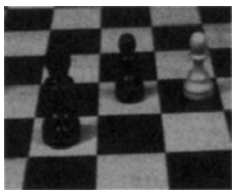
Image Processing - Lesson 10

Edge Detection

- Edge detection masks
- Gradient Detectors
- Compass Detectors
- Second Derivative - Laplace detectors
- Edge Linking
- Hough Transform



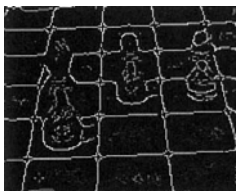


Image Processing - Computer Vision

Low Level

Image Processing

representation, compression,transmission

image enhancement

edge/feature finding

Computer Vision

image "understanding"

High Level

UFO - Unidentified Flying Object

Point Detection

Convolution with:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Large Positive values = light point on dark surround
Large Negative values = dark point on light surround

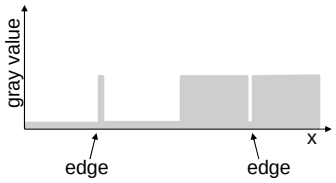
Example:

$$\begin{matrix} 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 100 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{matrix} \quad * \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

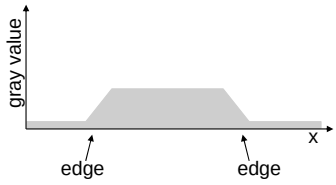
$$= \begin{matrix} 0 & 0 & -95 & -95 & -95 \\ 0 & 0 & -95 & 760 & -95 \\ 0 & 0 & -95 & -95 & -95 \end{matrix}$$

Edge Definition

Line Edge



Step Edge



Edge Detection

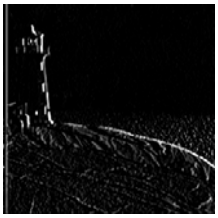
Line Edge Detectors

$$\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Step Edge Detectors

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Example

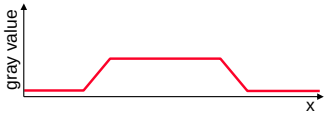


$$\begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Edge Detection by Differentiation

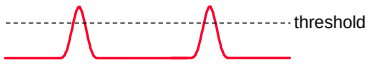
<u>Step Edge detection by differentiation:</u>

1D image f(x)



1st derivative f'(x)

|f'(x)| - threshold



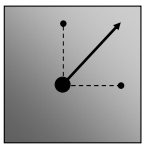
Pixels that passed the threshold are Edge Pixels

Gradient Edge Detection

Gradient $\nabla f((x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$

Gradient Magnitude $\sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$

Gradient Direction $tg^{-1} (\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x})$



Gradient Edge - Examples

$\nabla f((x,y) = \begin{bmatrix} \frac{\partial f}{\partial x}, 0 \end{bmatrix}$

$\nabla f((x,y) = \begin{bmatrix} 0, \frac{\partial f}{\partial y} \end{bmatrix}$

Differentiation in Digital Images

horizontal - differentiation approximation:

$F_A = \frac{\partial f(x,y)}{\partial x} = f(x,y) - f(x-1,y)$

convolution with $[\,1 \;\; -1\,]$

vertical - differentiation approximation:

$F_B = \frac{\partial f(x,y)}{\partial y} = f(x,y) - f(x,y-1)$

convolution with $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Gradient (F_A, F_B)

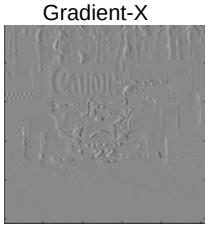
Magnitude $((F_A)^2 + (F_B)^2)^{1/2}$

Approx. Magnitude $|F_A| + |F_B|$

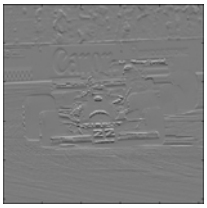
Example Edge

Original

Gradient-X



Gradient-Y



Gradient-Magnitude

Gradient-Direction

Roberts Edge Detector

$$F_A = f(x,y) - f(x-1,y-1)$$
$$F_B = f(x-1,y) - f(x,y-1)$$

$$A = \begin{bmatrix} \boxed{1} & 0 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} \boxed{0} & 1 \\ -1 & 0 \end{bmatrix}$$

Roberts and other 2x2 operators are sensitive to noise.

Prewitt Edge Detector

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & \boxed{0} & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -1 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Smoothed operators

Sobel Edge Detector

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -2 & \boxed{0} & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -2 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Isotropic Edge Detector

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & \boxed{0} & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & \boxed{0} & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

Edge Detector - Comparison

Original

Roberts

Sobel

Prewitt

Magnitude Negative Magnitude

Compass Operators

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

↑ N ↖ NW ← W ↙ SW

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

↓ S ↘ SE → E ↗ NE

Given k operators, $g_k(x,y)$ is the image obtained by convolving f(x,y) with the kth operator.

The gradient is defined as:

$$\boxed{g(x,y) = \max_k g_k(x,y)}$$

k defines the edge direction

Various Compass Operators:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \boxed{-2} & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & \boxed{0} & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & \boxed{0} & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

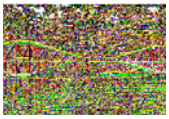
$$\begin{bmatrix} 5 & 5 & 5 \\ -3 & \boxed{0} & -3 \\ -3 & -3 & -3 \end{bmatrix}$$ Kirsch Edge Detector

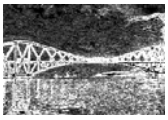
Edge Detector - Comparison

Original

Sobel

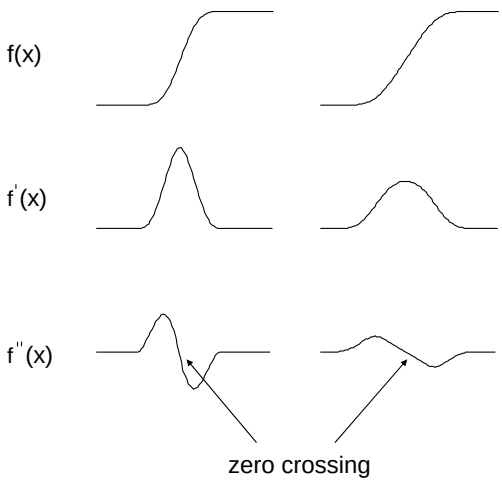
Kirsch (Compass)





Direction Map Magnitude Negative Magnitude

Derivatives



Laplacian Operators

Approximation of second derivative (horizontal):

$$\frac{\partial f^2(x,y)}{\partial^2 x} = f''(x,y) = f'(x+1,y) - f'(x,y) =$$
$$= [f(x+1,y) - f(x,y)] - [f(x,y) - f(x-1,y)]$$
$$= f(x+1,y) - 2\,f(x,y) + f(x-1,y)$$

convolution with: $[\,1 \;\; -2 \;\; 1\,]$

Approximation of second derivative (vertical):

convolution with: $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

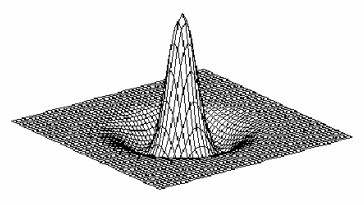
Laplacian Operator $\boxed{\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)}$

convolution with: $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

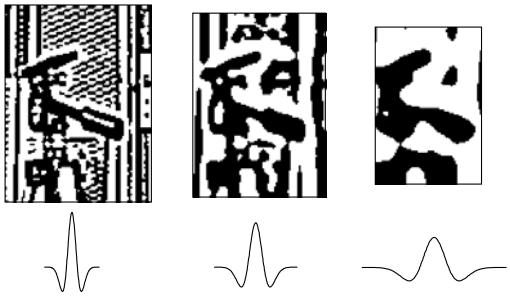
Variations on Laplace Operators:

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

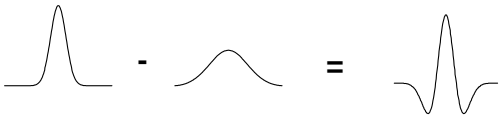
All are approximations of:



Example of Laplacian Edge Detection



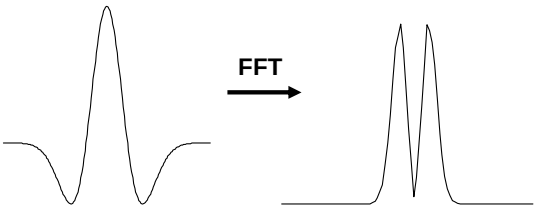
Laplacian ~ Difference of gaussians

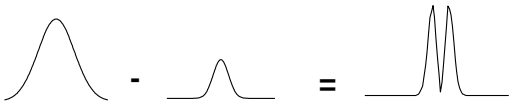


DOG = Difference of Gaussians

Laplacian Operator
(Image Domain)

Laplacian Filter
(Frequency Domain)

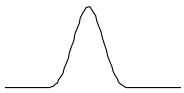




Enhancement Using the Laplacian Operator

Edge Image f(x)

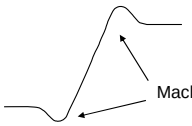
1st derivative $\frac{\partial f}{\partial x}$



2nd derivative (Laplacian) $\frac{\partial^2 f}{\partial x^2}$

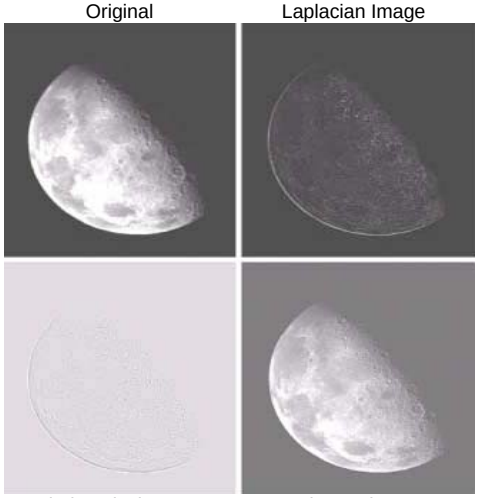
$-\frac{\partial^2 f}{\partial x^2}$

$f(x) - \frac{\partial^2 f}{\partial x^2}$



Examples of enhanced images

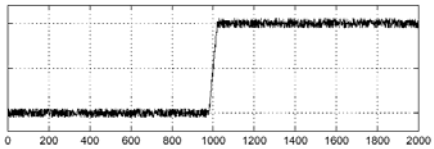
Original | Laplacian Image



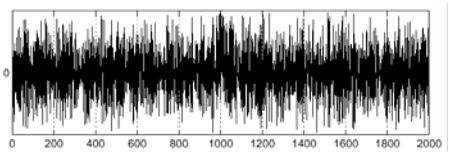
Scaled Laplacian Image | Enhanced Image

Edge Detection - Noise Issues

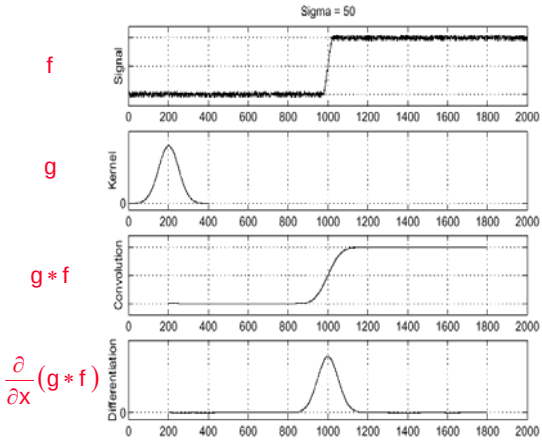
$f(x)$



$\frac{d}{dx} f(x)$



Edge Detection - Noise Issues

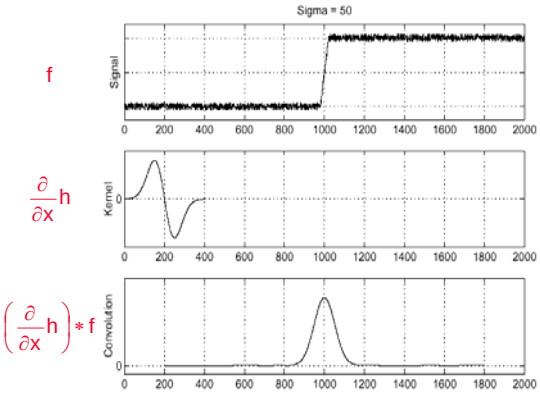


Peaks in $\frac{\partial}{\partial x}(g * f)$ mark the edge

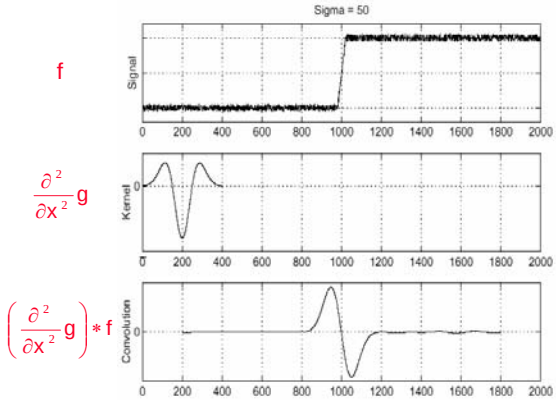
Edge Detection - Noise Issues

From the convolution theorem we have:

$$\frac{\partial}{\partial x}(h * f) = \left(\frac{\partial}{\partial x} h\right) * f$$



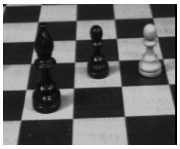
Edge Detection - Noise Issues

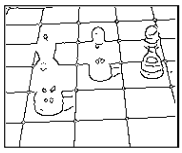


Zero Crossings in $\frac{\partial^2}{\partial x^2}(g * f)$ mark the edge

Canny Edge Detector

1) Convolve image with derivative of a Gaussian
2) Perform NonMaximum suppression.
3) Perform Hysteresis Thresholding.





Canny Edge Detector - Step 1

Convolve image with derivative of a Gaussian

$$f(x,y) * G'(x,y) \cong f(x,y) * G'_x(x,y) + f(x,y) * G'_y(x,y)$$

$$F_a = f(x,y) * G_x(x,y) * D_x(x,y)$$

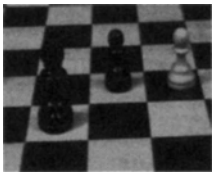
$$= f(x,y) * \begin{bmatrix} 1 & 2 & 4 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$F_b = f(x,y) * G_y(x,y) * D_y(x,y)$$

$$= f(x,y) * \begin{bmatrix} 1 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

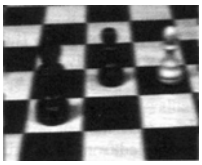
Magnitude $((F_A)^2 + (F_B)^2)^{1/2}$

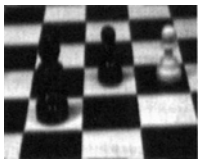
Canny Edge Detector - Step 1



Original

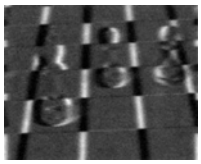
$f * G_x$

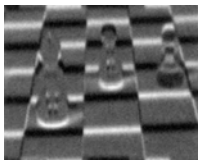




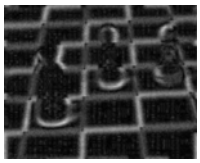
$f * G_y$

$F_a = f*G_x*D_x$





$F_b = f*G_y*D_y$



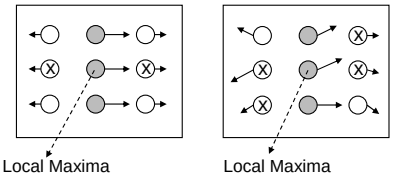
Magnitude

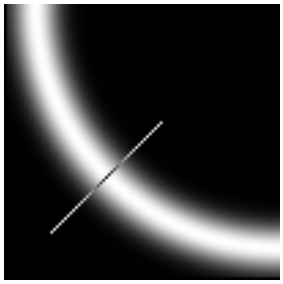
$((F_A)^2 + (F_B)^2)^{1/2}$

Canny Edge Detector - Step 2

NonMaximum suppression.

Remove edges that are NOT local Maxima in the gradient direction.

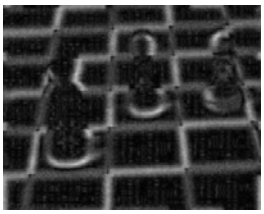




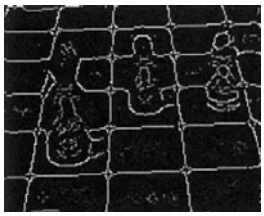
Canny Edge Detector - Step 2

NonMaximum Suppression.

Magnitude

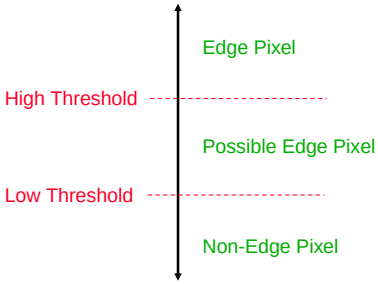


NonMaximum Supression



Canny Edge Detector - Step 3

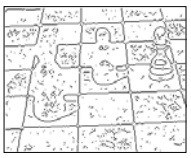
Hysteresis Thresholding.



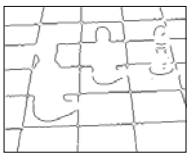
Include possible edge pixels that are adjacent to edge pixels.

Canny Edge Detector - Step 3

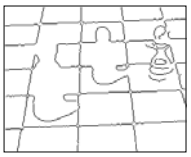
Magnitude (NonMax Suppressed)



Low Threshold



High Threshold

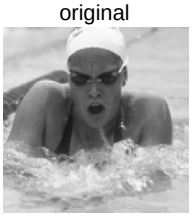


Hysteresis Threshold

Effect of Scale

Effect of σ (Gaussian kernel size)

original

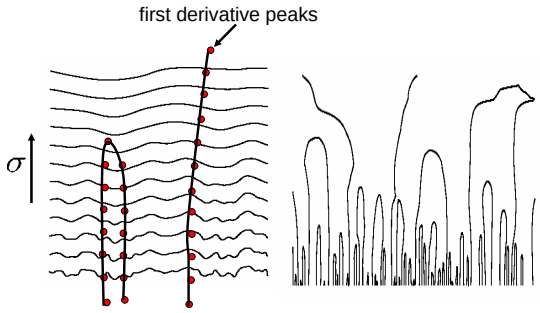


Canny with $\sigma = 1$

Canny with $\sigma = 2$

Scale Space

Witkin 83

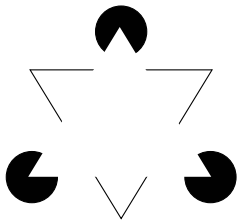


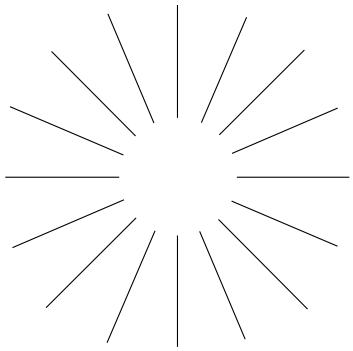
Gaussian filtered signal

Properties of Scale Space:

- Position of edges may change with scale.
- Edges may merge with increase in scale.
- Edges do NOT split with increase in scale.

Edge Completion





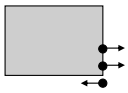
Edge Linking

(x,y) is an edge pixel.
Search for neighboring edge pixels that are "similar".

Similarity:

Similarity in Edge Orientation
Similarity in Edge strength (Gradient Amplitude)

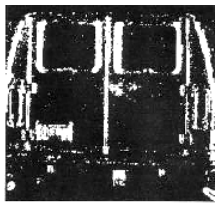
Perform Edge Following along similar edge pixels.
(as in Contour Following in binary images).

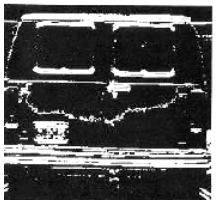


Examples of edge linking

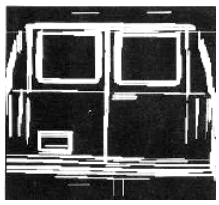
Original

Sobel Vertical





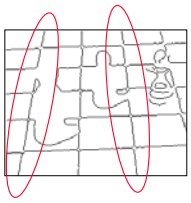
Sobel Horizontal



Linked Edges

Edge Points linked: Gradient Value > 25
Gradient direction within 15%

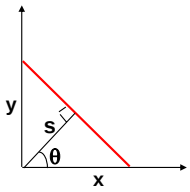
Problem:
Edges are not lines even when linked.



Edge pixels are not ellipses even when linked.

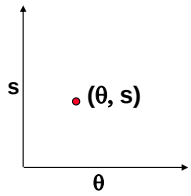
Hough Transform

Image Domain



straight line

Hough Domain



Hough Transform

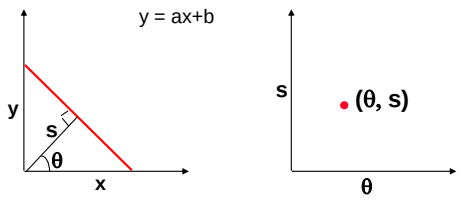
$$y = ax+b$$

$$\downarrow$$

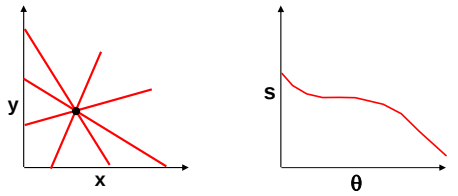
$$s = x\cos(\theta)+y\sin(\theta)$$

Hough Transform

Image Domain **Hough Domain**

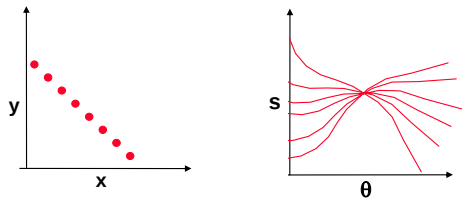


straight line Hough Transform



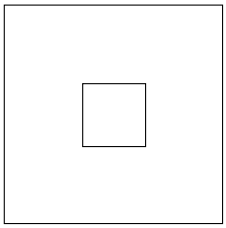
single point = many possible lines

Hough Transform

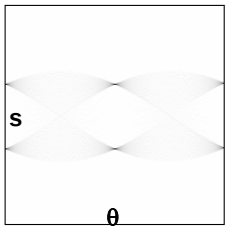


many points on a line =
many lines in the Hough transform space which intersect at 1 point.

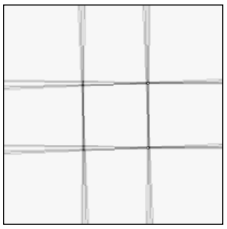
Hough Transform Example



Original
square image



Hough Transform
(s,θ) space



Reconstructed
line segments

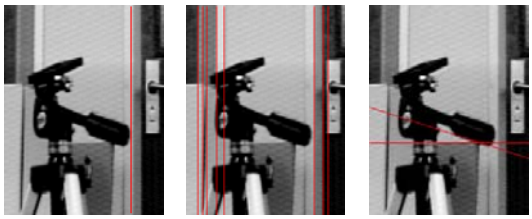
Hough Transform Example

Original

Edges

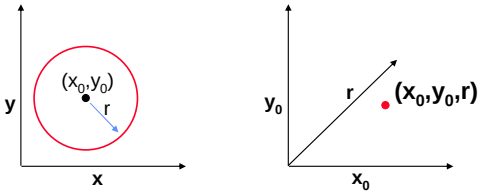
Hough Transform

Results1 Results2 Results3



Hough Transform for Circles

Image Domain **Hough Domain**

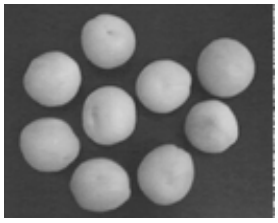


circle Hough Transform

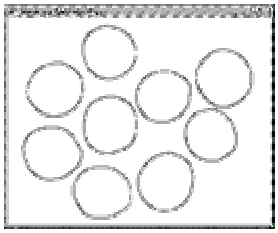
$$r^2 = (x-x_0)^2+(y-y_0)^2$$

Hough Transform Example

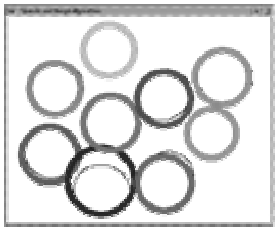
Original



Edges

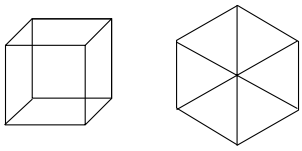


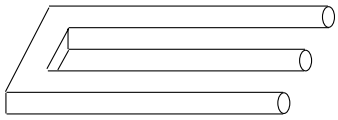
Result



Nice demo: http://www.markschulze.net/java/hough/index.html

3D Perception - Depth Perception





Impossible Figures
(Escher)