Hierarchical Symmetry

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Abstract

We view symmetry as a continuous feature and dependent on resolution. Combining a Continuous Symmetry Measure (CSM) with a multiresolution scheme, we present a method that hierarchically detects symmetric and almost symmetric patterns. Evaluation of symmetry at low frequencies guides the process to find the symmetry at higher frequencies.

1 Introduction

The world is rich in symmetries. However, the exact mathematical definition of symmetry is not adequate to describe most symmetries, which are very seldom exact. Faces, and the human body, are a classic example for inexact symmetry. The projection of the world onto an image plane (the retina or a digital image) creates additional deviations from exact symmetry (due to perspective projection, digitization, occlusion etc.). To handle those inexact symmetries, we use a "continuous symmetry measure" (CSM) that can measure different types of symmetries of objects [9]. Once symmetry is considered a continuous feature, it becomes dependent on the resolution by which the scene is viewed. Fig. 1 shows examples which are more symmetrical at low resolutions than at high resolutions. Julesz et. al. showed that separation into frequency channels precedes symmetry detection, thus different symmetries can be seen simultaneously in an image [1].

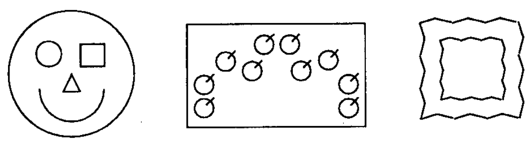

Figure 1: Figures which are more symmetric at low resolution than at high resolution.

In general, however, the symmetries found at high resolution also exist at low resolution. Studies [5, 3] have shown that human scanning of mirror-symmetric images is concentrated on one mirror-half of the image, implying that the symmetry detection process uses low spatial frequencies for guiding the scanning process. We use a similar approach by combining the Continuous Symmetry Measure with a multiresolution scheme.

In Sect. 2 we briefly review the definition of the Continuous Symmetry Measure for 3D shapes. In Sect. 4 we apply the CSM to finding the orientation of faces and combine the CSM with a multiresolution scheme to locate and segment symmetrical regions.

2 Continuous Symmetry

For a summary of symmetry studies in computer vision see [9]. Following are definitions of exact symmetry in 3D (see [6]):

A 3D object has a perfect **mirror-symmetry** if it is invariant under a reflection about a plane (called the reflection plane) passing through the centroid of the object.

A 3D object has a perfect **rotational-symmetry** of order n (C_n) if it is invariant under rotations of $2\pi/n$ radians about a line passing through its centroid.

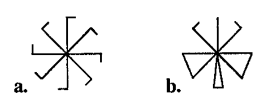

Figure 2: a) C_8-symmetry. b) mirror-symmetry

2.1 A Continuous Symmetry Measure

Denote by Ω the space of all 3D shapes, where each shape P is represented by a sequence of n points $\{P_i\}_{i=0}^{n-1}$.

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We define a metric d on this space as follows:

$$d : \Omega \times \Omega \rightarrow R$$

$$d(P, Q) = d(\{P_i\}, \{Q_i\}) = \frac{1}{n} \sum_{i=1}^{n} ||P_i - Q_i||^2$$

We define the **Symmetry Transform** ST as the symmetric shape closest to P in terms of the metric d.

The **Continuous Symmetry Measure (CSM)** is now defined as the distance to the closest symmetric shape:

$$CSM(P) = d(P, ST(P))$$

The CSM of a shape $P = \{P_i\}_{i=0}^{n-1}$ is evaluated by finding the symmetry transform $\hat{P}$ of P (Fig.3) and computing: CSM(P) $= \frac{1}{n}\Sigma_{i=0}^{n-1}||P_i - \hat{P}_i||^2$.

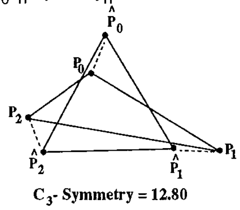

Figure 3: The symmetry transform of $\{P_0, P_1, P_2\}$ is $\{\hat{P}_0, \hat{P}_1, \hat{P}_2\}$. CSM $= \frac{1}{3}\Sigma_{i=0}^{2}||P_i - \hat{P}_i||^2$

2.2 3D Symmetry Transform

The geometrical algorithm for deriving the symmetry transform of 2D shapes was given in [9]. In this section we describe the geometrical algorithm for 3D. Mathematical derivations and proofs can be found in [8]. Following is the derivation of the C_n symmetry transform about a given rotation axis of a 3D shape P having n points. This transforms P into a regular n-gon lying in a single plane perpendicular to the rotation axis and passing through the centroid of P which remains invariant (Fig. 4).

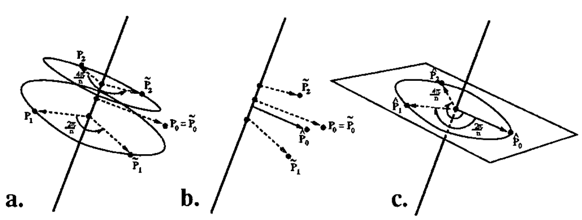

Figure 4: The C_3-symmetry Transform of 3 points:
a) Folding $\{P_i\}_{i=0}^{2}$ into $\{\tilde{P}_i\}_{i=0}^{2}$.
b) Averaging$\{\tilde{P}_i\}_{i=0}^{2}$ obtaining $\hat{P}_0$.
c) Unfolding $\hat{P}_0$ obtaining $\{\hat{P}_i\}_{i=0}^{2}$.

1. *Fold* the points $\{P_i\}_{i=0}^{n-1}$ by rotating each point P_i counterclockwise about the given rotation axis by $2\pi i/n$ radians.
2. Let $\hat{P}_0$ be the average of the points $\{\tilde{P}_i\}_{i=0}^{n-1}$.
3. *Unfold* the points, obtaining the C_n-symmetric points $\{\hat{P}_i\}_{i=0}^{n-1}$ by duplicating $\hat{P}_0$ and rotating clockwise about the given rotation axis by $2\pi i/n$ radians.

A 3D shape P having qn points is represented as q sets of n points. The C_n-symmetry transform of P about a given rotation axis is obtained by separately transforming each set of n points into a regular n-gon perpendicular to the rotation axis as described above (Fig. 5).

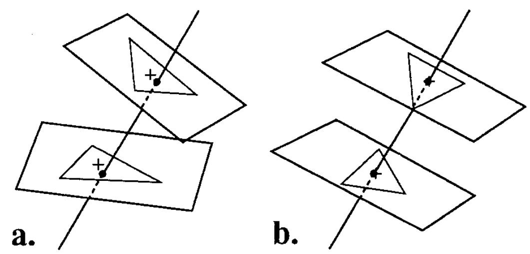

Figure 5: Geometric description of the C_3-symmetry transform for a warped 3-sided prism given a rotational axis. a) The original polyhydron shown as two sets of 3 points. b) Each set is transformed into a regular triangle such that its centroid (marked as +) coincides with the rotation axis. A 3D C_3-symmetric shape is obtained.

When the rotational axis is unknown, the axis can be found by minimizing the CSM over all rotational axes using a gradient descent method.

The procedure for evaluating the 3D symmetry transform for mirror-symmetry is similar (see [8]).

3 Selecting the Points of the Shape

As symmetry has been defined on a sequence of points, representing a given shape by points must precede the application of the symmetry transform. In 2D, points are selected along the contour using the natural ordering. In the case of 3D shapes, no underlying linear ordering of contour points can be assumed. However given an axis of rotational symmetry passing through the origin (or a plane of mirror-symmetry), selection of points can be performed on a perfect sphere about that origin. Selection of points are at the junctions of longitudinal and latitudinal lines on the sphere, defined by the axis. The number of longitudinal lines is a multiple of n (when measuring C_n-symmetry), while latitudinal lines are dependent on resolution parameters. To select points on the contour of a general 3D shape, we transform the shape into a sphere (gaussian image), select points on the sphere and back project them onto the original shape.

4 Applications

4.1 Finding Face Orientation

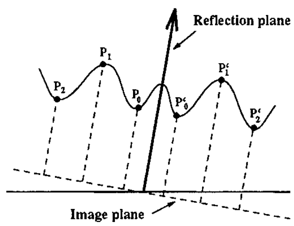

Figure 6: Finding 3D mirror-symmetry. The reflection plane minimizing the CSM indicates the closest mirror-symmetry.

When analyzing depth maps, pixel values denote elevation. We applied the CSM to find the orientation of faces by finding their mirror-symmetry transform.

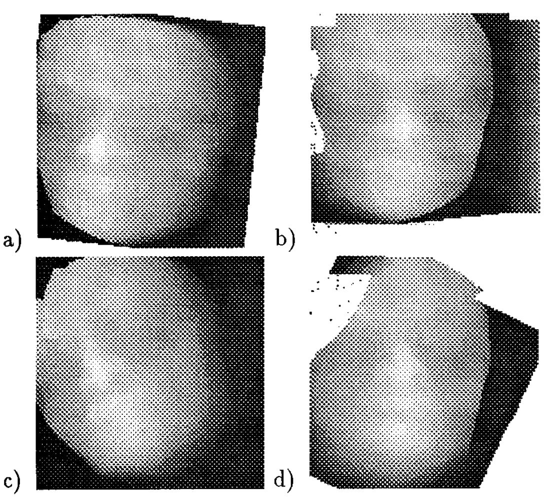

Figure 7: Applying the 3D mirror-symmetry to range data images.
a,c) original depth maps.
b,d) The symmetry reflection plane has been found and the image rotated to a frontal vertical view.

The 3D shape is represented by a set of 3D points selected using a variation of the method described in the previous section (Fig. 6). The mirror-symmetry transform of the image is obtained by minimizing the CSM over all possible reflection planes. Results are shown in Fig. 7.

4.2 Using a Multiresolution Scheme

In many images the process of finding the reflection plane did not converge to the correct solution. To overcome this problem we introduced a multiresolution scheme, where an initial estimation of the symmetry transform is obtained at low resolution (see Fig. 8) and is fine tuned using high resolution images.

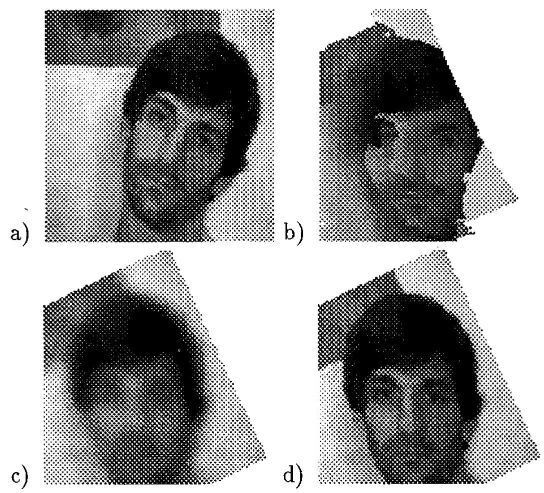

Figure 8: Applying the Multiresolution scheme on real face images. Results tested by rotating the image to a frontal vertical view.
a) Orginal image.
b) Applying the mirror-symmetry transform on (a) does not find correct reflection plane.
c) Applying the mirror-symmetry transform in low resolution finds a good estimation of the reflection plane and direction of gaze.
d) Using the reflection plane found in (d) as an initial guess, the process now converges to the correct symmetry plane.

4.3 Finding Localy symmetric Regions

To locate localy symmetric regions in digital images, a three stage process is used.

The first stage locates symmetry focals. Using the multiresolution scheme with a foveating scheme [7] where the image is subdivided recursively, we locate centers and reflection plane of regions having mirror-symmetry. Additional methods can be used [4].

Given a reflection plane, a symmetry map of the image is created where the value of each symmetric pixel pair denotes its continuous symmetry value.

Starting from the symmetry focals, regions are expanded using "snakes" [2] to include compact symmetric regions. The expansion is guided by the symmetry map and continues as long as the pixels included in the localy symmetric region do not degrade the symmetry of the region excessively.

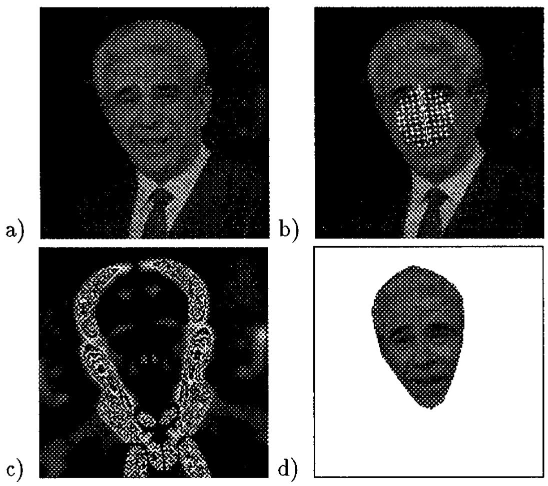

Figure 9: Applying the Multiresolution scheme on real face images:
a) Original image.
b) A mirror-symmetry focal was found.
c) Symmetry map of the image for the mirror-symmetry found in b).
d) Extracted localy symmetric region.

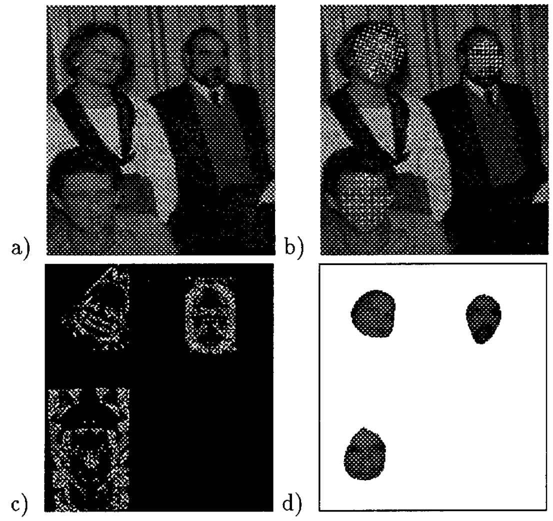

Figure 10: Applying the Multiresolution scheme on real face images:
a) Original image.
b) The mirror-symmetry focals found.
c) Symmetry maps of for each mirror-symmetry are merged into a single image.
d) Extracted localy symmetric regions.

5 Conclusion

We viewed symmetry as a resolution dependent continuous feature. Using the observation that high resolution symmetries usually exist at low resolutions, we combined a Continuous Symmetry Measure (CSM) with a multiresolution scheme, to hierarchically detect symmetric and almost symmetric patterns. Evaluation of symmetry at low frequencies guides the process to find the inexact symmetry at higher frequencies.

References

[1] B. Julesz and J. Chang. Symmetry perception and spatial frequency channels. *Perception*, 8:711–718, 1979.

[2] M. Kass, A. Witkin, and D. Terzopoulos. Snakes: active contour models. *Int. J. Comp. Vis.*, 1:322–332, 1988.

[3] P. Locher and C. Nodine. Influence of stimulus symmetry on visual scanning patterns. *Perception and Psychophysics*, 13(3):408–412, 1973.

[4] D. Reisfeld, H. Wolfson, and Y. Yeshurun. Detection of interest points using symmetry. In *3-ICCV*, pages 62–65, Osaka, December 1990.

[5] E. Thomas. Eye movements and fixations during initial viewing of rochard cards. *J. Projective Techniques and Personal Assesment*, 27:345–353, 1963.

[6] H. Weyl. *Symmetry*. Princeton Univ. Press, 1952.

[7] H. Zabrodsky and S. Peleg. Atentive transmission. *J. Vis. Comm. Im. Rep.*, 1(2), 1990.

[8] H. Zabrodsky, S. Peleg, and D. Avnir. A continuous symmetry measure. Technical Report TR-91-9, Hebrew University, August 1991.

[9] H. Zabrodsky, S. Peleg, and D. Avnir. A measure of symmetry based on shape similarity. In *CVPR-92*, Champaign, 1992.