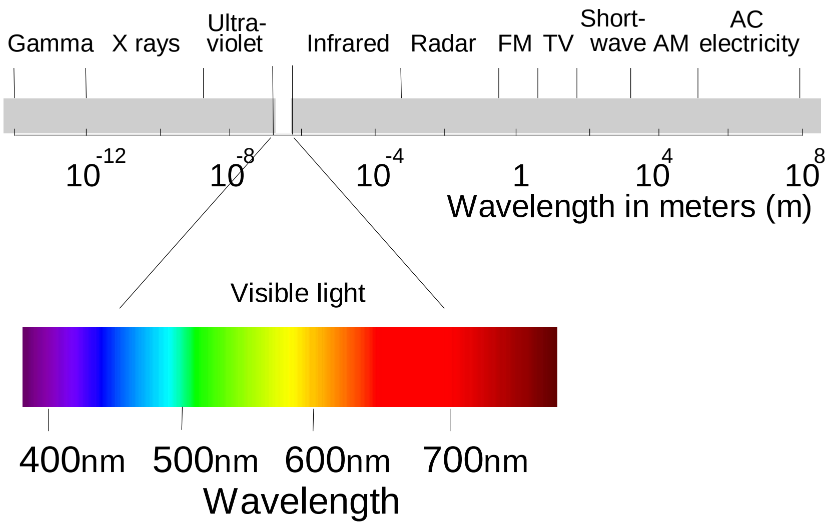
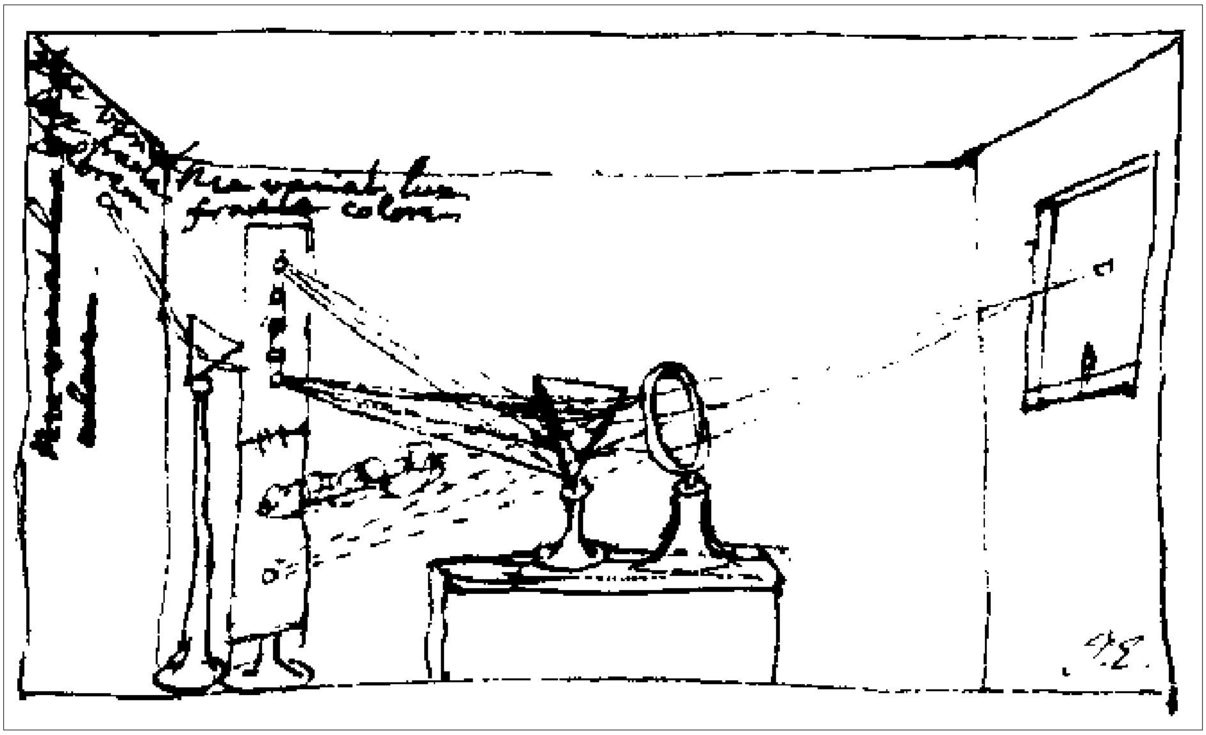
Lecture 2

Color Basics

Wavelength Encoding
Trichromatic Color Theory
Color Matching Experiments

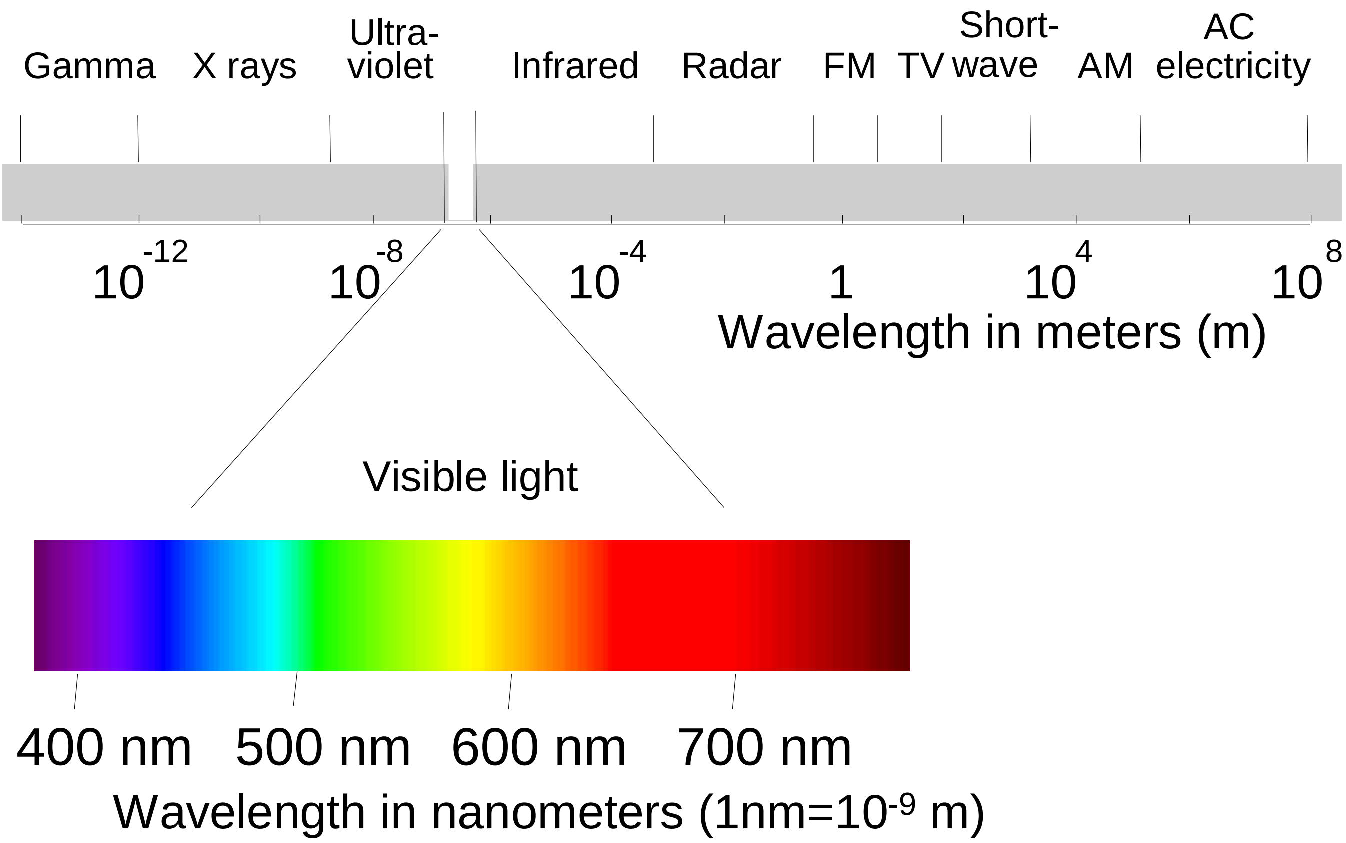




Newton's Experiment

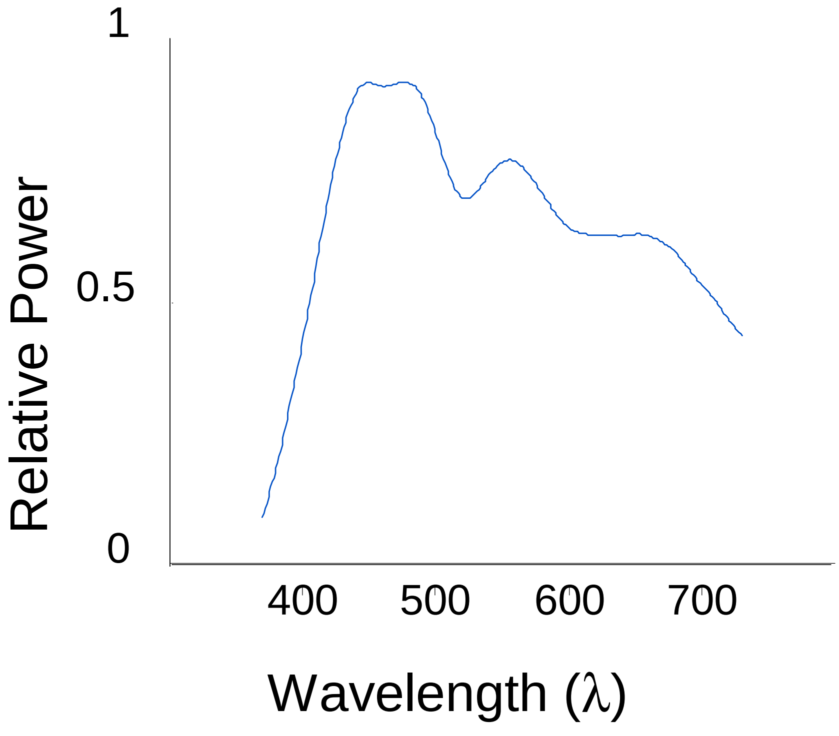
1665, Cambridge University

Electromagnetic Radiation - Spectrum



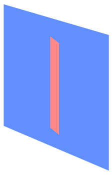
Spectral Power Distribution

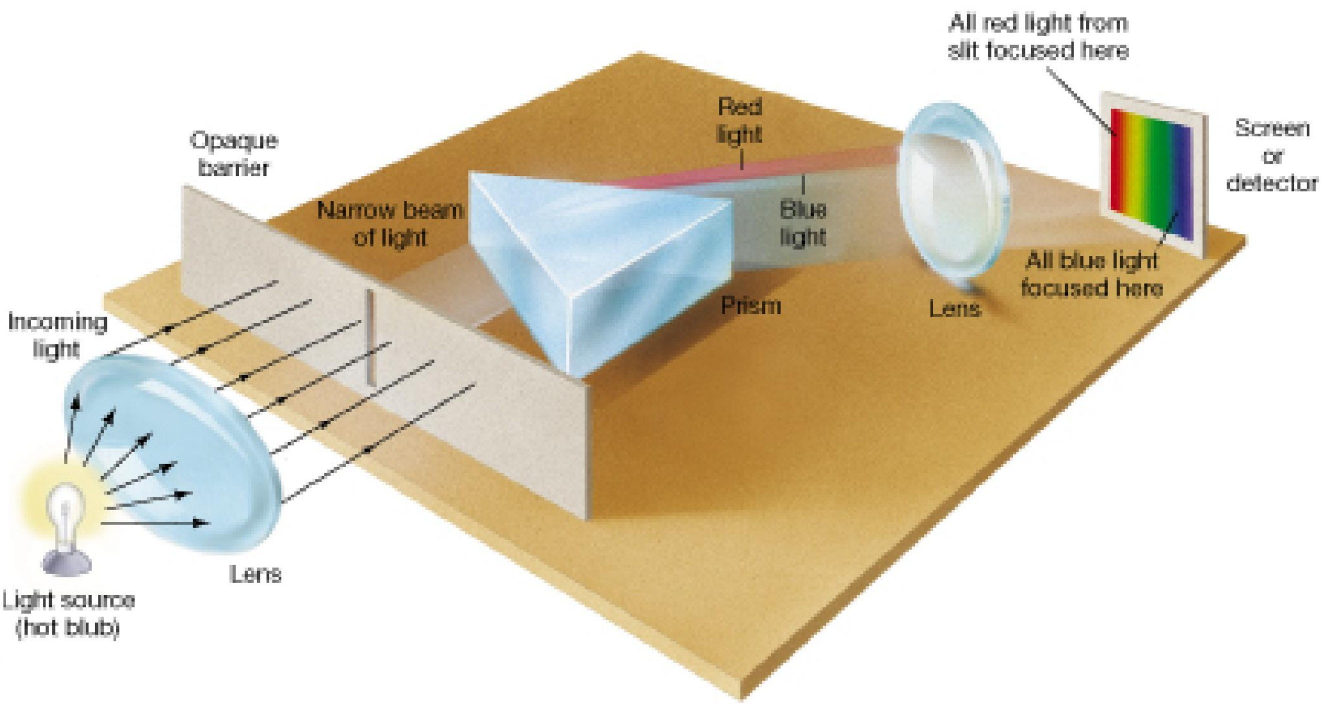
The **Spectral Power Distribution** (SPD) of a light is a function f(l) which defines the energy at each wavelength.

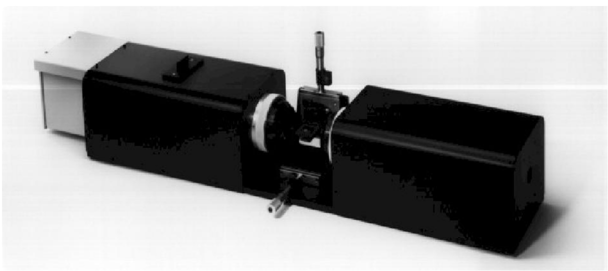


Monochromators

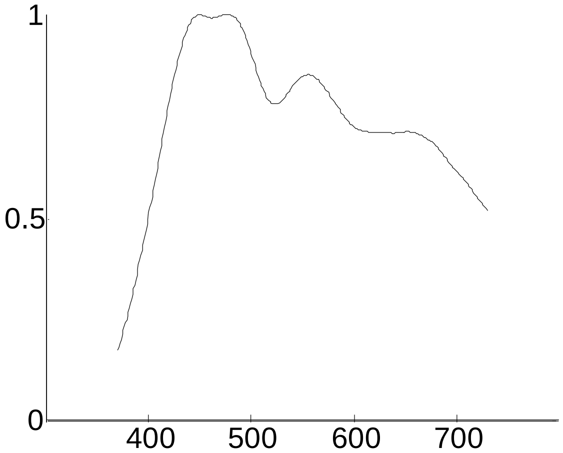
Monochromators measure the power or energy at different wavelengths



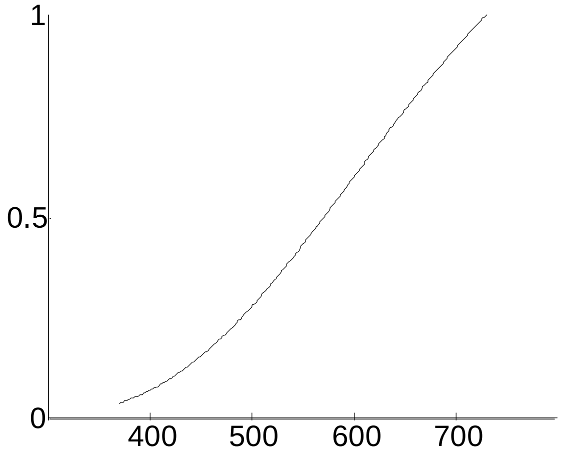




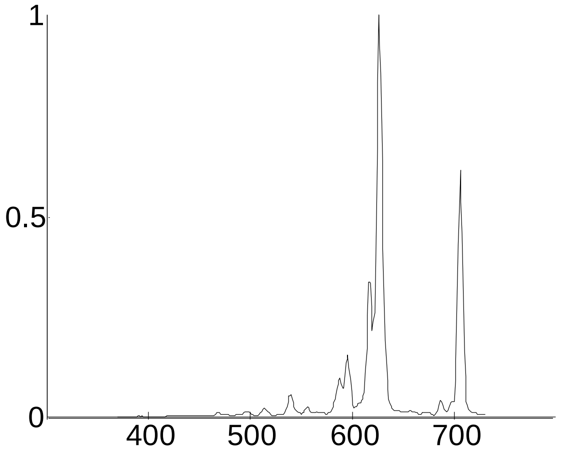
Examples of Spectral power Distributions



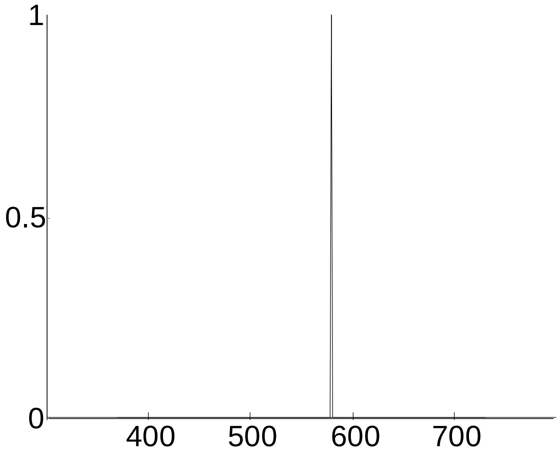
Blue Skylight



Tungsten bulb

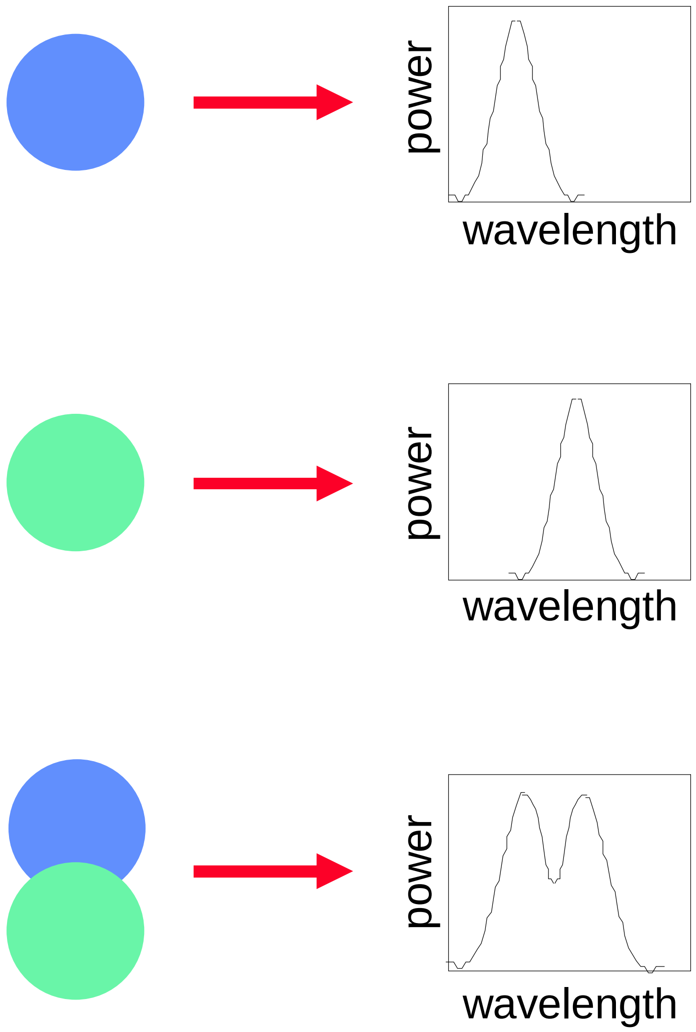


Red monitor phosphor



Monochromatic light

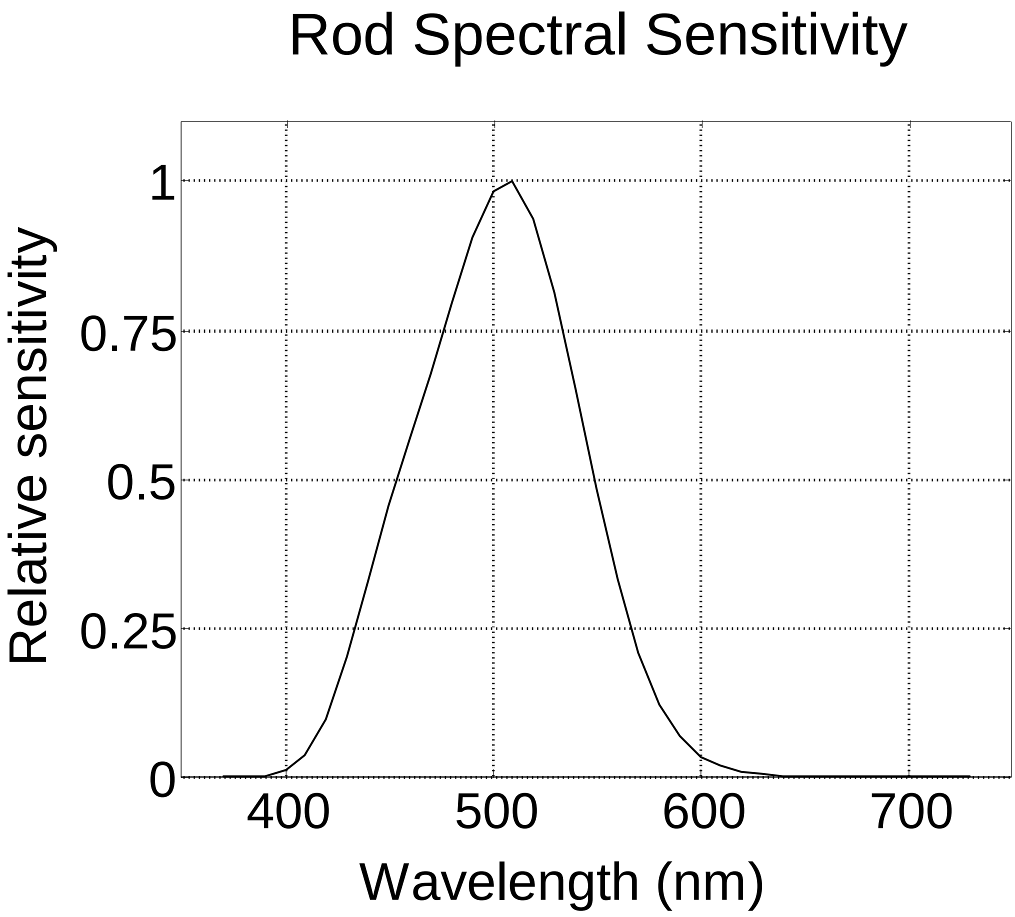
Superposition of Light SPDs



Retinal Photoreceptors

Rods -

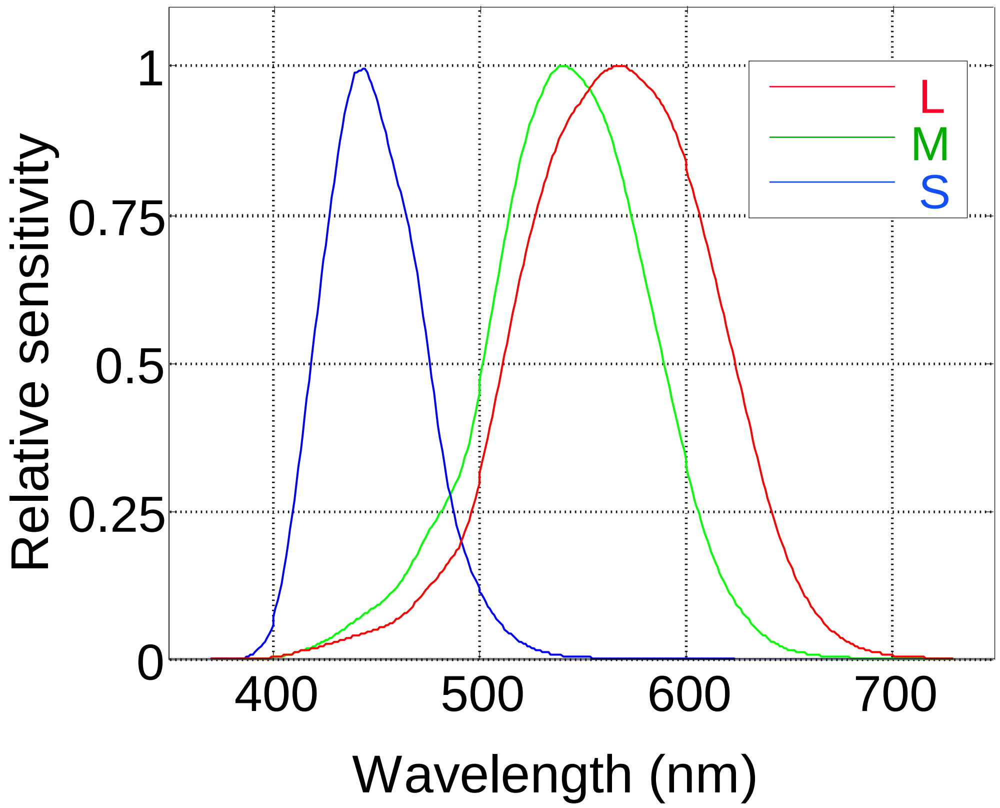
- Low illumination levels (Scotopic vision).
- Highly sensitive (respond to a single photon).
- 100 million rods in each eye.
- No rods in fovea.



Retinal Photoreceptors

Cones -
- High illumination levels (Photopic vision)
- Less sensitive than rods.
- 5 million cones in each eye.
- Only cones in fovea (aprox. 50,000).
- Density decreases with distance from fovea.
- 3 cone types differing in their spectral sensitivity: L , M, and S cones.

Cone Spectral Sensitivity



L and M Cones -

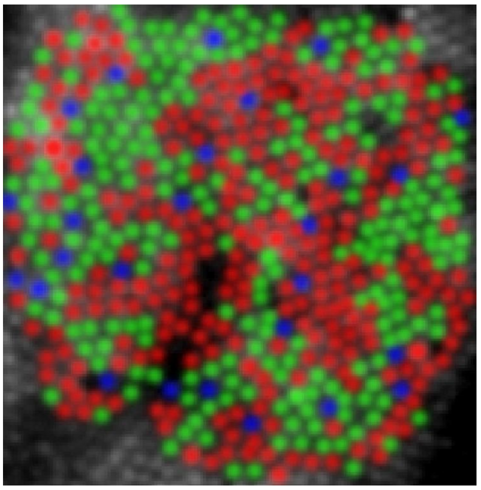
Density decreases with distance from fovea.

None past 40 deg.

S Cones -

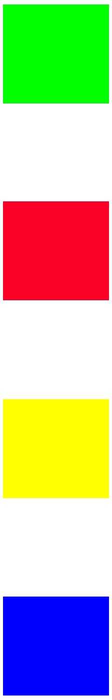
None in the fovea (central 25’).

Very sparse elsewhere.



Sensitivity to color decreases with distance from fovea in the order: green, red, yellow, blue.

Color Sensitivity test – distance from fovea



Color Deficiency

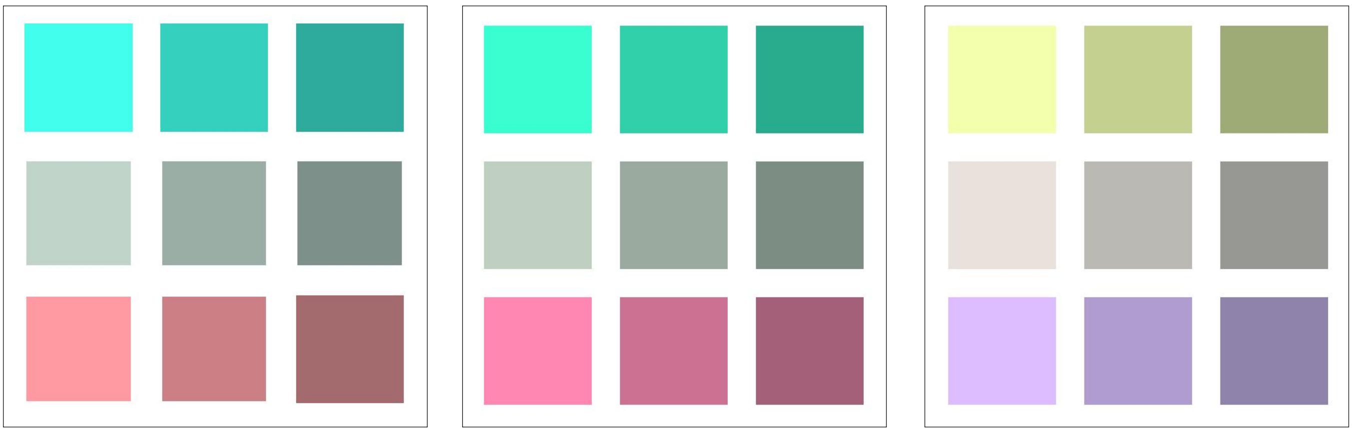
Trichromats - use 3 sensors

Dichromats - use 2 sensors (8% males, 0.05% females)

- protanopia - missing red cone
- deuteranopia - missing green cone
- tritanopia - missing blue cone

Monochromats - use 1 sensor.

Dichromatic confusions:

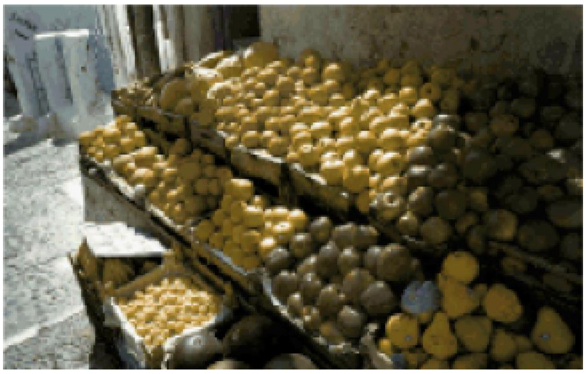


protanopia deuteranopia tritanopia

Color Deficiency

Normal

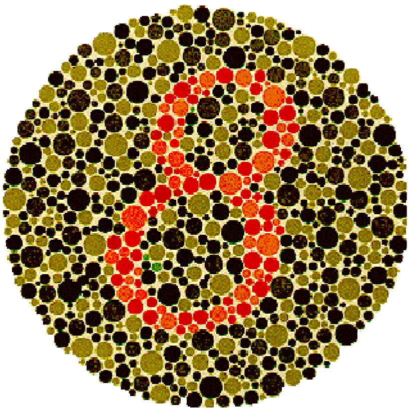
Protanope

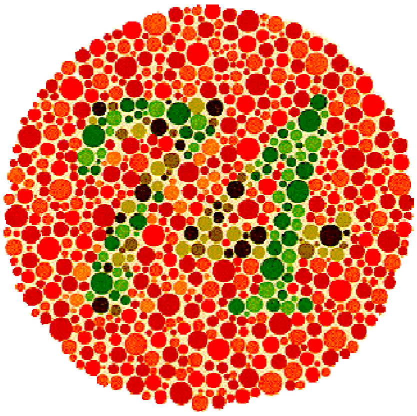


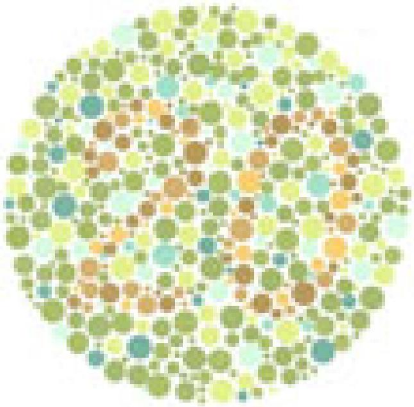
Deuteranope

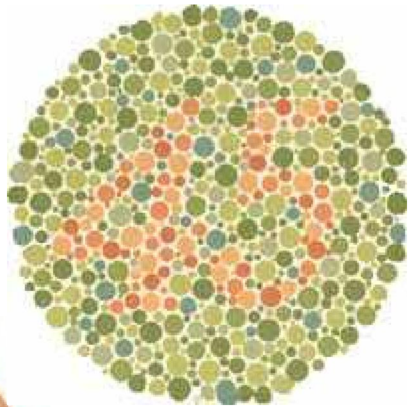
Tritanopia

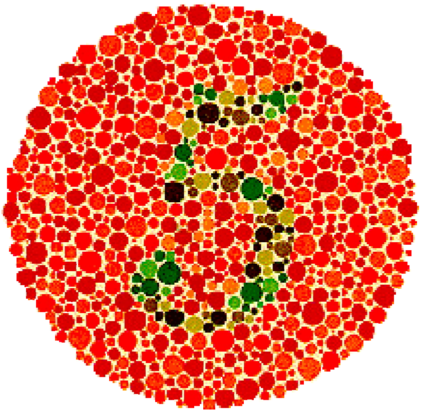
Ishihara Plates (1917).

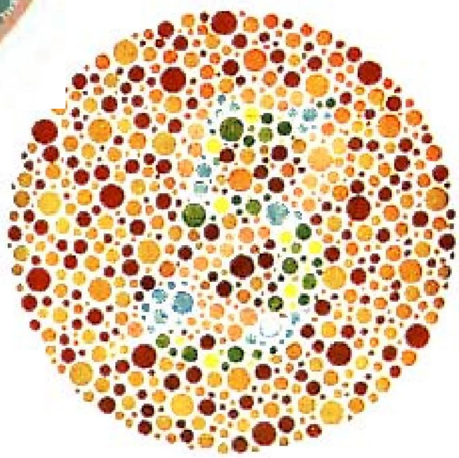




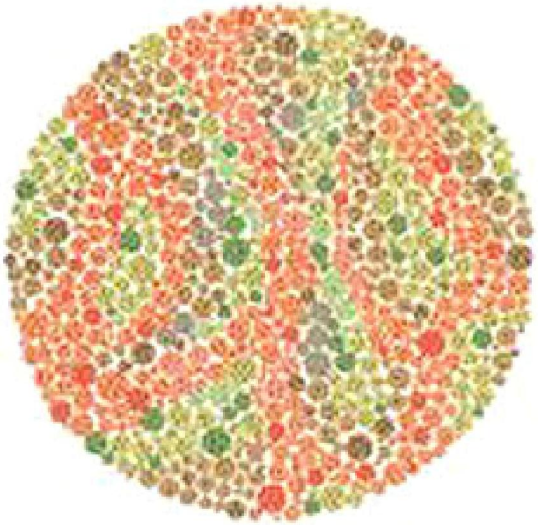




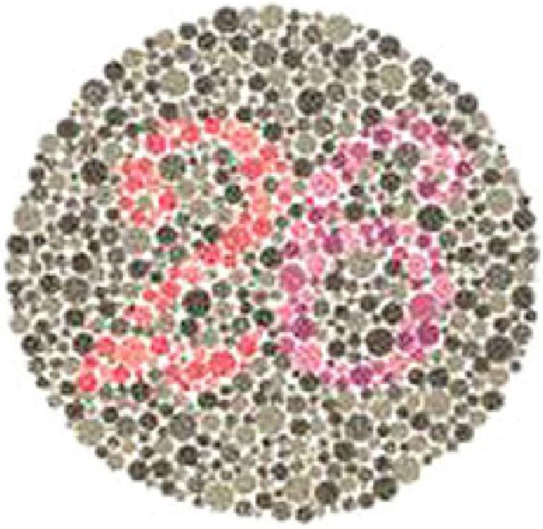




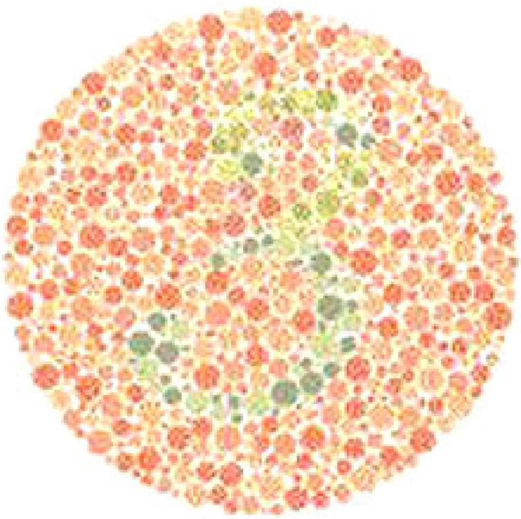
Reverse Ishihara Plates



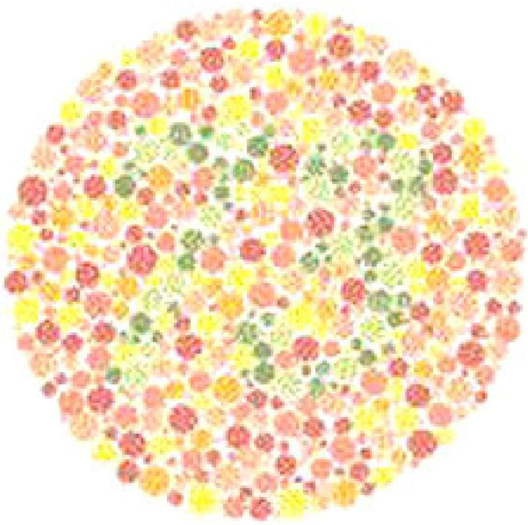
Normal Should see Nothing.
CVD should see 5



Normal Should see both 2 and 6
Deutanopes should see 2 more easily
Protanopes should see 6 more easily



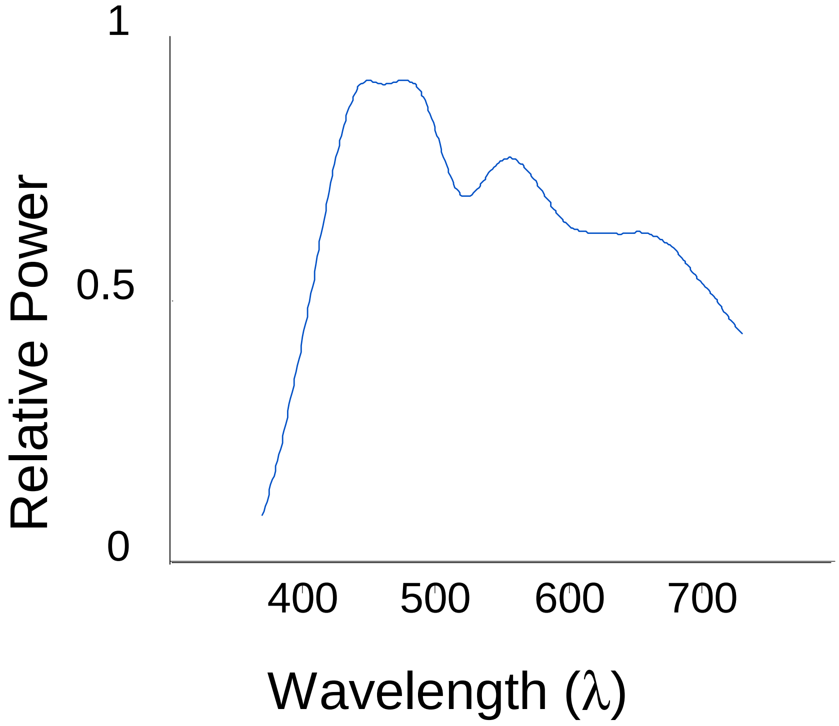
Normal Should see 3.
CVD should see 5



Normal Should see 73.
CVD should nothing.

Spectral Power Distribution

High dimensional data



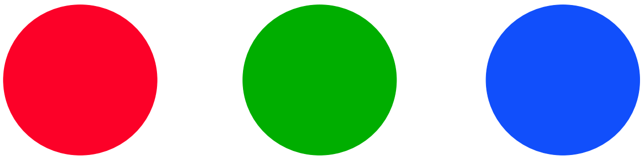
RGB – 3 dimensional

Trichromatic Color Theory

Trichromatic: "tri"=three "chroma"=color
color vision is based on three primaries
(i.e., it is 3 dimensional).

Thomas Young (1773-1829) -
A few different retinal receptors operating with different wavelength sensitivities will allow humans to perceive the number of colors that they do.
Suggested 3 receptors.

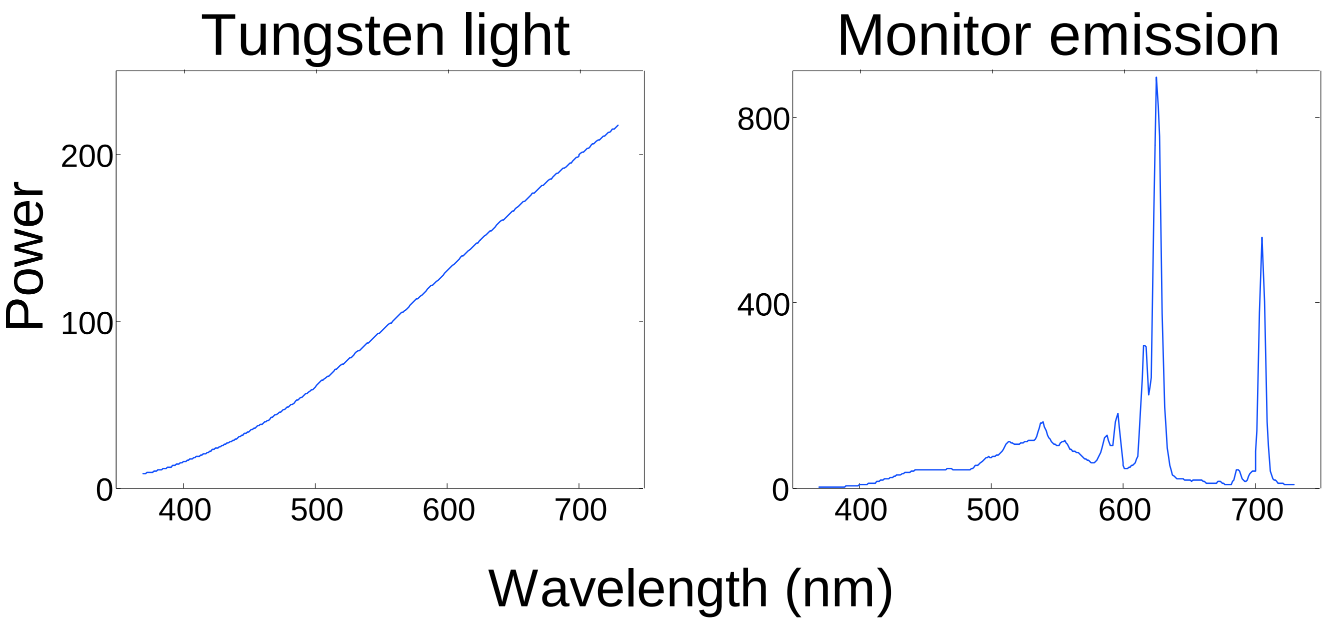
Helmholtz & Maxwell (1850) -
Color matching with 3 primaries.



Color Matching Experiment

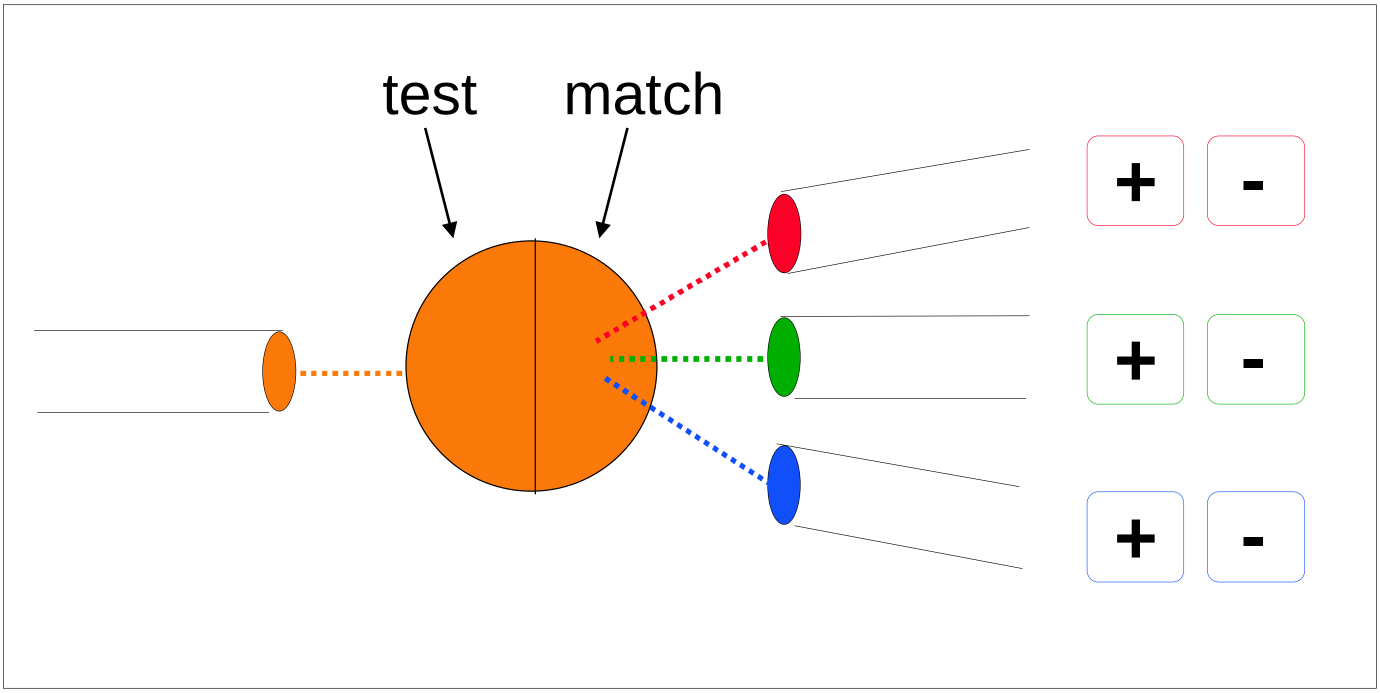
Thomas Young 1802
Helmholtz & Maxwell 1850
Wright 1929
Stiles & Burch 1959
Judd & Wyszeki 1975

Metamer - two lights that appear the same visually. They might have different SPDs (spectral power distributions).

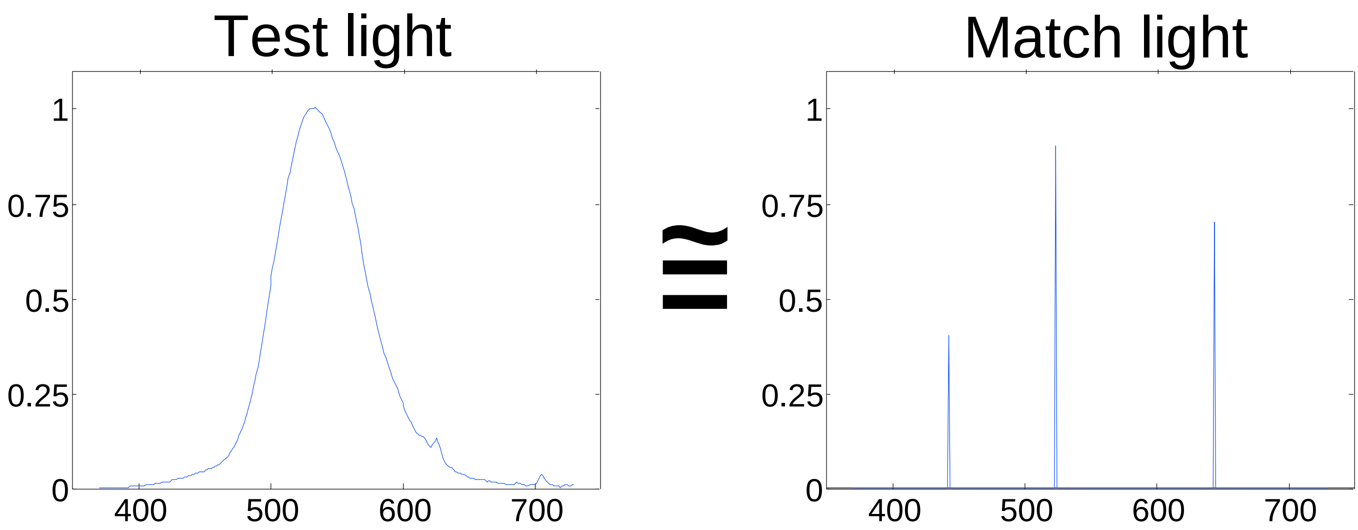


The phosphors of the monitor were set to match the tungsten light.

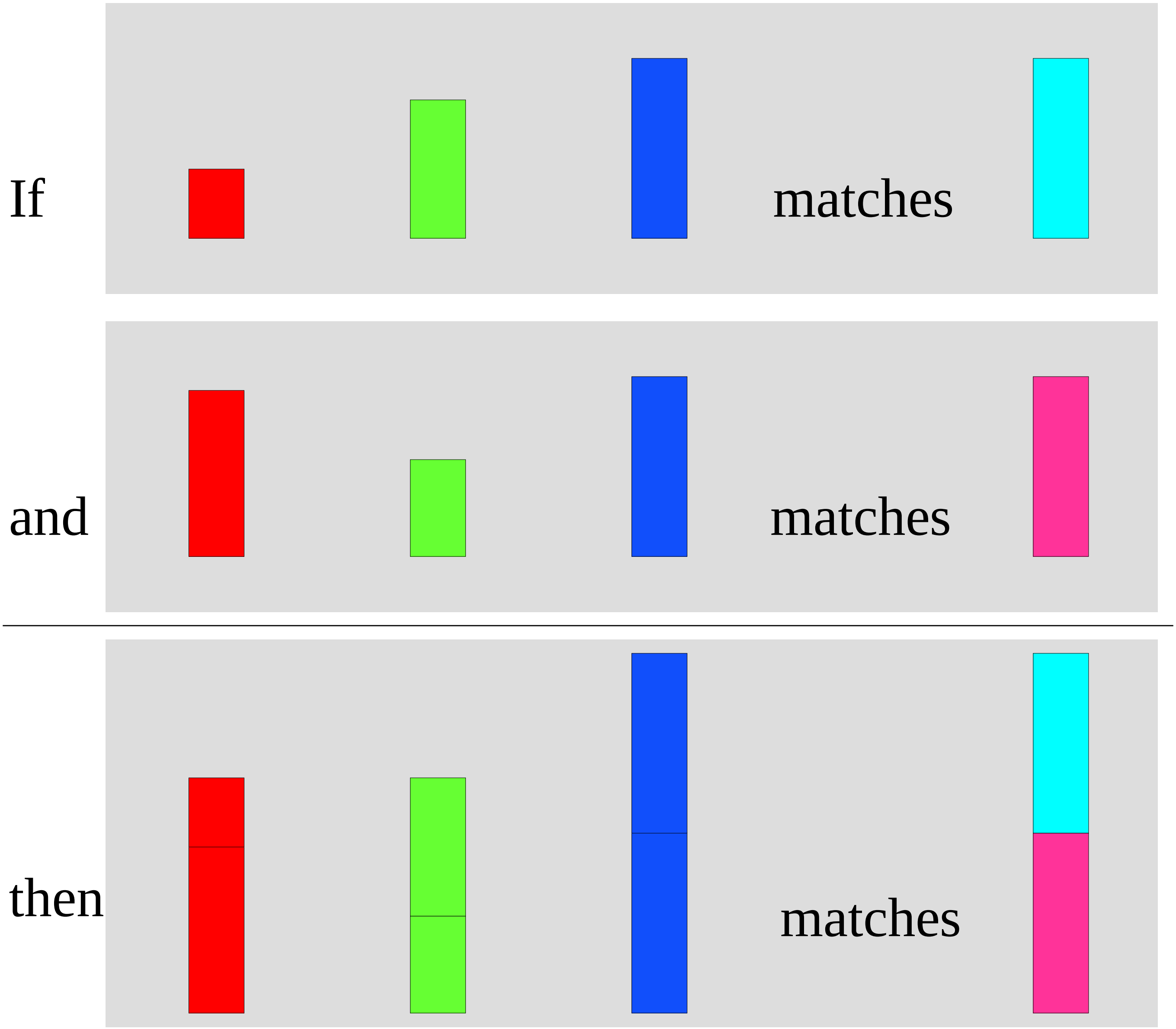
Color Matching Experiment



Three primary lights are set to match a test light.



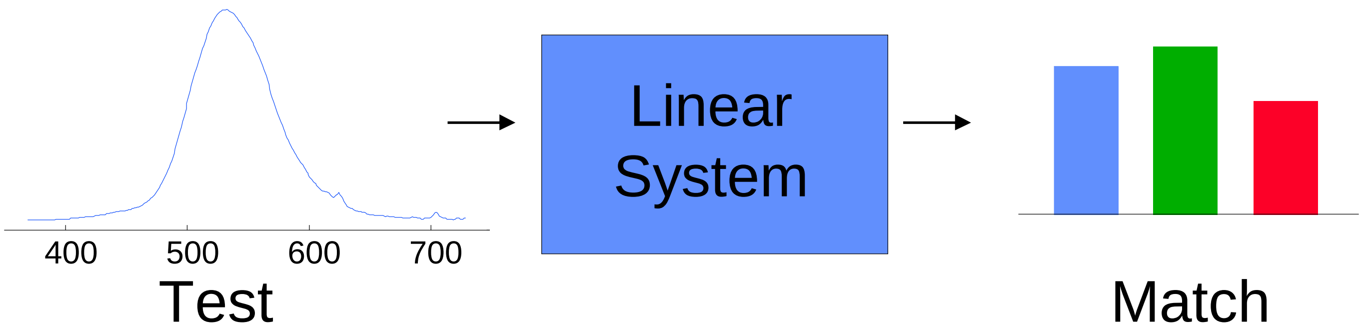
Color Matching Experiment is Linear



Homogeneity + additivity

Color Matching Experiment is a Linear System

There exists a system matrix that maps test SPD to Match intensities.

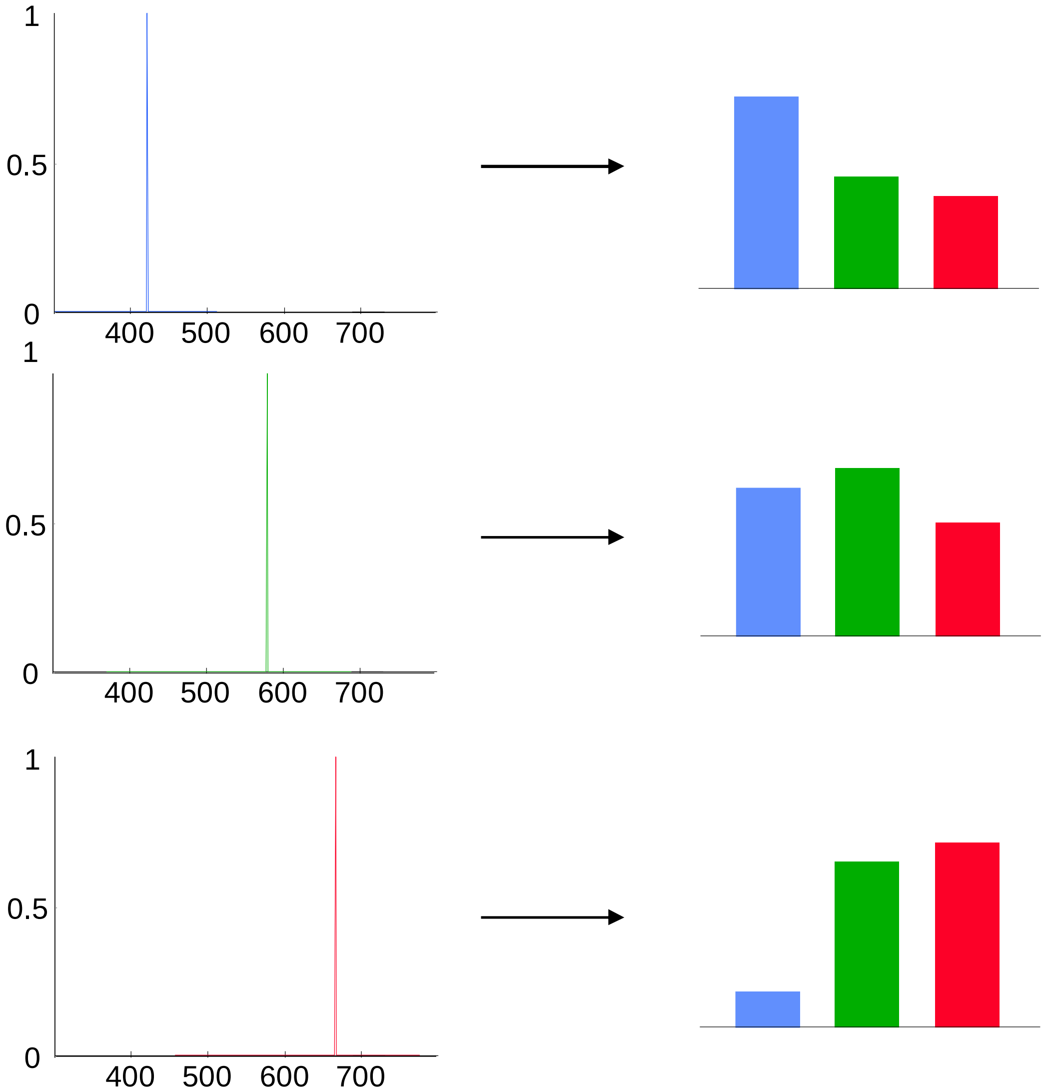


$$C\,T = M$$

Color Matching Functions

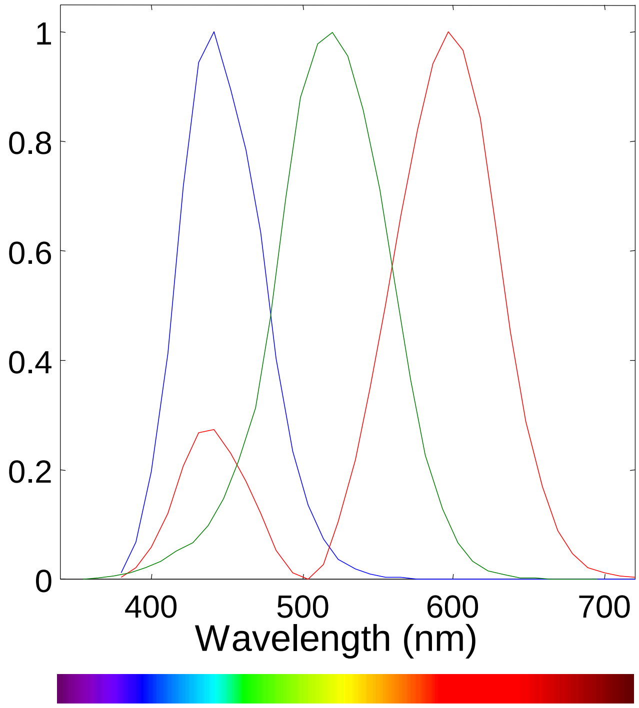
Given a set of primaries, one can determine for every spectral wavelength, the intensity of the guns required to match a monochromatic light of that spectral wavelength.

Monochromatic lights | Primary Intensities



Color Matching Functions

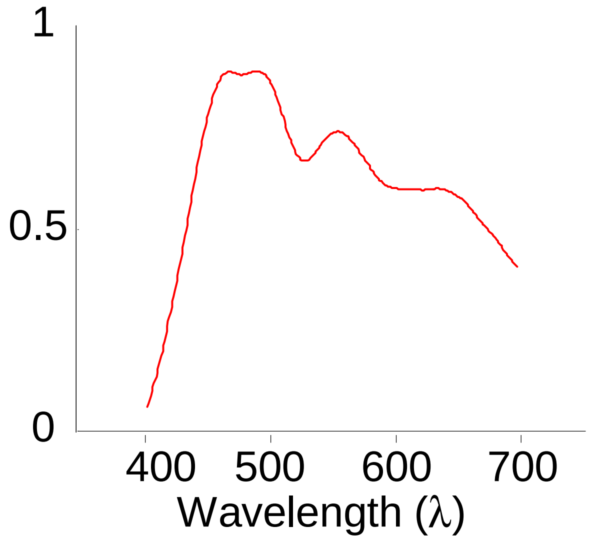
These values form the Color Matching Functions associated with the primaries.



The intensity of the primaries required to match any spectra can then be determined by inner product of the spectra with the 3 color matching functions.

Color Matching Functions

Test Light = sum of monochromatic lights



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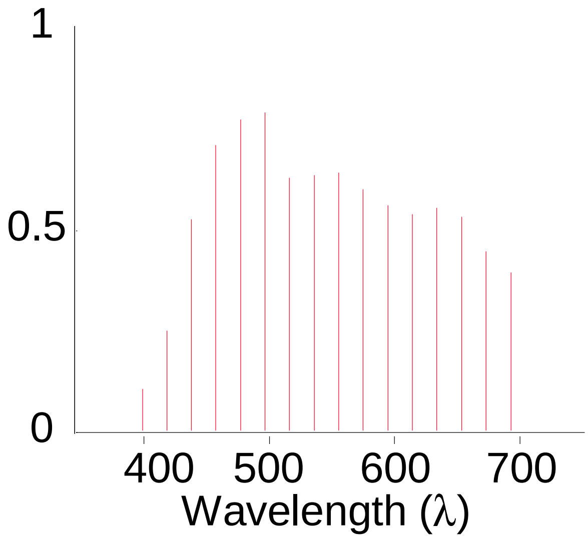
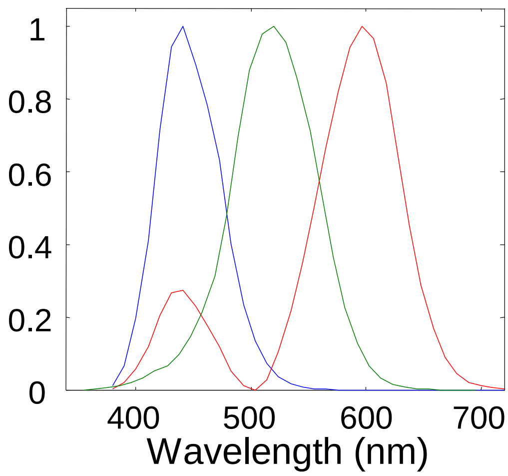
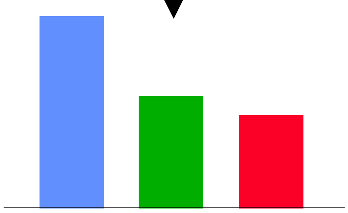


Color Matching Functions

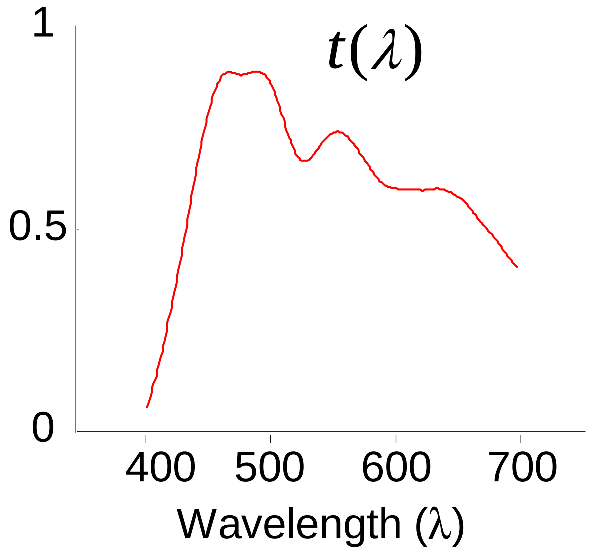


Tristimulus Values

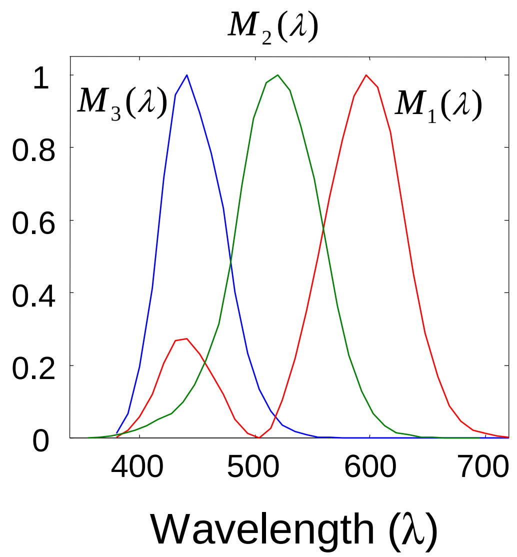


Tristimulus Calculation

Tristimulus Values = Inner product of SPD and CMF



● dot product



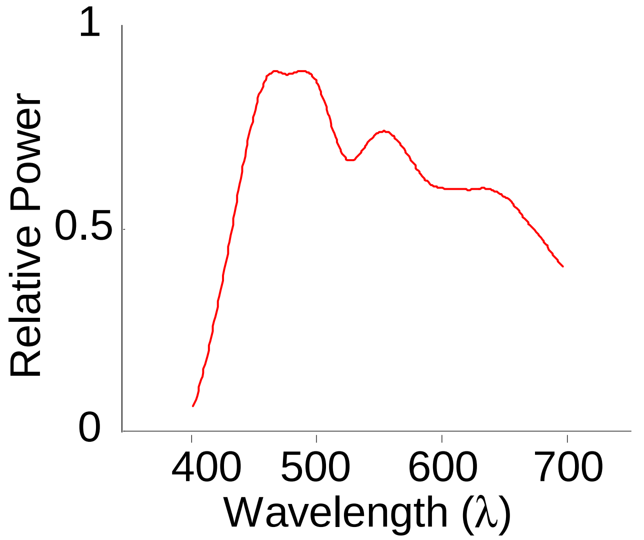
$$e_1 = \int_\lambda t(\lambda) M_1(\lambda) d\lambda$$

$$e_2 = \int_\lambda t(\lambda) M_2(\lambda) d\lambda$$

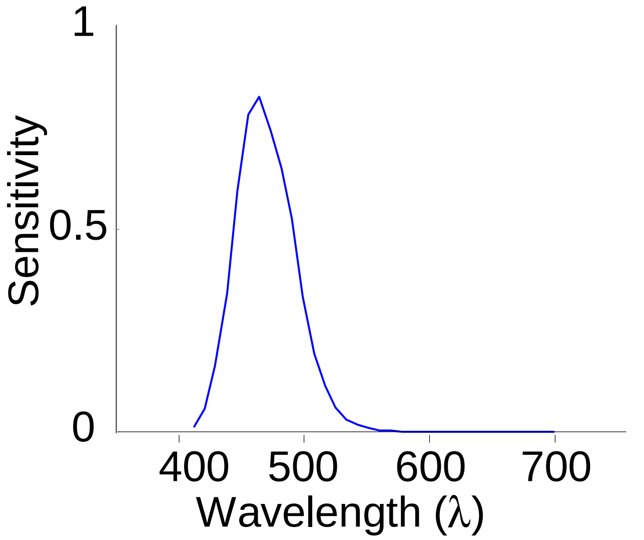
$$e_3 = \int_\lambda t(\lambda) M_3(\lambda) d\lambda$$

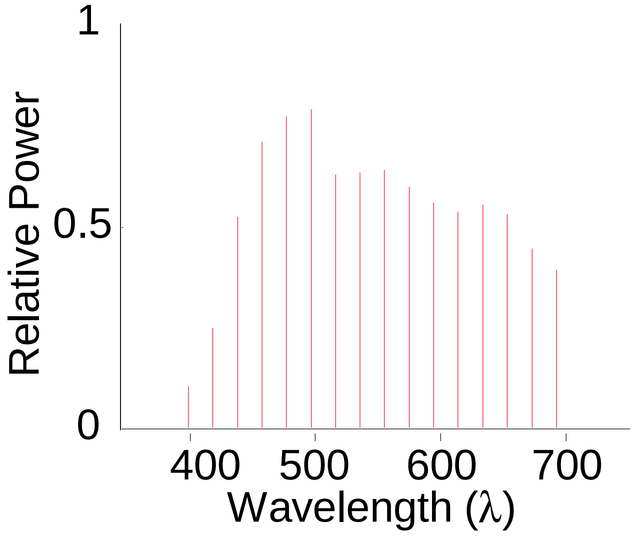
Matrix Representation of the color matching system

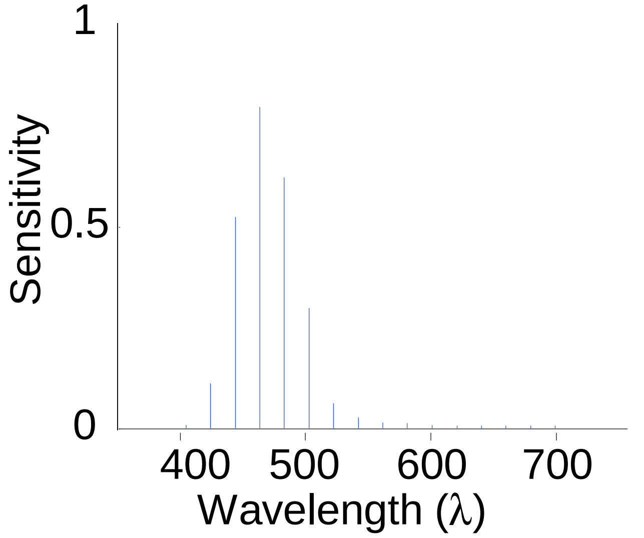
Light

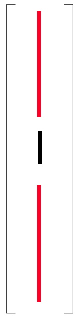


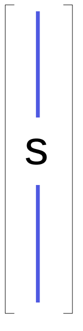
Sensor



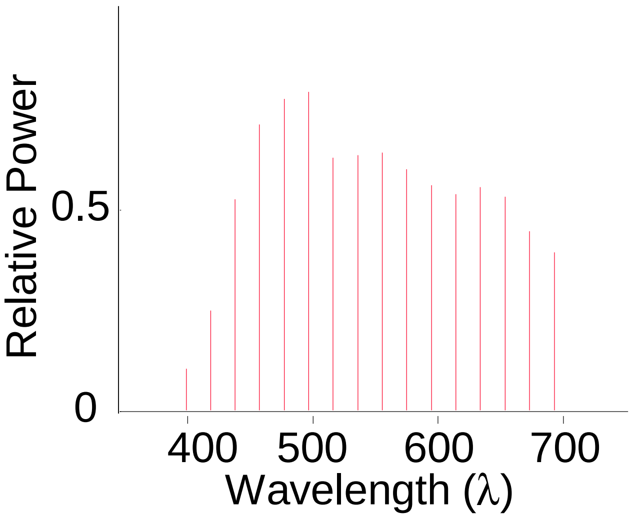


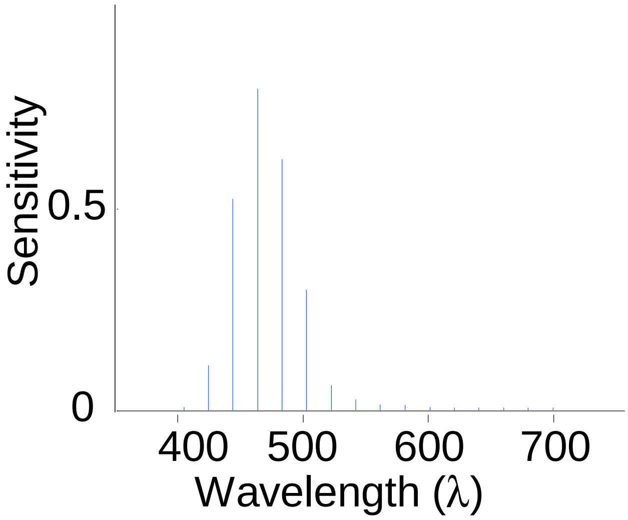




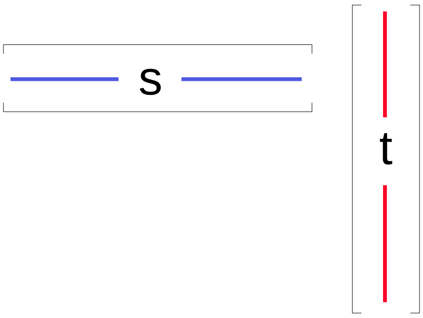


Sensor Response Calculation

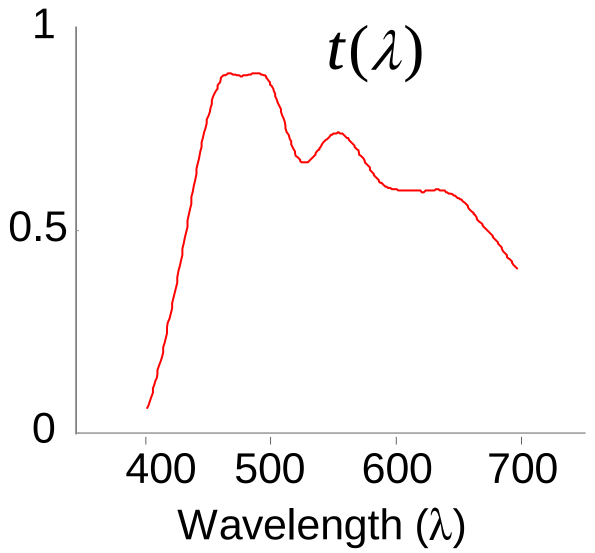




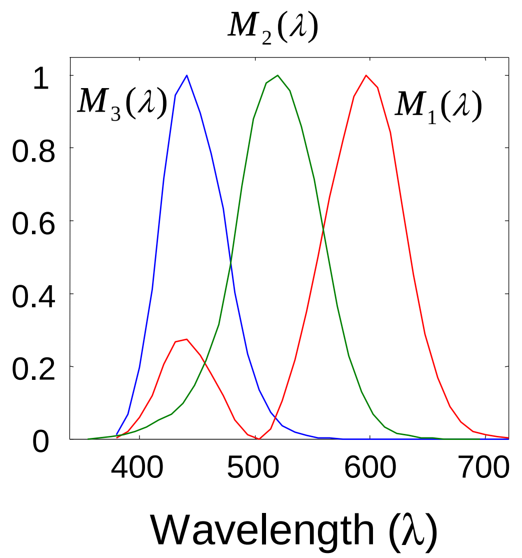
$e = \langle s,t \rangle = s^T t =$



Tristimulus Calculation

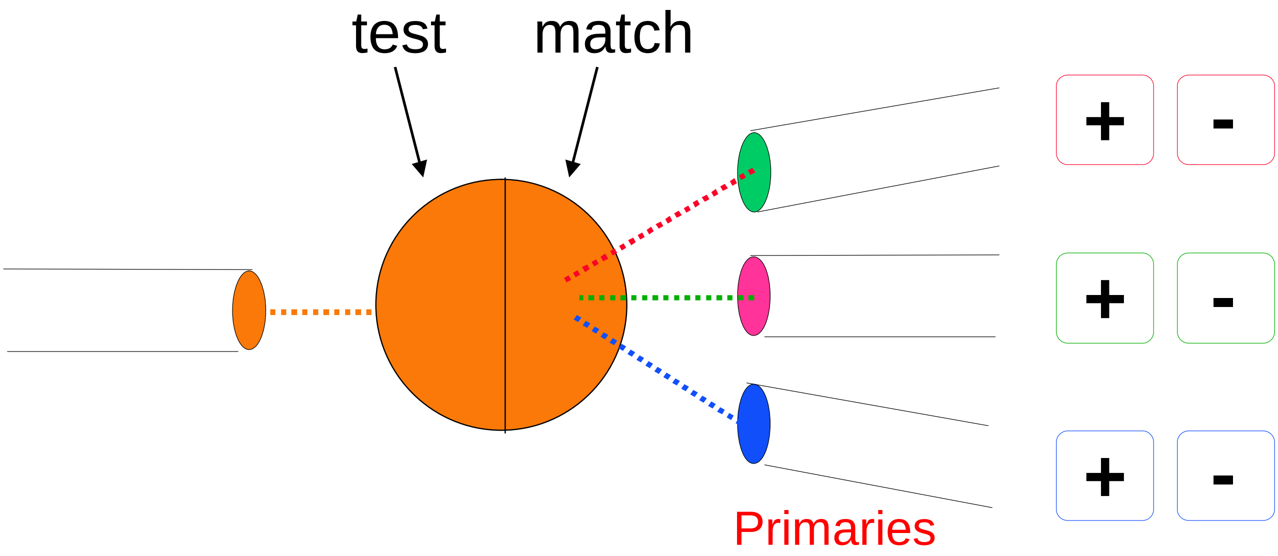


●
dot
product



$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} — M_1 — \\ — M_2 — \\ — M_3 — \end{bmatrix} \begin{bmatrix} | \\ t \\ | \end{bmatrix}$$

Color Matching Experiment



$$\begin{bmatrix} | \\ t \\ | \end{bmatrix} \cong \begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} \bullet \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Spectra | Primaries | Tristimulus values

Color Matching is Linear

Color Matching is a linear system.
i.e.
Color Matching defines a linear mapping from the test spd (n_λ x 1 vector) to 3 primary intensities (3 x 1 vector).

$$\begin{bmatrix} | \\ t \\ | \end{bmatrix} \longrightarrow \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Spectra

Tristimulus values

Thus there must exist a 3 x n_λ system matrix that maps input to output:

$$\boxed{\mathbf{e = C\ t}}$$

$$\boxed{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \text{---} & M1 & \text{---} \\ \text{---} & M2 & \text{---} \\ \text{---} & M3 & \text{---} \end{bmatrix} \bullet \begin{bmatrix} | \\ t \\ | \end{bmatrix}} \quad \updownarrow n_\lambda$$

$$\longleftarrow n_\lambda \longrightarrow$$

Calculating the system matrix

Measure intensities of primaries for monochromatic test lights:

$$\mathbf{t_1} = [1\ 0\ .\ .\ .\ 0\ ]^t \longrightarrow \mathbf{e_1}$$
$$\vdots \qquad\qquad \vdots$$
$$\mathbf{t_2} = [0\ 0\ .\ .\ .\ 1\ ]^t \longrightarrow \mathbf{e}_{\mathbf{n}_\lambda}$$

$\mathbf{e}_\mathbf{i}$ are the columns of the color matching system matrix **C.**

$$\mathbf{C} = [\,\mathbf{e_1} \cdots\ \mathbf{e}_{\mathbf{n}_\lambda}\,]$$

Now, given the color matching functions, we can calculate the response to any test light **t** :

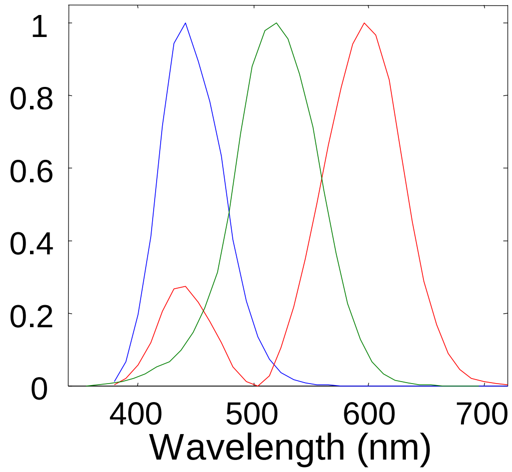
$$\mathbf{e = C\ t}$$

Color Matching Functions

The rows of the system matrix are the
color matching functions
with respect to the given primaries.

$$\mathbf{C} = \begin{bmatrix} \text{——} & M_1 & \text{——} \\ \text{——} & M_2 & \text{——} \\ \text{——} & M_3 & \text{——} \end{bmatrix}$$

Color
Matching
Functions



Note an important relationship between Primaries and their Matching Functions:

$$\begin{bmatrix} \text{——— } M_1 \text{ ———} \\ \text{——— } M_2 \text{ ———} \\ \text{——— } M_3 \text{ ———} \end{bmatrix} \bullet \begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

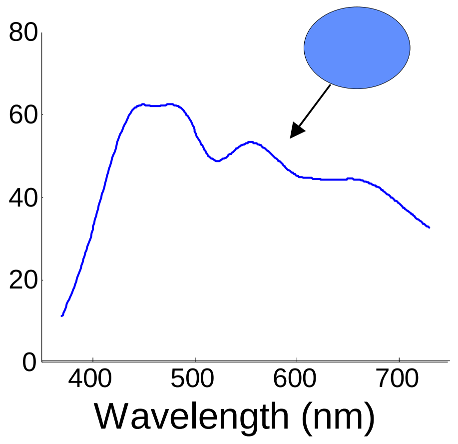
Matching Functions

Primaries

Trichromatic Color Theory

“tri”=three “chroma”=color

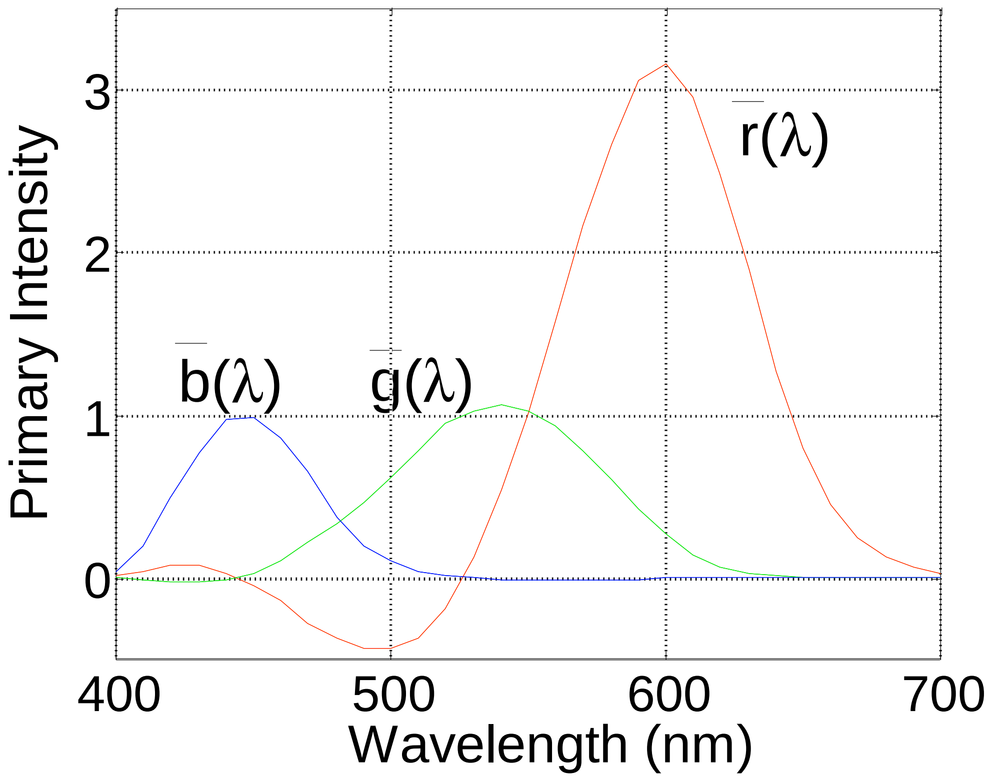
Every color can be represented by 3 values.



$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

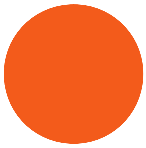
Space of visible colors is 3 Dimensional.

Color Representation

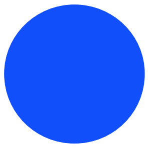


Stiles & Burch (1959) Color matching functions.
Primaries are: 444.4 525.3 645.2
10 deg field..

Given the color matching functions, we can describe any light with 3 values (CIE-RGB):



(85, 38, 10)



(21, 45, 72)

(65, 54, 73)

Caveat: For some matches e_i may be negative.

e.g.:

$$\mathbf{t} \cong e_1\,\mathbf{p_1} + e_2\,\mathbf{p_2} + e_3\,\mathbf{p_3} \qquad e_1 < 0$$

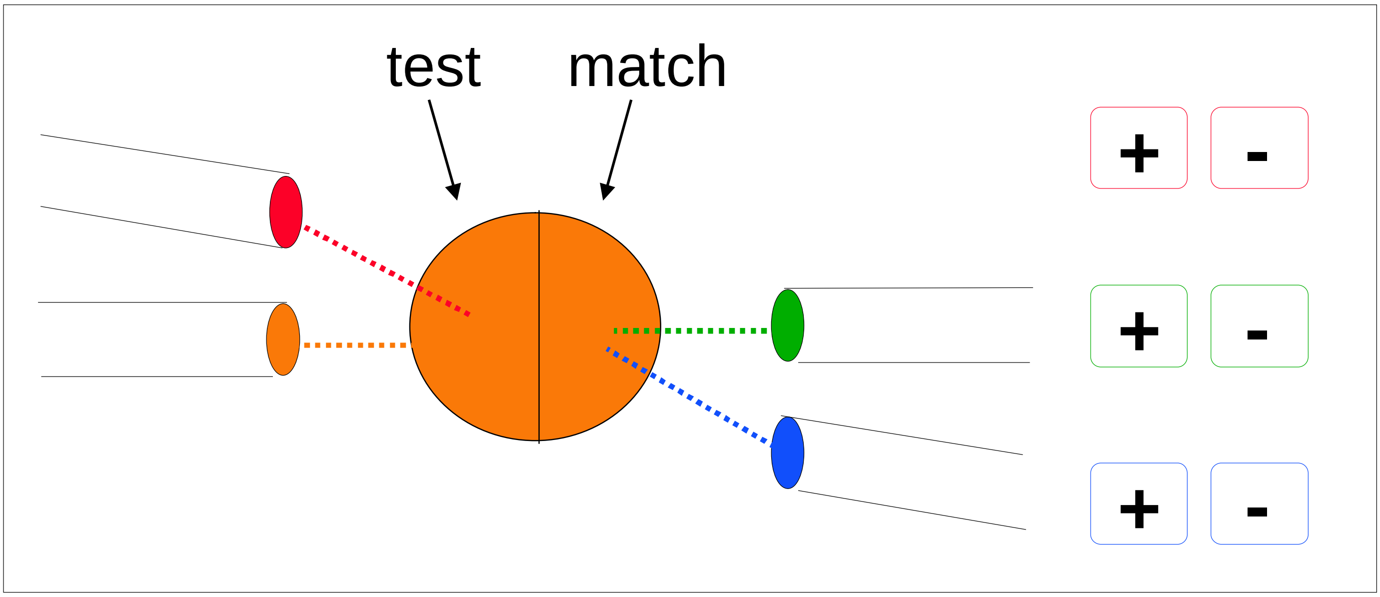
This does not make sense physically, however mathematically:

$$\mathbf{t} \cong -|e_1|\,\mathbf{p_1} + e_2\,\mathbf{p_2} + e_3\,\mathbf{p_3}$$

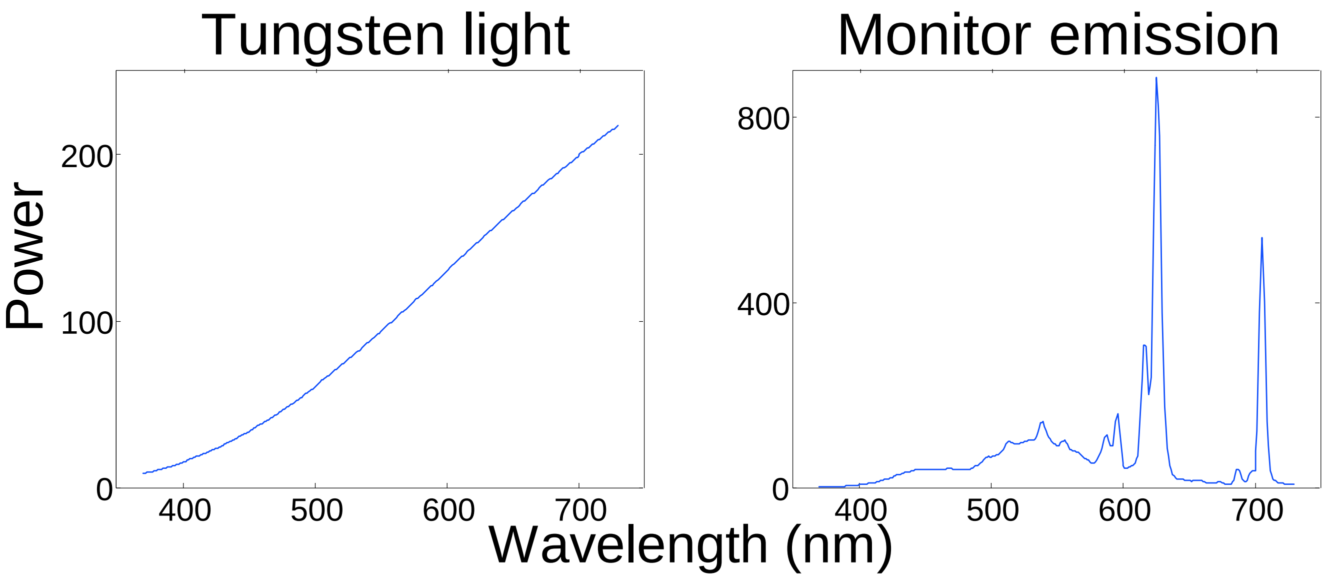
then $$\mathbf{t} + |e_1|\,\mathbf{p_1} \cong e_2\,\mathbf{p_2} + e_3\,\mathbf{p_3}$$

with all positive coefficients.

Physically this can be interpreted as adding primary light $\mathbf{p_1}$ to the test :



The Color Matching Experiment Predicts Metameric Matches



$$\begin{bmatrix} \text{---} \; M_1 \; \text{---} \\ \text{---} \; M_2 \; \text{---} \\ \text{---} \; M_3 \; \text{---} \end{bmatrix} \bullet \begin{bmatrix} | \\ t \\ | \end{bmatrix} = \begin{bmatrix} \text{---} \; M_1 \; \text{---} \\ \text{---} \; M_2 \; \text{---} \\ \text{---} \; M_3 \; \text{---} \end{bmatrix} \bullet \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

S and T match perceptually; they are **metamers**

Using different primary lights

1) Primary lights must be visually independent.

2) Uniqueness of the color matching functions:

primaries	$p_1 \quad p_2 \quad p_3$	$p\grave{}_1 \quad p\grave{}_2 \quad p\grave{}_3$
denote	$\mathbf{P} = [p_1 \; p_2 \; p_3]$	$\mathbf{P}\grave{} = [p\grave{}_1 \; p\grave{}_2 \; p\grave{}_3]$
cmf	$\mathbf{C}$	$\mathbf{C}\grave{}$
given a test light **t**:	$\mathbf{e} = \mathbf{C}\,\mathbf{t}$	$\mathbf{e}\grave{} = \mathbf{C}\grave{}\,\mathbf{t}$
we have	$\mathbf{t} \cong \mathbf{P}\,\mathbf{e}$	$\mathbf{t} \cong \mathbf{P}\grave{}\,\mathbf{e}\grave{}$

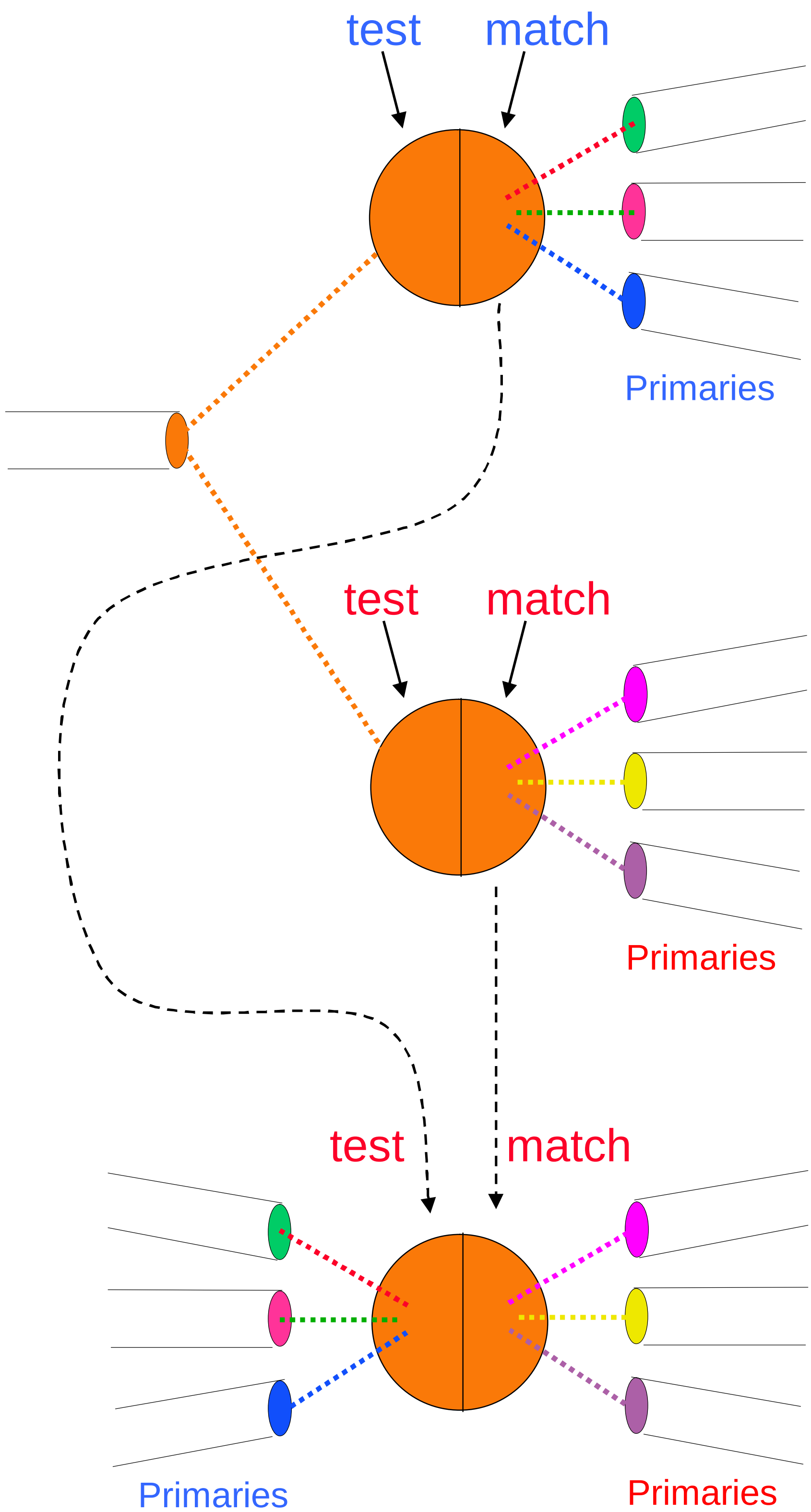
Since **t** is metameric to **Pe** and to **P`e`** :

$$\mathbf{C}\,\mathbf{t} = \mathbf{C}\,\mathbf{P}\,\mathbf{e} = \mathbf{C}\,\mathbf{P}\grave{}\,\mathbf{e}\grave{}$$

or

$$\boxed{\mathbf{e} = \mathbf{C}\,\mathbf{P}\grave{}\,\mathbf{e}\grave{}}$$

This is true for all **t.**


test
match
Primaries
test
match
Primaries
test
match
Primaries
Primaries

$$\boxed{\mathbf{e} = \mathbf{C}\,\mathbf{P}^{\grave{}}\,\mathbf{e}^{\grave{}}} \qquad \text{for all } \mathbf{t}.$$

For monochromatic **t**:

e are the columns of **C**,
e`** are the columns of **C`.

$$\boxed{\mathbf{C} = \underbrace{(\mathbf{C}\,\mathbf{P}^{\grave{}})}_{3 \times 3 \text{ matrix}}\,\mathbf{C}^{\grave{}}}$$

CP' is a 3 x 3 matrix relating the two sets of color matching functions.

i.e.

The color matching functions are *unique* up to a free 3 x 3 linear transformation.

The color matching functions are unique up to a free linear transformation.

$$\begin{bmatrix} \text{---} & M_1 & \text{---} \\ \text{---} & M_2 & \text{---} \\ \text{---} & M_3 & \text{---} \end{bmatrix} \bullet \begin{bmatrix} | \\ t \\ | \end{bmatrix} = \begin{bmatrix} \text{---} & M_1 & \text{---} \\ \text{---} & M_2 & \text{---} \\ \text{---} & M_3 & \text{---} \end{bmatrix} \bullet \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

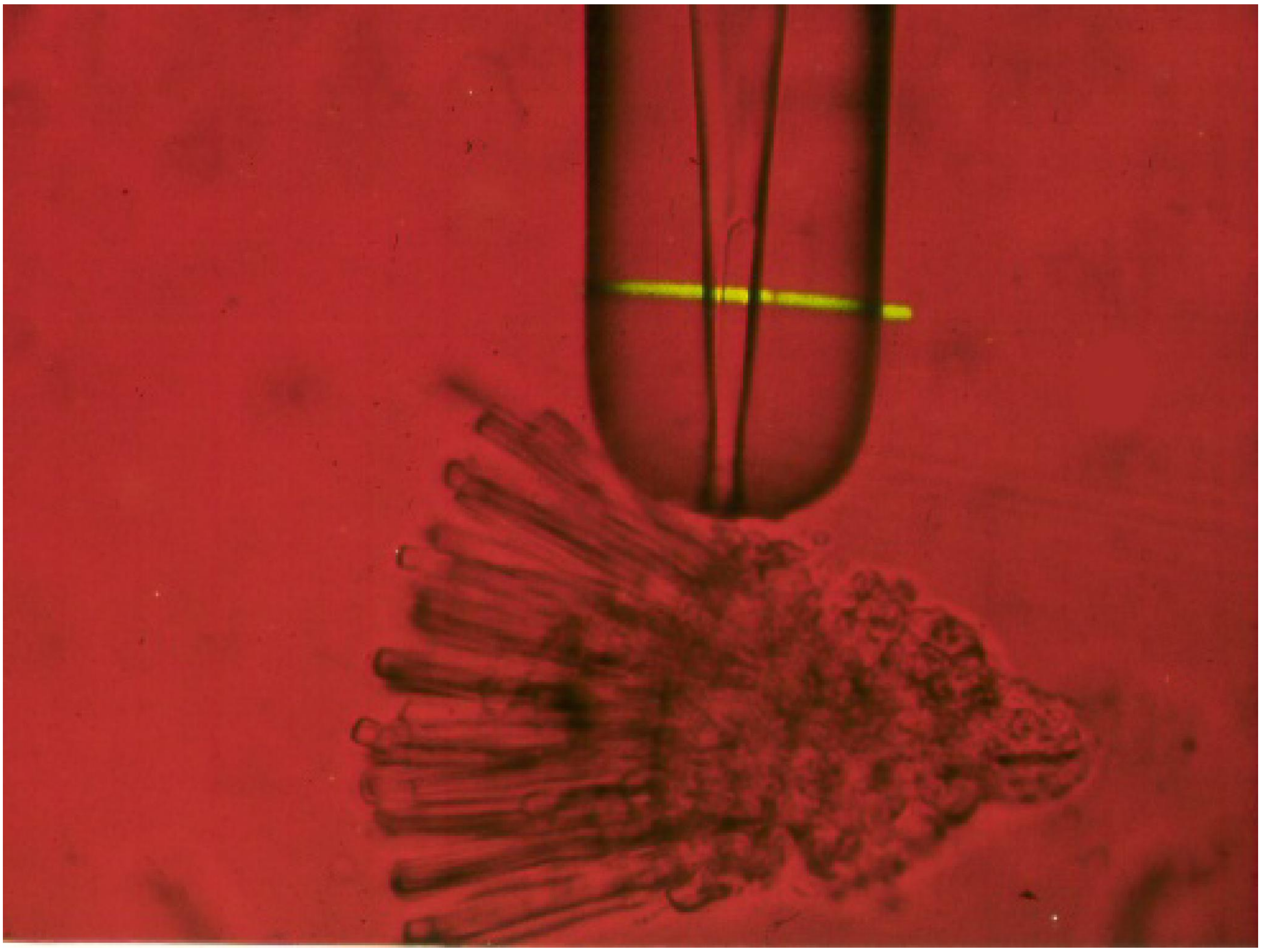
S and T are **metamers**

Linear Transformation of The CMFs Predict the Same Metamers:

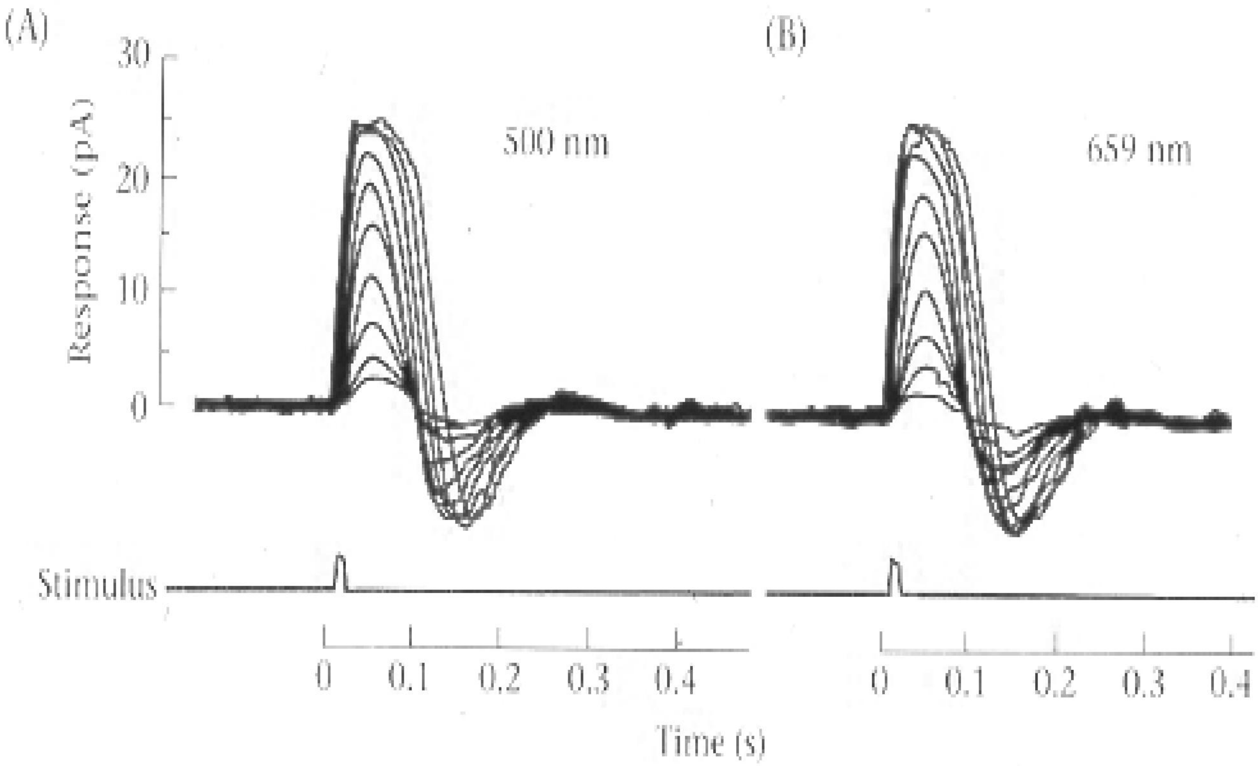
$$\begin{bmatrix} L \end{bmatrix} \begin{bmatrix} \text{---} & M_1 & \text{---} \\ \text{---} & M_2 & \text{---} \\ \text{---} & M_3 & \text{---} \end{bmatrix} \begin{bmatrix} | \\ t \\ | \end{bmatrix} = \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} \text{---} & M_1 & \text{---} \\ \text{---} & M_2 & \text{---} \\ \text{---} & M_3 & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

CMF vs Human Photoreceptors

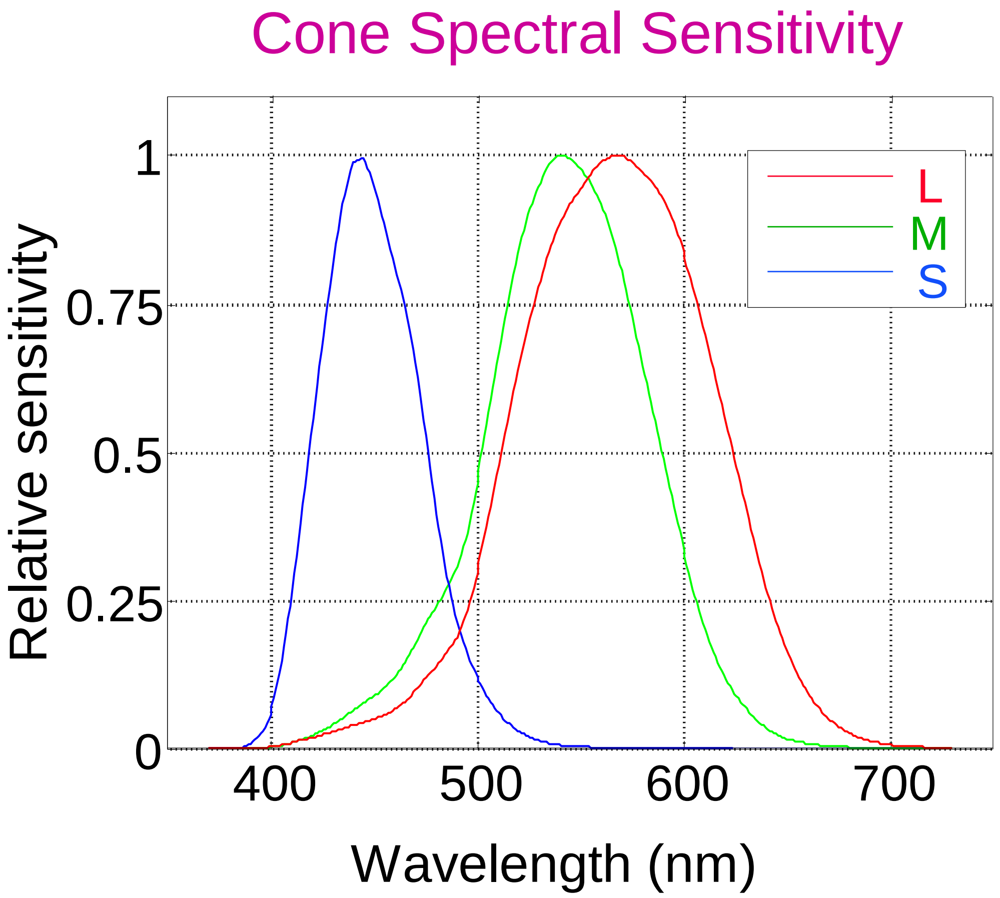
Single Unit Cone Photocurrent Measurements (Schnapf, Baylor et al – 1987).



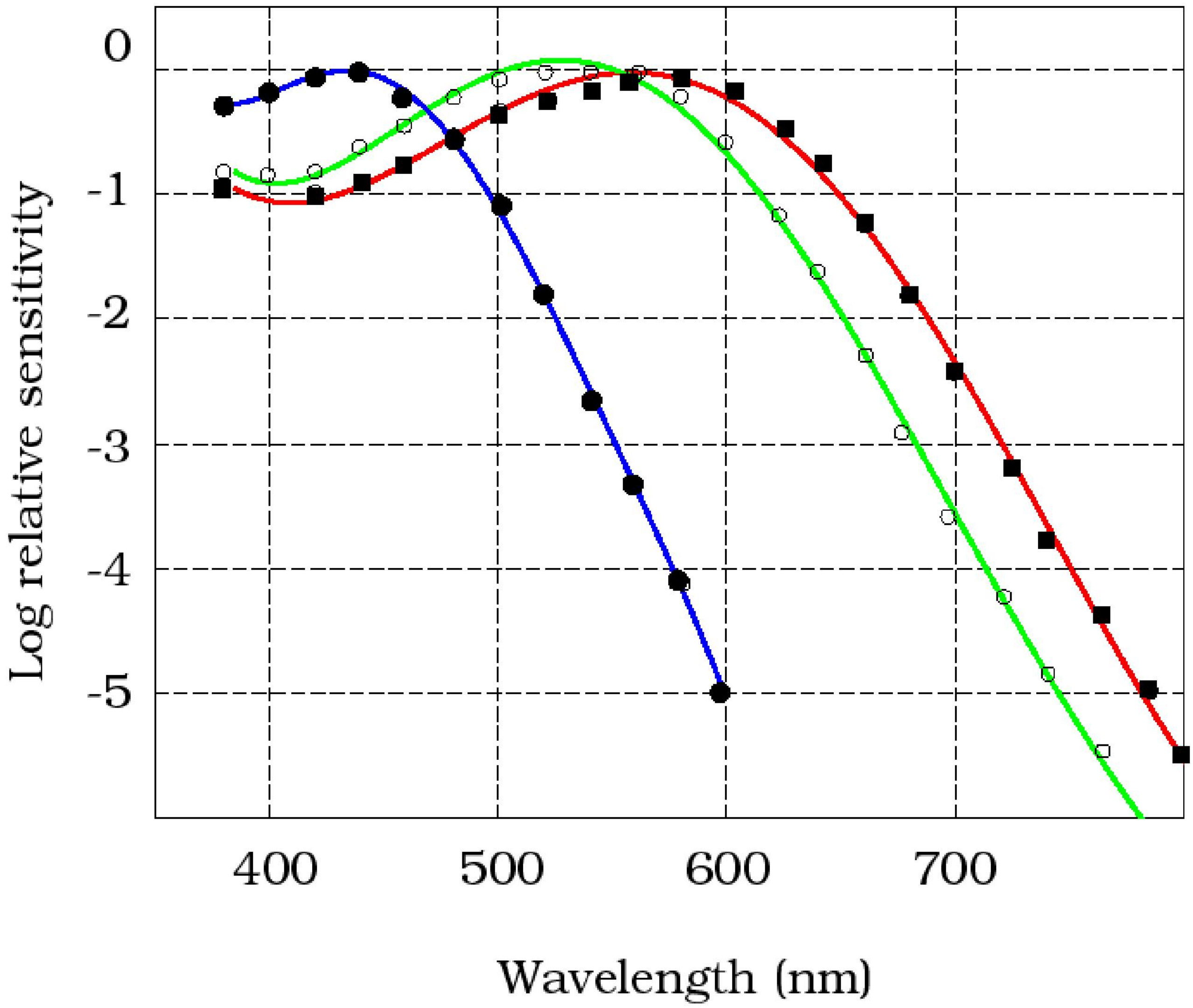
Current Recordings From a Cone



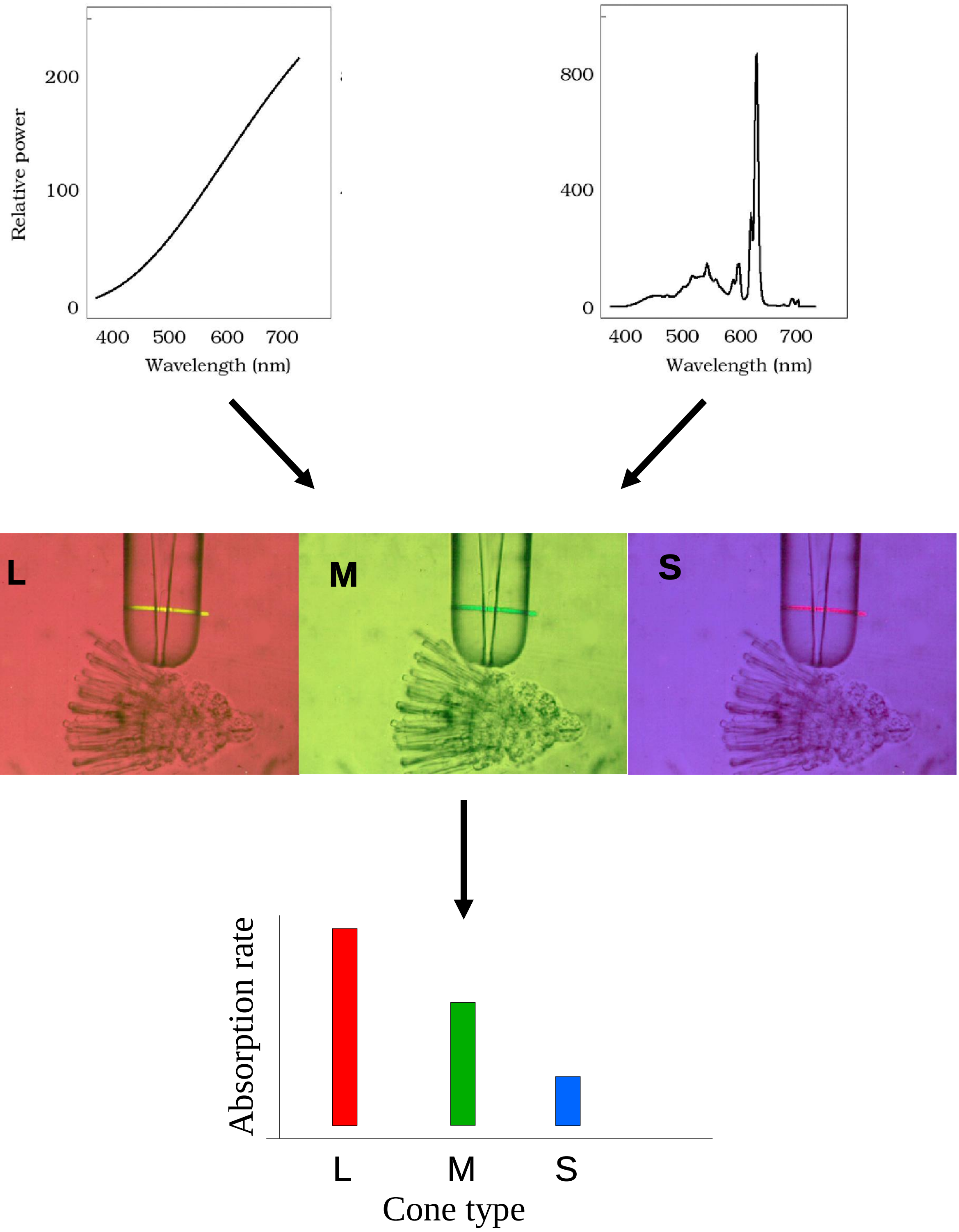
Cone Spectral Sensitivities



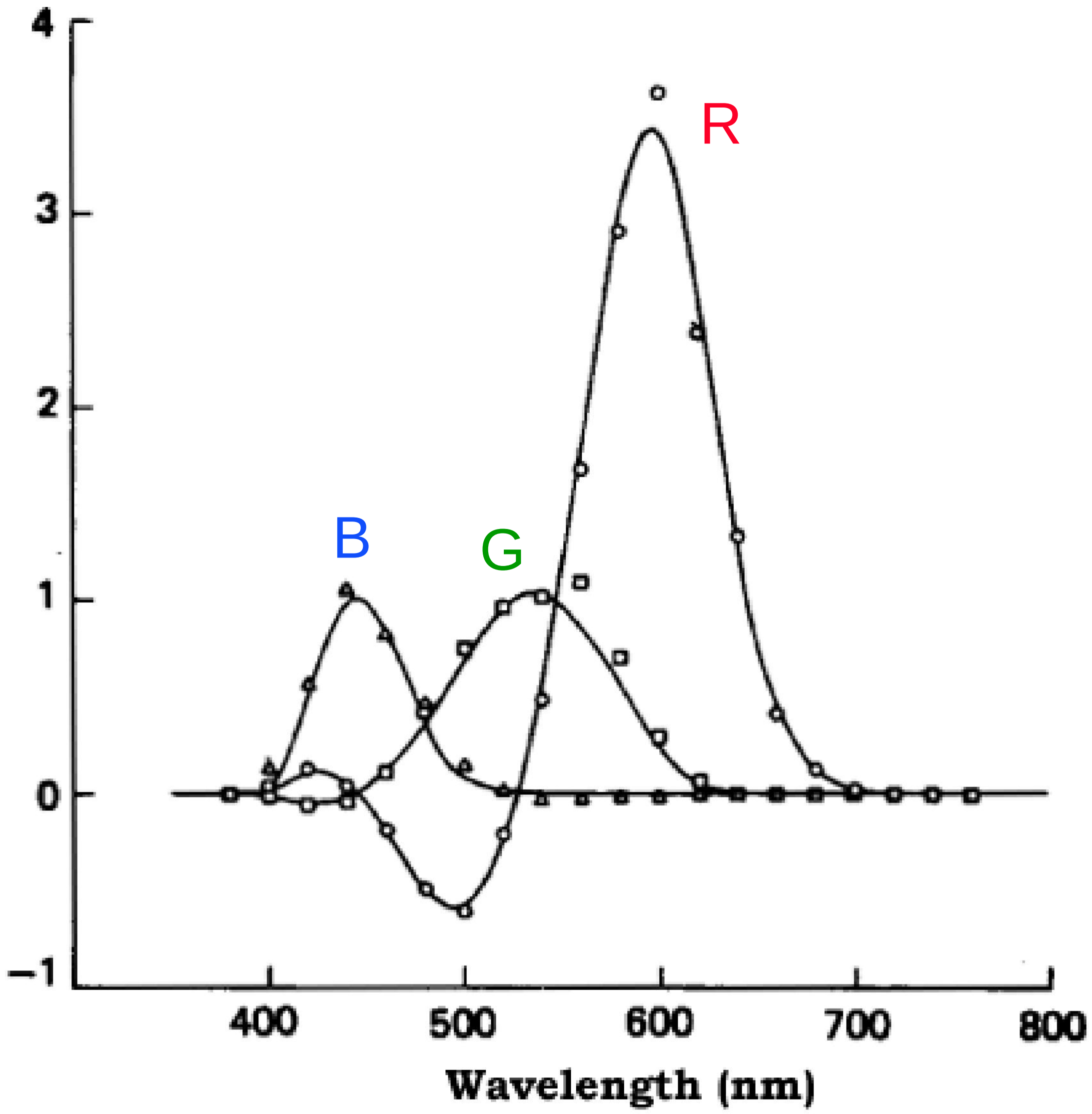
There are Three Types of Cone Wavelength Responsivity



Stimuli Causing Equal Cone Signals Match Perceptually



Behavioral CMFs are accurately predicted by cone responsivities



The cone responsivities are a linear transformation from the CMFs

Sensor Responsivities

Color Matching Functions

$$\begin{bmatrix} \text{———} \; L \; \text{———} \\ \text{———} \; M \; \text{———} \\ \text{———} \; S \; \text{———} \end{bmatrix} = \underbrace{\begin{bmatrix} & & \\ & L & \\ & & \end{bmatrix}}_{\text{3x3 matrix}} \begin{bmatrix} \text{———} \; M_1 \; \text{———} \\ \text{———} \; M_2 \; \text{———} \\ \text{———} \; M_3 \; \text{———} \end{bmatrix}$$

Sensor Respones

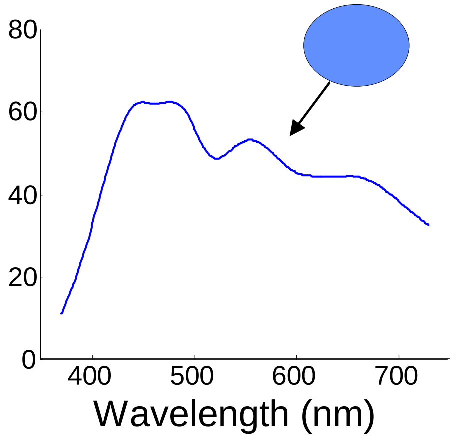
Tristimulus Values

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \underbrace{\begin{bmatrix} & & \\ & L & \\ & & \end{bmatrix}}_{\text{3x3 matrix}} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Trichromatic Color Theory

"tri"=three "chroma"=color

Every color can be represented by 3 values.

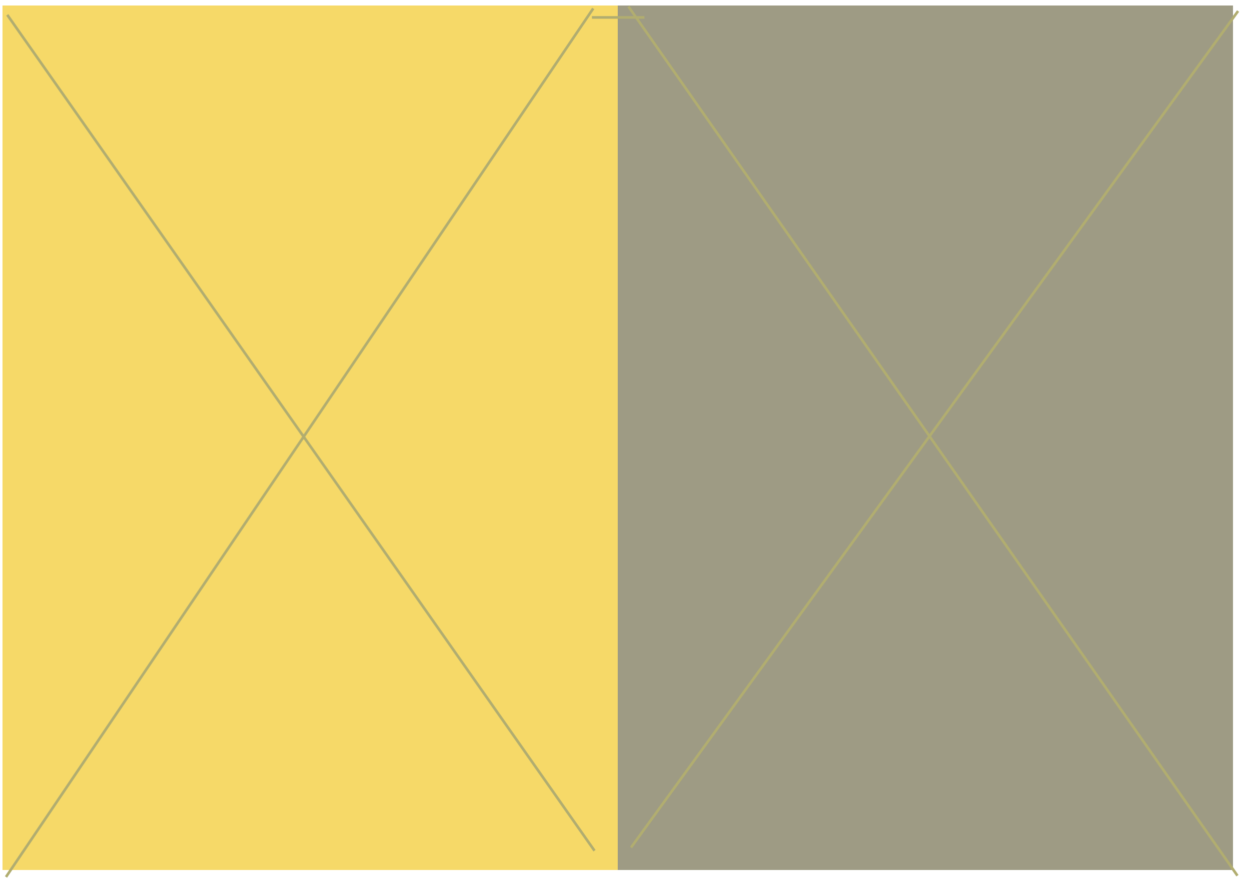


$$\longleftrightarrow \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Space of visible colors is 3 Dimensional.

Color Representation ?

Color Matching Predicts Matches, Not Appearance



Albers (1975)