



Radiance and Irradiance Units				
	<u>Term</u>	Defining Equation	Application	<u>SI Unit</u>
	Radiant Flux	$F = \frac{\Delta Q}{\Delta t}$	Total quantity of light emitted from a point	watt
	Radiant Intensity	$I = \frac{\Delta F}{\Delta \omega}$	Total quantity of light emitted from a point in a given solid angle	watt /sr
	Irradiance	$E = \frac{\Delta F}{\Delta A_r}$	Light density incident on a plane	watt /m ²
	Radiance	$L = \frac{\Delta I}{\Delta A_s \cos(\theta)}$	Light emitted or reflected from an extended source in a given direction	watt /sr/m ²
Q = energy (joules) t = time (sec) $\omega = solid angle (steradian)$ A = area (meter2) $\theta = angle incident to plane$ watt=joule/sec				
http://www.calculator.org/properties/luminance.prop				



Radiometry – Photometry
$X_{v} = K_{m} \int X_{1}(\lambda) V(\lambda) d\lambda$ Photometric term Radiometric term
$V(\lambda)$ is the Photopic Luminous Efficiency function ($Y(\lambda)$).
This equation represents a weighting, wavelength by wavelength of the radiant spectral term by the visual response at that wavelength. The constant K_m is a scaling factor = 683 lm/W.
Basic Unit in Photometry is the Lumen and the Candela
Monochromatic light 555nm with radiant intensity of 1 Watt = 683 Lumens.
Monochromatic light 555nm with radiant intensity of 1 Watt/sr = 683 Candela.

Radiometry – Photometry

$X_v = K_m \int X_1(\lambda) V(\lambda) d\lambda$

For *X*, we can pair the Radiometric and Photometric pairs:

QUANTITY	RADIOMETRIC	PHOTOMETRIC
power	Radiant Flux watt (W)	Luminous Flux lumen (Im)
power per unit solid angle	Radiant Intensity W/sr	Luminous Intensity Im/sr = candela (cd)
power per unit area	Irradiance W/m ²	Illuminance Im/m ² = lux (lx)
power per area per solid angle	Radiance W/m²/sr	Luminance Im/m²/sr = cd/m² = nit

Luminance and Illuminance Units			
<u>Term</u>	Defining Equation	Application	<u>SI Unit</u>
Luminous Flux	$F_{v} = K_{m} \int F_{e}(\lambda) V(\lambda) d\lambda$	Total quantity of light emitted from a point	lumen
Luminous Intensity	$I_{v} = \frac{\Delta F_{v}}{\Delta \omega}$	Total quantity of light emitted from a point in a given solid angle	candela (cd)
Illuminance	$E_{v} = \frac{\Delta L_{v}}{\Delta A_{r}}$	Light density incident on a plane	lumens/m ² (lux)
Luminance	$L_{v} = \frac{\Delta I}{\Delta A_{s} \cos(\theta)}$	Light emitted or reflected from an extended source in a given direction	cd/m ²
Q = energy (joules) t = time (sec) $\omega = \text{solid angle (steradian)}$ $A = \text{area (meter}^2)$ $\theta = \text{ angle incident to plane}$ watt=joule/sec			
watt-joure/sec http://www.electro-optical.com/whitepapers/candela.htm			



Photometry of	Scenes: Lum	inance
Luminance of outside sc	enes (cd/m²)	
Sun	6x10 ⁸	
Visual saturation	49,000	
Just below saturation	25,000	2
Outdoor building façad	le 10,000 🛛 🚿	~
Blue sky (morning)	4,600	
Concrete sidewalk		
in sun	3,200	
in shadow	570	
in deep shado	w 290	
Lumina	ance of interior scene	es (cd/m²)
Interi	or room (fluorescent	lighting)
<u>e</u>	floor/walls	90
	in shadow	10
Interi	or room (no lighting)	
i l	floor/walls	30
	in shadow	5
	in closet door	1





















































Illuminant + Surface reflectance Estimation

Linear Models

Judd et al '64 Cohen '64 Maloney '86 Marimont & Wandell '93

Assume: Likely Illuminants and Surfaces

Represent Illuminants and Surfaces using low dimensional linear representation



Linear Model - Illuminants
Find a basis of SPDs
$e_1(\lambda) e_2(\lambda) e_3(\lambda) \dots$
such that a linear combination gives a good approximation for every illuminant.
Chose a linear model that minimizes:
$\sum_{\lambda} [\mathbf{e}(\lambda) - \sum_{i=1}^{n} \omega_{i} \mathbf{e}_{i}(\lambda)]^{2}$
For all illuminants $e(\lambda)$.
n = dimensions of the linear model (# of basis finctions)















Simple Illuminant Estimation
e(I) = illuminant to be estimated
3 color sensors: $R_1(\lambda) R_2(\lambda) R_3(\lambda)$ $B_e = [e_1(\lambda) e_2(\lambda) e_3(\lambda)]$ matrix of illuminant basis
Given the sensor responses: $r_i = \sum_i R_i(\lambda) e(\lambda)$ $i = 1,2,3$
In matrix representation:
$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} \hline & R_1 \\ \hline & R_2 \\ \hline & R_3 \end{bmatrix} \begin{bmatrix} e \\ e \end{bmatrix}$
r = R e
$\mathbf{r} = \mathbf{R}(\mathbf{B}_{e}\boldsymbol{\omega}) = (\mathbf{R}\mathbf{B}_{e})\boldsymbol{\omega}$ $\mathbf{R}\mathbf{B}_{e} = \text{matrix } 3x3$
estimate $\boldsymbol{\omega}$: $\boldsymbol{\omega} = (\mathbf{RB}_{e})^{-1}\mathbf{r}$
estimate \mathbf{e} : $\mathbf{e} = \mathbf{B}_{\mathbf{e}} \boldsymbol{\omega}$









Surface and Illumination Estimation of a Scene

Simplifying Assumptions:

- 1) Likelihood of surfaces and illuminants are given (for example using linear models).
- 2) Illuminant does not change rapidly over the scene.
- 3) Sensor sensitivities are known.

Problem: assuming a 3D linear model for surface reflection and for illuminants, find 3 surface reflection coefficients for each of the p points in the scene.

Case 1: Illuminant is known

If the illuminant **e** is given, then there are 3p measurements and 3p unknowns. For every point :

$$\begin{split} \mathbf{r}_{1} &= \sum_{\lambda} \mathsf{R}_{1}(\lambda) \mathbf{e}(\lambda) \; \mathbf{s}(\lambda) = \sum_{\lambda} \mathsf{R}_{1}(\lambda) \mathbf{e}(\lambda) \sum_{j=1}^{3} \sigma_{j} \mathbf{s}_{j}(\lambda) \\ \mathbf{r}_{2} &= \sum_{\lambda} \mathsf{R}_{2}(\lambda) \mathbf{e}(\lambda) \; \mathbf{s}(\lambda) = \sum_{\lambda} \mathsf{R}_{2}(\lambda) \mathbf{e}(\lambda) \sum_{j=1}^{3} \sigma_{j} \mathbf{s}_{j}(\lambda) \\ \mathbf{r}_{3} &= \sum_{\lambda} \mathsf{R}_{3}(\lambda) \mathbf{e}(\lambda) \; \mathbf{s}(\lambda) = \sum_{\lambda} \mathsf{R}_{3}(\lambda) \mathbf{e}(\lambda) \sum_{j=1}^{3} \sigma_{j} \mathbf{s}_{j}(\lambda) \end{split}$$

Surface and Illumination Estimation of a Scene $r_{1} = \sum_{\lambda} R_{1}(\lambda)e(\lambda) \sum_{j=1}^{3} \sigma_{j}s_{j}(\lambda)$ $r_{2} = \sum_{\lambda} R_{2}(\lambda)e(\lambda) \sum_{j=1}^{3} \sigma_{j}s_{j}(\lambda)$ $r_{3} = \sum_{\lambda} R_{3}(\lambda)e(\lambda) \sum_{j=1}^{3} \sigma_{j}s_{j}(\lambda)$ As a matrix equation: $\mathbf{r} = \mathbf{\Lambda}_{\mathbf{e}} \mathbf{\sigma}$

where the (i,j) entry of matrix Λ_e is

$$\sum \mathsf{R}_{i}(\lambda) \mathbf{e}(\lambda) \mathbf{s}_{i}(\lambda)$$

Solving for σ :

if n=3 then $\sigma = \Lambda_{e}^{1} \mathbf{r}$

if n>3 then $\sigma = \Lambda_e^* r$ (Λ_e^* is the pseudo-inverse)

 Λ_{e} is the surface matrix for illumination e.

Case 2: Illuminant is unknown, 1 surface is known if illuminant is unknown: p points in scene 3p sensor responses are given 3p + 3 unknowns Assume surface $s(\lambda)$ is known. $\mathbf{r}_{1} = \sum_{\lambda} \mathbf{R}_{1}(\lambda) \ \mathbf{e}(\lambda) \ \mathbf{s}(\lambda) = \sum_{\lambda} \mathbf{R}_{1}(\lambda) \sum_{j=1}^{3} \mathbf{0}_{j} \mathbf{e}_{j}(\lambda) \ \mathbf{s}(\lambda)$ $\mathbf{r}_2 = \sum_{\lambda} \mathbf{R}_2(\lambda) \mathbf{e}(\lambda) \mathbf{s}(\lambda) = \sum_{\lambda} \mathbf{R}_2(\lambda) \sum_{j=1}^{3} \omega_j \mathbf{e}_j(\lambda) \mathbf{s}(\lambda)$ $r_3 = \sum_{\lambda} R_3(\lambda) \ e(\lambda) \ s(\lambda) = \sum_{\lambda} R_3(\lambda) \sum_{j=1}^{3} \omega_j e_j(\lambda) \ s(\lambda)$ As a matrix equation: $\mathbf{r} = \Lambda_s \boldsymbol{\omega}$ where the (i,j) entry of matrix Λ_s is $\sum_{\lambda} \mathsf{R}_{i}(\lambda) \mathsf{e}_{i}(\lambda) \mathsf{s}(\lambda)$ Compute Λ_s for the known s(\lambda), then solve for $_{\omega}:$ $\omega=\Lambda_s^{^{-1}}r$ $\mathbf{e}(\lambda) = \sum_{i=1}^{3} \boldsymbol{\omega}_{i} \mathbf{e}_{i}(\lambda)$ Calculate the illuminant: Proceed as in Case 1.





Option 3: Variations on Gray world assumption Average of all surface in scene is not exactly gray. The more colors in scene and larger std - the more likely to average to gray.

(Lee '01, Lam '04)

























Illumination Correction Examples:
Tungsten ───→ Blue Sky
$\mathbf{r} = \begin{bmatrix} 0.8119 & 0.2271 & 0.0550 \\ -0.0803 & 1.1344 & 0.1282 \\ 0.0429 & -0.0755 & 1.8091 \end{bmatrix} \mathbf{r}'$
Notice diagonal elements are dominant.
$ \begin{bmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{bmatrix} $ Compensation for illuminants using pure diagonal transformation = scaling of sensor responses = Von Kries Coefficient Law

Illumination Correction

Given two images, find the illuminant transformation M between them.

Assume r_i and r'_i are sensor outputs in the two images corresponding to same surface reflectance.

$$r_i = M r'_i$$

Build sensor response matrixes:

$$\begin{bmatrix} | & | & | \\ r_1 & r_2 & \cdots & r_n \\ | & | & | \end{bmatrix} = \mathbf{M} \begin{bmatrix} | & | & | \\ r_1^* & r_2^* & \cdots & r_n^* \\ | & | & | \end{bmatrix}$$

Solve

$$M = \begin{bmatrix} | & | & | \\ r_1 & r_2 & \cdots & r_n \\ | & | & | \end{bmatrix} \begin{bmatrix} | & | & | \\ r_1^* & r_2^* & \cdots & r_n^* \\ | & | & | \end{bmatrix}^*$$









