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Completion of Occluded Shapes using Symmetry

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Abstract

Symmetry is usually viewed as a discrete feature: an object is either symmetric or non-symmetric. Following the view that symmetry is a continuous feature, a Continuous Symmetry Measure (CSM) has been developed to evaluate symmetries of shapes and objects. In this paper we extend the symmetry measure to evaluate the symmetry of occluded shapes. Additionally, using the symmetry measure, we reconstruct occluded shapes by locating the center of symmetry of the shape.

1 Introduction

One of the basic features of shapes and objects is symmetry. Symmetry is considered a pre-attentive feature which enhances recognition and reconstruction of shapes and objects [2, 3, 1]. Symmetry is also an important parameter in physical and chemical processes and is an important criterion in medical diagnosis.

Figure 1: Faces are not perfectly symmetrical. a) Original image. b) Right half of original image and its reflection. c) Left half of original image and its reflection.

The exact mathematical definition of symmetry [4, 5] is inadequate to describe and quantify the symmetries found in the natural world nor those found in the visual world (a classic example is that of faces - see Figure 1). Furthermore, even perfectly symmetric objects loose their exact symmetry when projected onto an image plane or retina due to occlusion, self-occlusion, digitization, etc.

Previous work [7, 6] introduced a symmetry measure to quantify the deviation of shapes and objects from perfect symmetry. In this paper we deal with evaluating the deviation from perfect symmetry of incomplete data such as occluded shapes and uncertain data.

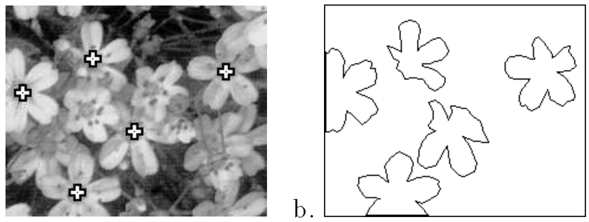

Figure 2: a) A collection of occluded asymmetric flowers. b) The contours of some occluded flower marked by '+' in a.

Figure 2a shows a collection of flowers. Each flower is not only imperfectly symmetric but also partially occluded. Figure 2b shows the contours of some occluded flowers (marked by '+' in Figure 2a). In this paper we describe a method for evaluating the symmetry of such occluded shapes, locating their symmetry centers and reconstructing the symmetric shape closest to the unoccluded original.
In the next section we briefly review the symmetry measure as applied to 2D shapes. In Section 3 we describe a method based on the symmetry measure, that deals with the symmetry of occluded shapes.

2 A Symmetry Measure

The **Symmetry Measure** as described in [7, 6] quantifies the minimum effort necessary to turn a given shape into a symmetric shape. This effort is measured by the sum of the square distances each point is moved from its location in the original shape to its location in the symmetric shape. Note that no a priori symmetric reference shape is assumed.

A shape P will be represented by a sequence of n

points $\{P_i\}_{i=0}^{n-1}$. We define a distance between every two shapes P and Q:

$$d(P,Q) = d(\{P_i\},\{Q_i\}) = \frac{1}{n}\sum_{i=1}^{n} ||P_i - Q_i||^2$$

We define the **Symmetry Transform** $\hat{P}$ of P as the symmetric shape closest to P in terms of distance d.

The **Symmetry Measure** of P denoted $s(P)$ is now defined as the distance to the closest symmetric shape:

$$s(P) = d(P, \hat{P})$$

The CSM of a shape $P = \{P_i\}_{i=0}^{n-1}$ is evaluated by finding the symmetry transform $\hat{P}$ of P (Fig.3) and computing: $s(P) = \frac{1}{n}\Sigma_{i=0}^{n-1}||P_i - \hat{P}_i||^2$. Following is a geometrical algorithm for deriving the symmetry transform of a shape P having n points with respect to rotational symmetry of order n (C_n-symmetry). Mathematical derivation and proof can be found in [8]. This method transforms P into a regular n-gon, keeping the centroid in place.

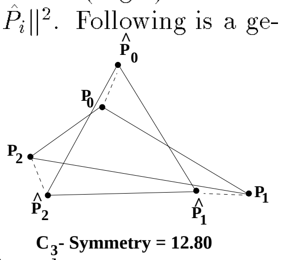

Figure 3: The symmetry transform of $\{P_0, P_1, P_2\}$ is $\{\hat{P}_0, \hat{P}_1, \hat{P}_2\}$. CSM $= \frac{1}{3}\Sigma_{i=0}^{2}||P_i - \hat{P}_i||^2$

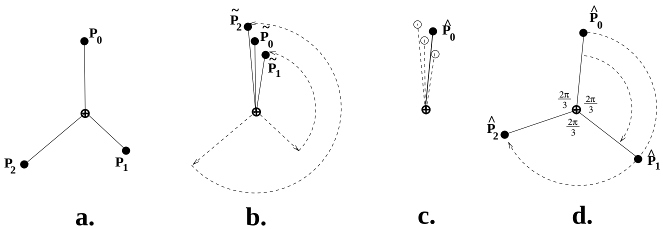

Figure 4: The C_3-symmetry Transform of 3 points: a) original 3 points $\{P_i\}_{i=0}^{2}$. b) Fold $\{P_i\}_{i=0}^{2}$ into $\{\tilde{P}_i\}_{i=0}^{2}$. c) Average $\{\tilde{P}_i\}_{i=0}^{2}$ obtaining $\hat{P}_0 = \frac{1}{3}\sum_{i=0}^{2}\tilde{P}_i$. d) Unfold the average point obtaining $\{\hat{P}_i\}_{i=0}^{2}$.

1. *Fold* the points $\{P_i\}_{i=0}^{n-1}$ by rotating each point P_i counterclockwise about the centroid by $2\pi i/n$ radians (Fig. 4a).
2. Let $\hat{P}_0$ be the average of the points $\{\tilde{P}_i\}_{i=0}^{n-1}$.
3. *Unfold* the points, obtaining the C_n-symmetric points $\{\hat{P}_i\}_{i=0}^{n-1}$ by duplicating $\hat{P}_0$ and rotating clockwise about the centroid by $2\pi i/n$ radians (Fig. 4b).

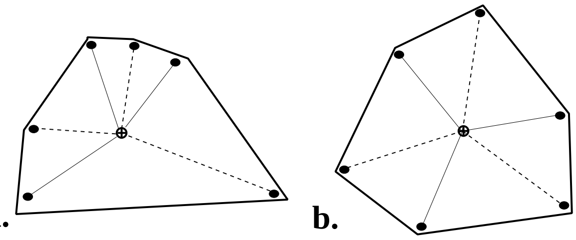

Figure 5: Geometric description of the C_3-symmetry transform for a 6-sided polygon. The centroid of the polygon is marked by $\oplus$. a) The original polygon shown as two sets of 3 points. b) The C_3-symmetric shape obtained.

A 2D shape P having qn points is represented as q sets $\{S_r\}_{r=0}^{q-1}$ of n interlaced points $S_r = \{P_{rn+i}\}_{i=0}^{n-1}$. The C_n-symmetry transform of P is obtained by applying the above algorithm to each set of n points seperately, where the folding is performed about the centroid of all the points (Fig. 5). The procedure for evaluating the symmetry transform for mirror-symmetry is similar (see [7]).

As symmetry has been defined on a sequence of points, representing a given shape by points must precede the application of the symmetry transform. There are several ways to select a sequence of points to represent general 2D shapes (see [7]). One method represents a shape by points selected along its contour at equal angular intervals about the centroid of the shape (Figure 6).

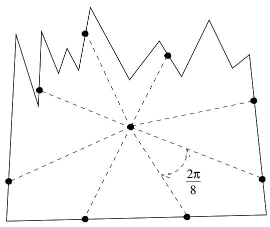

Figure 6: Points are distributed along the contour at regular angular intervals around the centroid.

3 Symmetry of Occluded Shapes

When a symmetric object is partially occluded, we use the symmetry measure to evaluate the symmetry of the occluded shapes, locate the center of symmetry and reconstruct the symmetric shape most similar to the unoccluded original.
As described in the previous section, a shape is represented by points selected at regular angular intervals (angular selection) about the centroid. Angular selection of points about a point other than the centroid will give a different symmetry measure value. We define the **center of symmetry** of a shape as that point

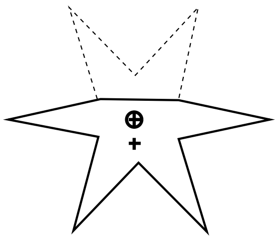

Figure 7: The symmetry value obtained by angular selection about the center of mass (marked by +) is greater than the symmetry value obtained by angular selection about the center of symmetry (marked by ⊕).

about which angular selection gives the minimum symmetry measure value. When a symmetric shape is not occluded the center of symmetry aligns with the centroid of the shape. However, the center of symmetry of truncated or occluded objects does not align with its centroid but aligns with the (unknown) centroid of the unoccluded shape. Thus the center of symmetry of a shape is robust under truncation and occlusion.

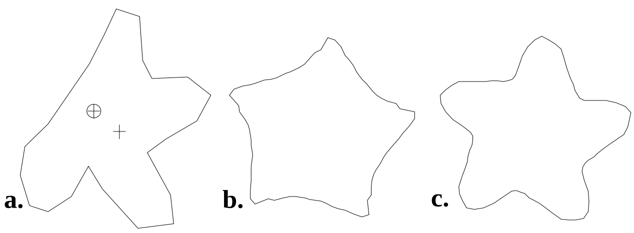

Figure 8: a) Original occluded shape, its centroid (+) and its center of symmetry(⊕). b,c) The closest C_5-symmetric shapes following angular selection about the centroid (b) and about the center of symmetry (c).

To locate the center of symmetry, we use an iterative procedure of gradient descent that converges from the centroid of an occluded shape to the center of symmetry. Denote by center of selection that point about which points are selected using angular selection. We initialize the iterative process by setting the centroid as the center of selection. At each step we compare the symmetry value of points angularly selected about the center of selection and about points in its immediate neighborhood. That point about which angular selection gives minimum symmetry value, is set to be the new center of selection. If center of selection does not change the neighborhood size is decreased. The process is terminated when neighborhood size reaches a predefined minimum size. The center of selection at the end of the process is taken as the center of symmetry.
The closest symmetric shape obtained by angular se-

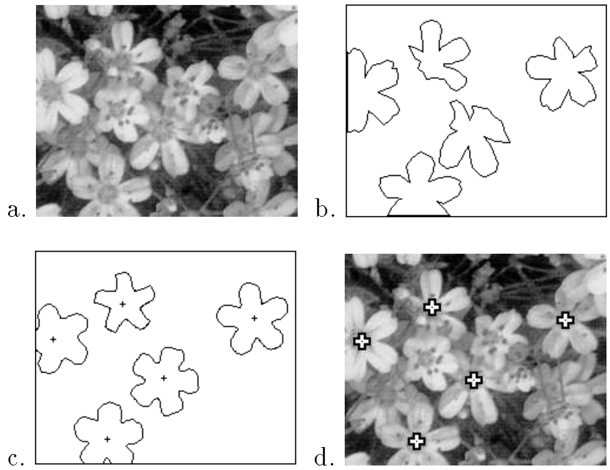

Figure 9: a-b) A collection of occluded asymmetric flowers. c) The closest symmetric shapes and their center of symmetry. d) The center of symmetry of the occluded flowers are marked by '+'.

lection about the center of symmetry is visually more similar to the original than that obtained by angular selection about the centroid of the occluded shape (Fig. 8). We can reconstruct the symmetric shape closest to the unoccluded shape by obtaining the symmetry transform of the occluded shape following angular selection about the center of symmetry (see Figure 8c). In Figure 9 the center of symmetry and the closest symmetric shapes were found for several occluded flowers.

The process of reconstructing the occluded shape can be improved by altering the method of evaluating the symmetry of a set of points. As described in Section 2 the symmetry of a set of points is evaluated by folding about the centroid, averaging and unfolding about the centroid. We alter the method as follows:

1. The folding and unfolding (steps 1 and 3) will be performed about the center of selection rather than about the centroid.

2. Rather than averaging the folded points (step 2), we apply other clustering methods. In practice we average over the folded points ommitting the extremes (see Figure 10).

The improvement in reconstruction of an occluded shape is shown in Figure 11. This method improves both the shape and the localization of the reconstruction.

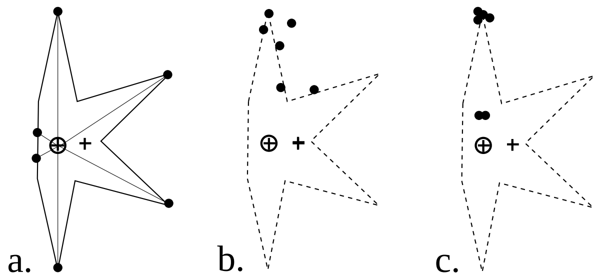

Figure 10: Improving the averaging of folded points:
a) An occluded shape and 6 points selected using angular selection about the center of symmetry (marked as $\oplus$).
b) folding the points about the centroid of the shape (marked as $+$), points are clustered sparsely.
c) folding the points about the center of symmetry of the shape, points are clustered tightly. Eliminating the extremes (cluster of two points) and averaging will result in smaller averaging error and better reconstruction.

4 Conclusion

In this paper we evaluated the deviation from perfect symmetry of incomplete data. We described a method for finding the center of symmetry of occluded shapes and reconstructing the symmetric shape closest to the unoccluded original. The methods can be easily extended to higher dimensions and to more complex symmetry classes.

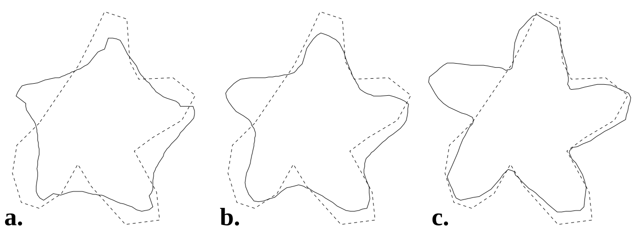

Figure 11: Reconstruction of an occluded almost symmetric shape. The original shape is shown as a dashed line. The reconstructed shape is shown as a solid line.
a) The closest symmetric shape following angular selection about the centroid. b) The closest symmetric shape following angular selection about the center of symmetry. c) The closest symmetric shape following angular selection about the centroid and altered symmetry evaluation (see text).

References

[1] F. Attneave. Symmetry information and memory for patterns. *American Journal of Psychology*, 68:209–222, 1955.

[2] M. Bornstein and J. Stiles-Davis. Discrimination and memory for symmetry in young children. *Developmental Psychology*, 17:82–86, 1984.

[3] E. Howe, J. Powell, K. Jung, and C. Brandau. Time course of encoding of patterns varrying in array size and symmetry. *Bulletin of the Psychonomic Society*, 27(5):437–440, 1989.

[4] W. Miller. *Symmetry Groups and their Applications.* Academic Press, 1972.

[5] H. Weyl. *Symmetry.* Princeton Univ. Press, 1952.

[6] H. Zabrodsky, S. Peleg, and D. Avnir. Hierarchical symmetry. In *International Conference on Pattern Recognition*, volume C: Image, Speech, and Signal Analysis, pages 9–12, The Hague, August-September 1992.

[7] H. Zabrodsky, S. Peleg, and D. Avnir. A measure of symmetry based on shape similarity. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 703–706, Champaign, June 1992.

[8] H. Zabrodsky, S. Peleg, and D. Avnir. Continuous symmetry measures. *J. Am. Chem. Soc.*, 114:7843–7851, Sept 1992.