. c) Original Gray-scale image. b,d) The symmetry reflection plane was found and the object in the image rotated to a frontal vertical view.

3.4 CSM of 3D Objects

Measuring the CSM of 3D objects is straight forward given a collection of 3D points as representatives [11, 3, 7, 9]. A variation of this scheme involves measuring the symmetry of the projection of a 3D object

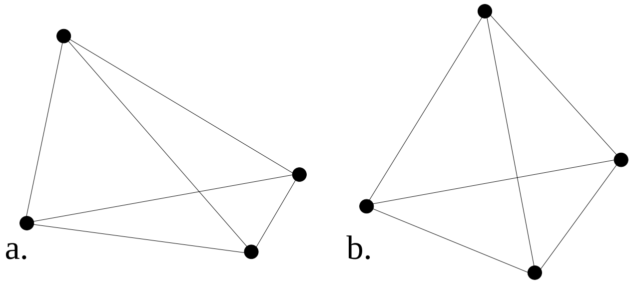

Figure 4: A 3D-object (a) and its symmetry transform (b) with respect to Tetrahedral-symmetry.

Figure 5: Reconstruction of a 3D-mirror-symmetric object from 2D images of different view points.

onto the image plane [12]. This approach is used in reconstruction of 3D objects from their 3D projections. Figure 4 shows a 3D object and its symmetry transform with respect to T_d-symmetry (tetrahedral). Figure 5 shows an example of 3 projections of a 3D object which was reconstructed using the CSM approach.

4 Exploiting the Characteristics of the CSM

The CSM approach to measuring symmetry can be embedded in a scheme that takes advantage of the multi-resolution characteristics of symmetry The symmetry transform of a low resolution version of an image is used to evaluate the symmetry transform of the high resolution version. This technique was used in estimating face orientation [11, 7].

In many cases the source data is noisy. The CSM method can be exploited to deal with noisy and missing data. Considering noisy data, where the collection of representative points are given as probability distributions, the CSM approach can evaluate the following properties [11, 10]:

- The most probable symmetric configuration represented by the points.
- The probability distribution of CSM values for the points.

Figure 6 shows the effect of varying the probability distribution of the object points on the resulting symmetric shape.

Figure 7 displays a fuzzy image of points (a Laue photograph which is an interference pattern created by projecting X-ray beams onto crystals). and the probability distribution of the CSM values obtained for the pattern.

Finally, the CSM approach to measuring symmetry allows the hierarchical nature of symmetry to be expressed and quantified thus, global and local features can be evaluated for their symmetry content. Figure 8 shows an example where the CSM approach measures local mirror symmetry to find faces in an image.

This issue of global vs local symmetry is important in the case of measuring symmetry of human faces.

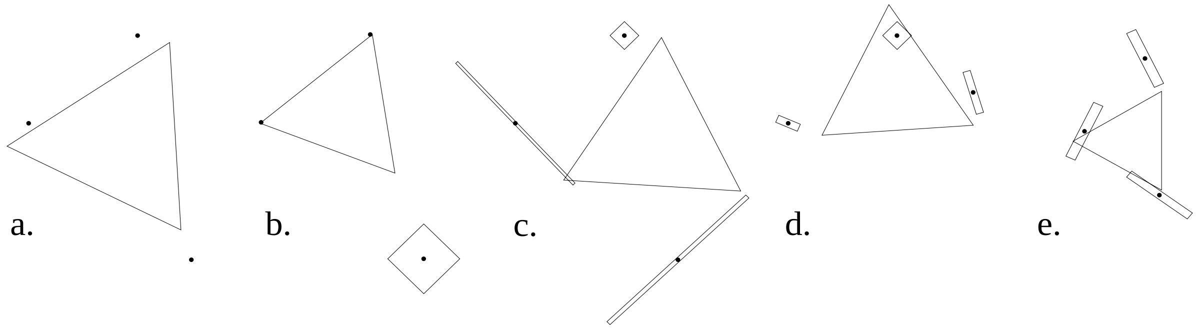

Figure 6: The most probable C_3-symmetric shape for a set of measurements after varying the probability distribution and expected locations of the measurements.
a-c) Changing the uncertainty (s.t.d.) of the measurements.
d-e) Changing both the uncertainty and the expected location of the measurements.

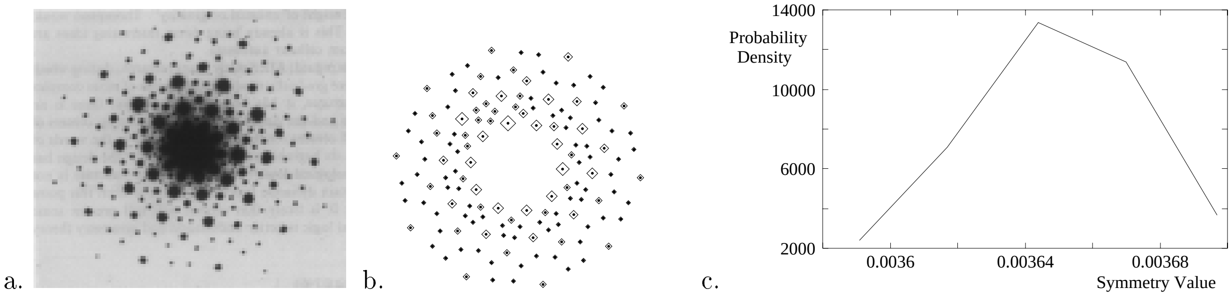

Figure 7: Probability distribution of symmetry values
a) Interference pattern of crystals. b) Probability distribution of point locations corresponding to a. c) Probability distribution of symmetry distance values with respect to C_{10}-symmetry was evaluated. Expectation value = 0.003663.

5 Symmetry of Human Faces

The human face is a complex structure comprising of a number of facial features. It can be modeled at several levels of hierarchy: at the top level is the global structure of the face (the face contour) and at lower levels facial details (such as facial features) are revealed. Accordingly, symmetry of the human face should be considered hierarchically. The CSM approach to measuring symmetry can be employed to evaluate mirror-symmetry of a face at several different levels (Figure 9):

- Face Contour - symmetry is evaluated by considering the contour of the face alone.
- Facial Features Centroids - the centroids of each of the facial features serve as the set of input points for symmetry evaluation.
- Facial Features Contours - the face contour and the contours of all the facial features are considered in the evaluation of symmetry.

Recent studies have proposed that there is a positive correlation between symmetry of faces and physical attraction [1]. However the methods used for evaluating symmetry do not capture the complexity of this characteristic. We propose that using the CSM approach in a hierarchical scheme will provide a more flexible reliable and meaningful measure of symmetry of human faces.

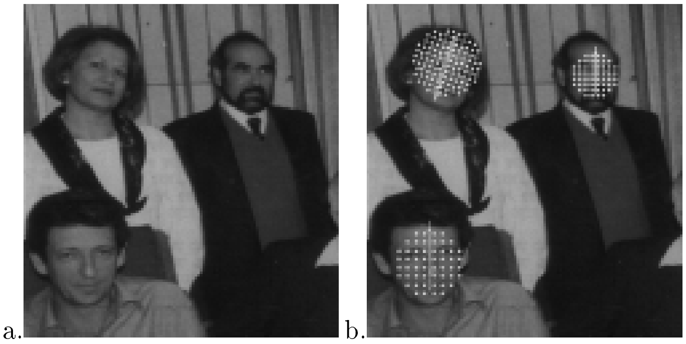

Figure 8: Multiple mirror-symmetric regions in images.
a) Original image. b) Faces are found as locally symmetric regions.

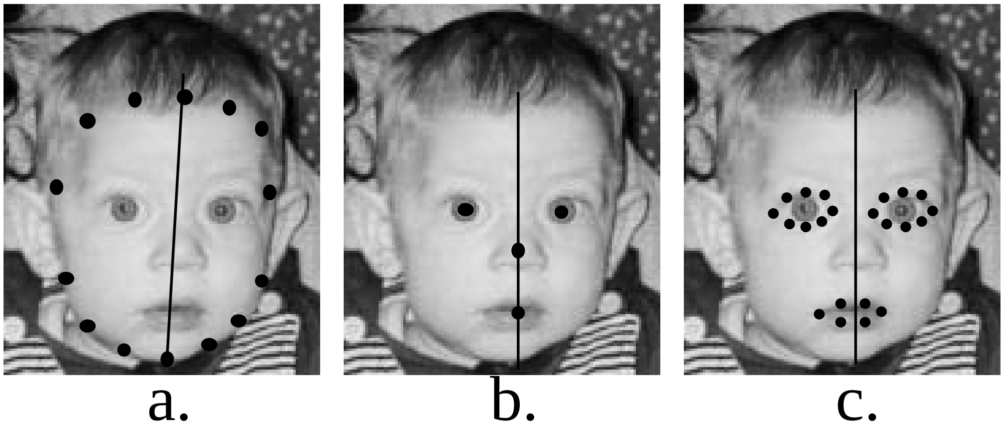

Figure 9: Measuring symmetry of human faces at 3 levels:
a) Face Contour. b) Facial Features Centroids c) Facial Features Contours

6 Conclusion

We view symmetry as a continuous feature and adopt the Continuous Symmetry Measure (CSM) to evaluate it. The CSM can evaluate any symmetry in any dimension and has been applied to shapes, images and 3D objects. The CSM can deal with noisy data, such as fuzzy and occluded data. The method of evaluating symmetry using the CSM can be applied to global and local symmetries. This can be extended to deal with symmetry in a hierarchical manner as in the case of measuring symmetry of human faces. The CSM is currently being used in numerous fields including Chemistry, Physics, Archaeology, Botany and more.

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