Image Representation

- Gaussian pyramids
- Laplacian Pyramids
- Wavelet Pyramids
- Applications

Image Pyramids

Image features at different resolutions require filters at different scales.

Edges (derivatives):

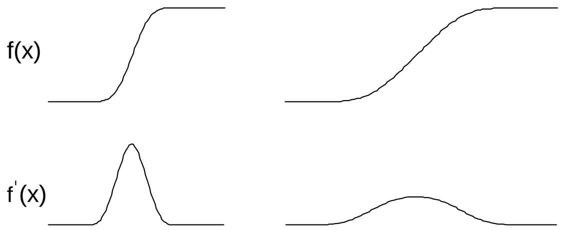
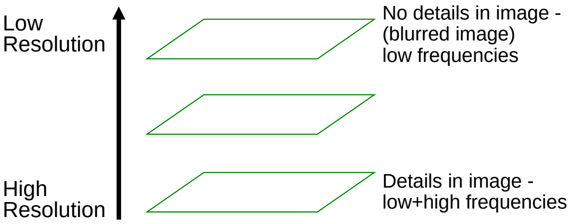


Image Pyramids

Image Pyramid = Hierarchical representation of an image



A collection of images at different resolutions.

Image pyramids

- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

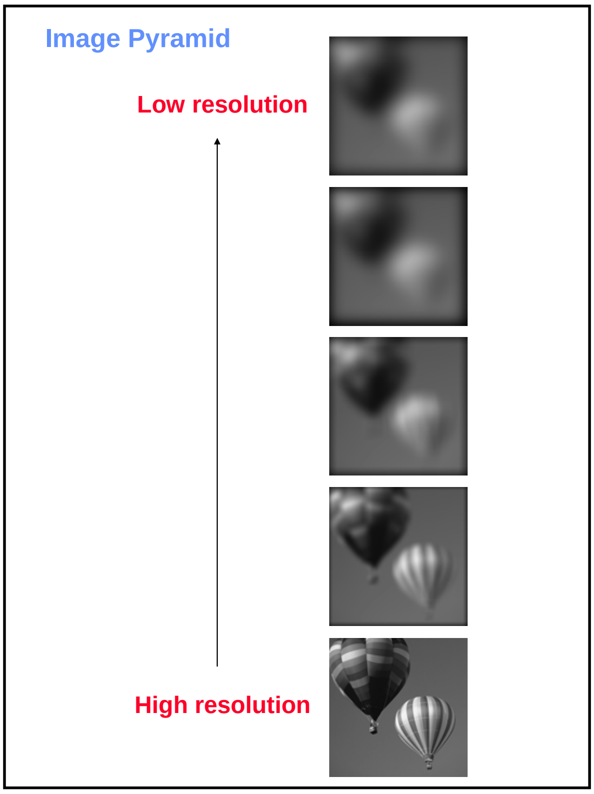


Image Pyramid
Low resolution
High resolution

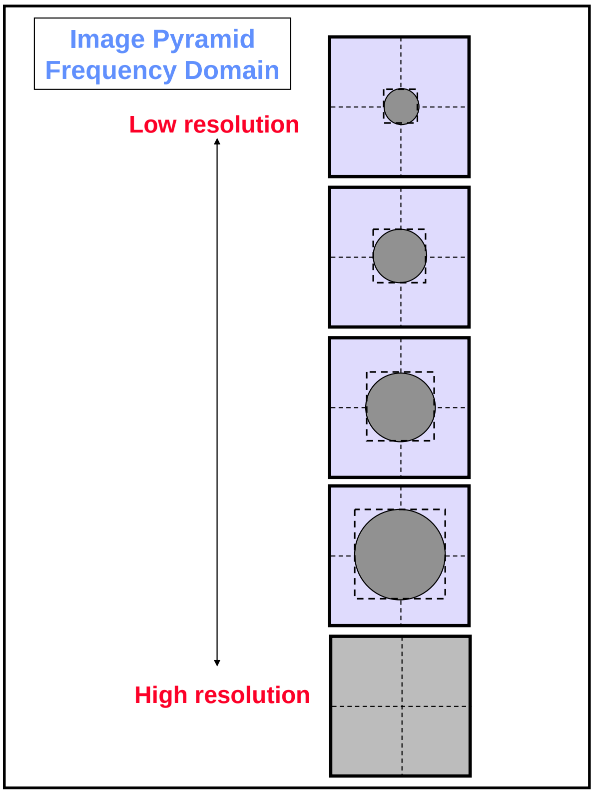


Image Pyramid
Frequency Domain
Low resolution
High resolution

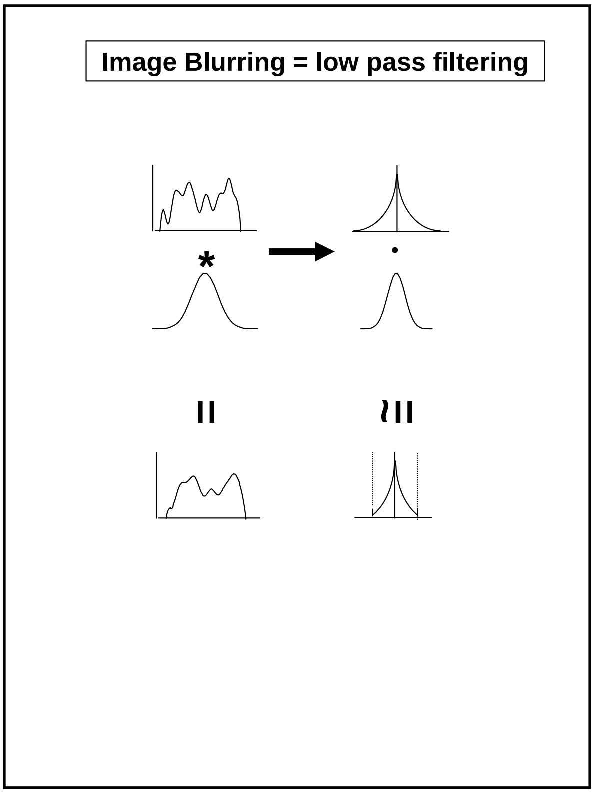


Image Blurring = low pass filtering

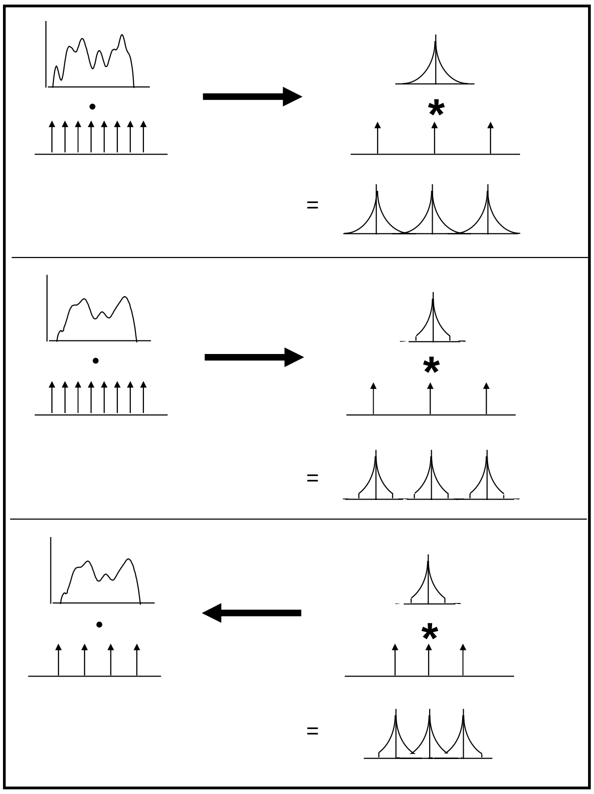
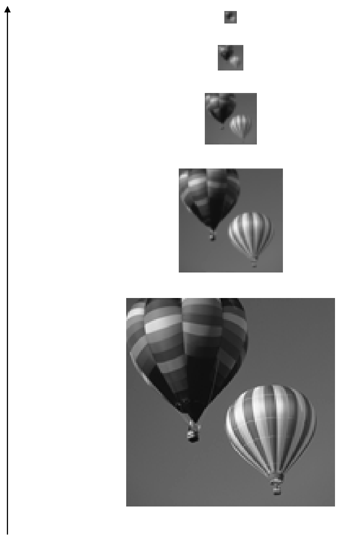


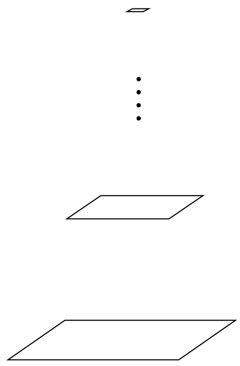
Image Pyramid

Low resolution



High resolution

Gaussian Pyramid

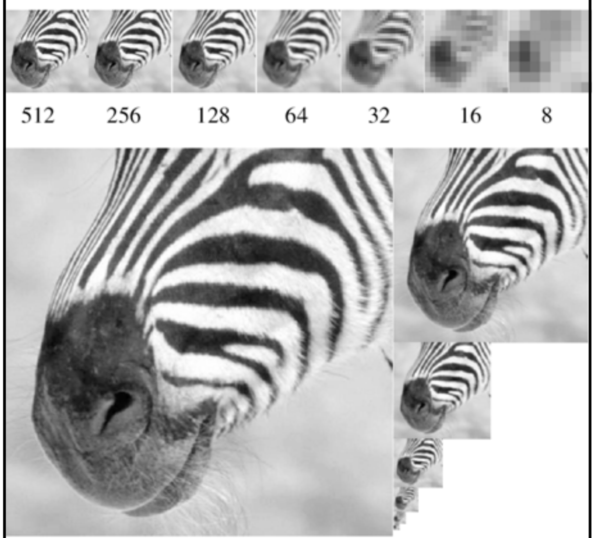


Level n
1×1

Level 1
$2^{n-1} \times 2^{n-1}$

Level 0
$2^n \times 2^n$

Gaussian Pyramid



Gaussian Pyramid

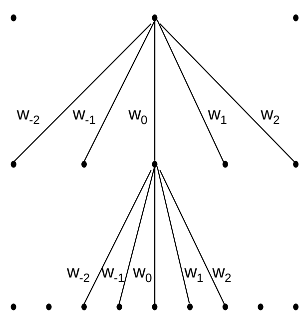
Burt & Adelson (1981)

Normalized: $\Sigma w_i = 1$

Symmetry: $w_i = w_{-i}$

Unimodal: $w_i \geq w_j$ for $0 < i < j$

Equal Contribution: for all j Σw_{j+2i} = constant



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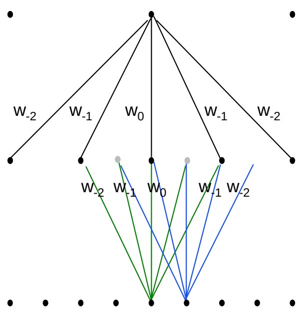

Gaussian Pyramid

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Normalized: $\Sigma w_i = 1$

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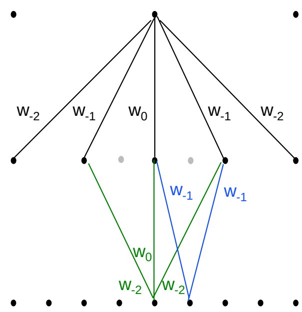
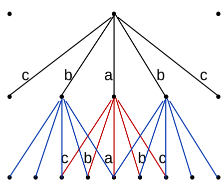
Unimodal: $w_i \geq w_j$ for $0 < i < j$

Equal Contribution: for all j Σw_{j+2i} = constant



Gaussian Pyramid

Burt & Adelson (1981)



$$a + 2b + 2c = 1$$
$$a + 2c = 2b$$

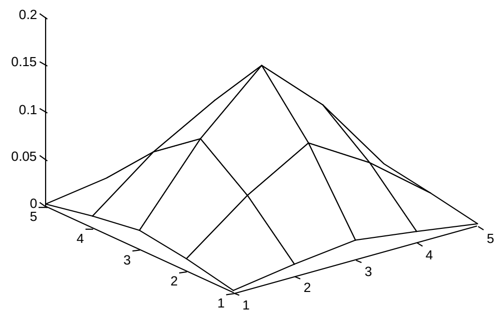
$$\boxed{\begin{aligned} &a > 0.25 \\ &b = 0.25 \\ &c = 0.25 - a/2 \end{aligned}}$$

For a = 0.4 most similar to a Gauusian filter

g = [0.05 0.25 0.4 0.25 0.05]

low_pass_filter = g' * g =

$$\begin{bmatrix} 0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \\ 0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\ 0.0200 & 0.1000 & 0.1600 & 0.1000 & 0.0200 \\ 0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\ 0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \end{bmatrix}$$



Gaussian Pyramid - Computational Aspects

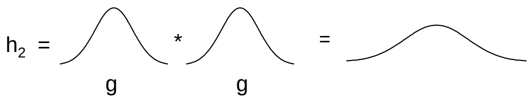
Memory:

$2^N X 2^N (1 + 1/4 + 1/16 + \ldots) = 2^N X 2^N * 4/3$

Computation:

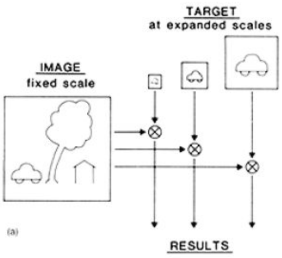
Level i can be computed with a single convolution with filter: $h_i = \underbrace{g * g * g * \ldots\ldots}_{\text{i times}}$

Example:

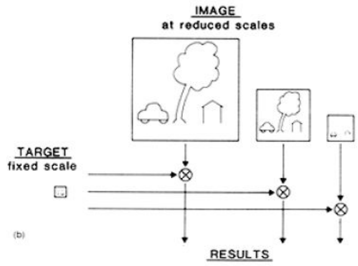


MultiScale Pattern Matching

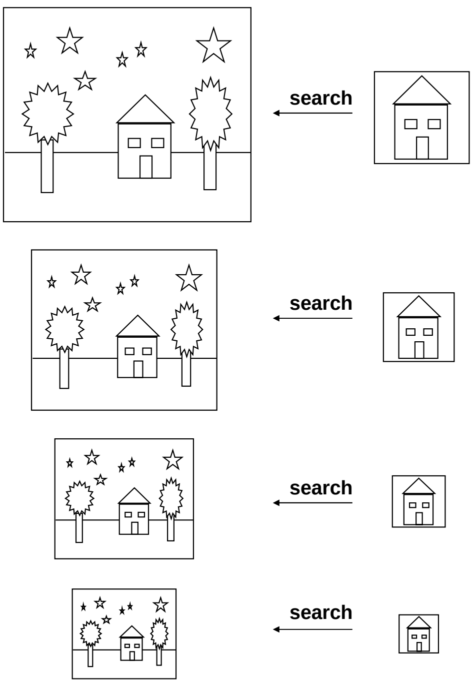
Option 1:
Scale target and search for each in image.



Option 2:
Search for original target in image pyramid.



Hierarchical Pattern Matching



Pattern matching using Pyramids - Example

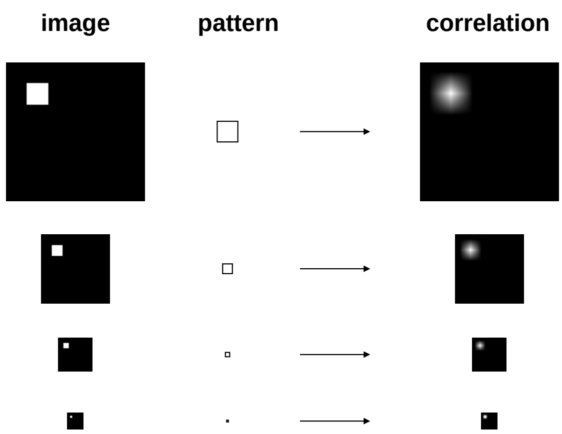


Image pyramids

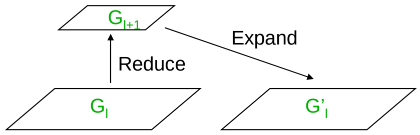
- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

Laplacian Pyramid

Motivation = Compression, redundancy removal.
compression rates are higher for predictable values.
e.g. values around 0.

$G_0, G_1, \ldots.$ = the levels of a Gaussian Pyramid.

Predict level G_l from level G_{l+1} by Expanding G_{l+1} to G'_l



Denote by L_l the error in prediction:

$$\boxed{L_l = G_l - G'_l}$$

$L_0, L_1, \ldots.$ = the levels of a Laplacian Pyramid.

What does blurring take away?

original

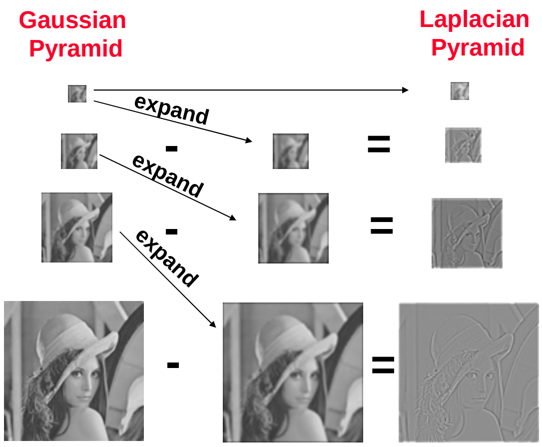
What does blurring take away?

smoothed (5x5 Gaussian)

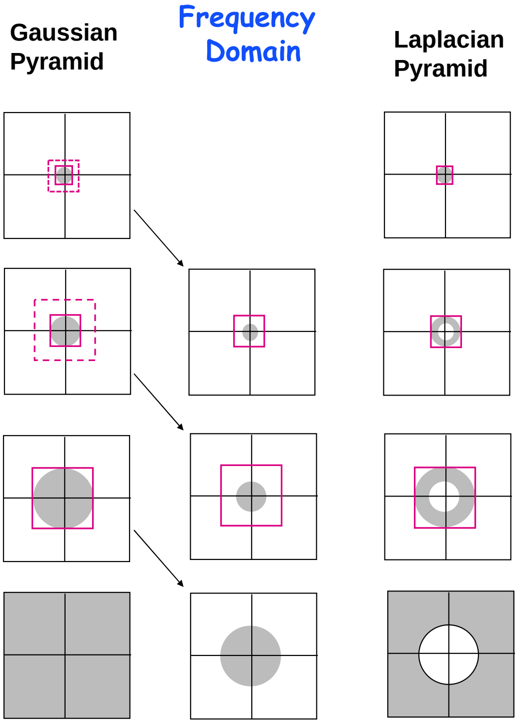
What does blurring take away?

smoothed – original

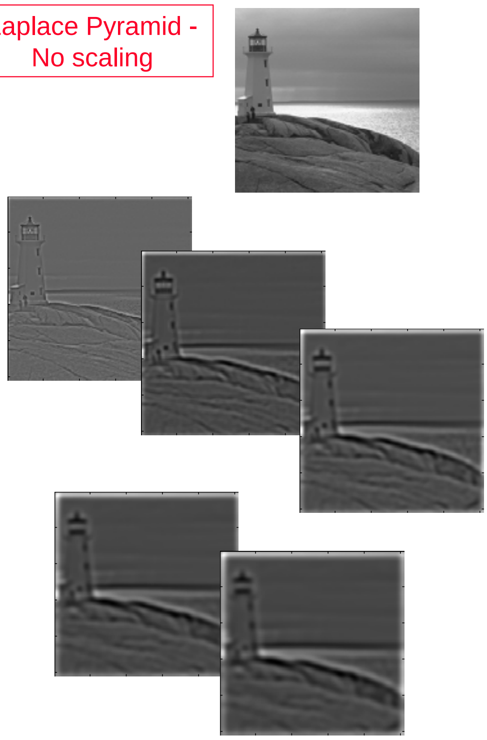
Laplacian Pyramid

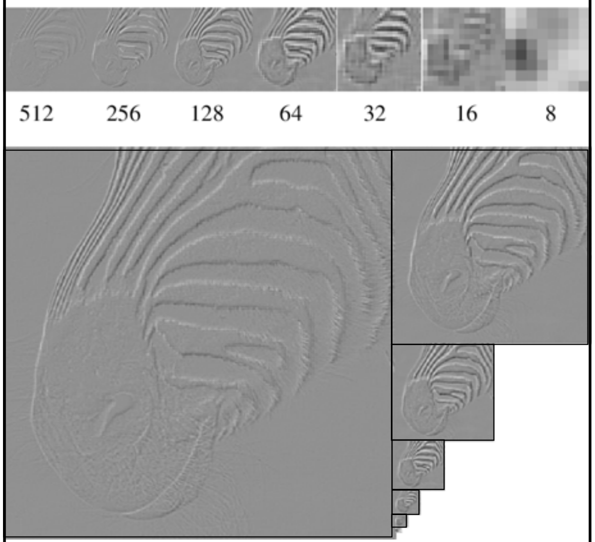


Frequency Domain



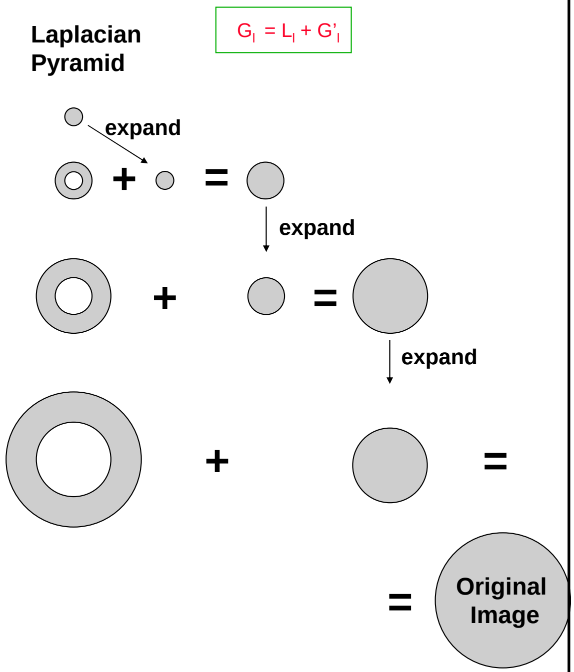
Laplace Pyramid - No scaling





from: B.Freeman

Reconstruction of the original image from the Laplacian Pyramid



Laplacian Pyramid - Computational Aspects

Memory:

$2^N X 2^N (1 + 1/4 + 1/16 + \dots) = 2^N X 2^N * 4/3$

However coefficients are highly compressible.

Computation:

L_i can be computed from G_0 with a single convolution with filter: $k_i = h_{i-1} - h_i$

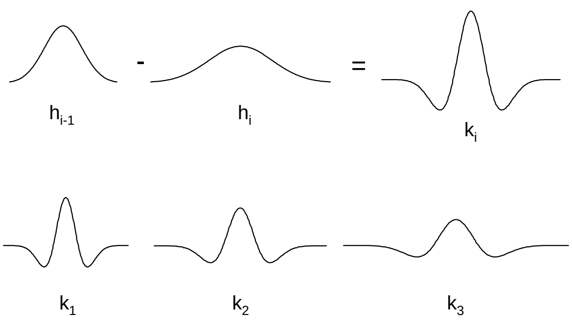


Image Mosaicing

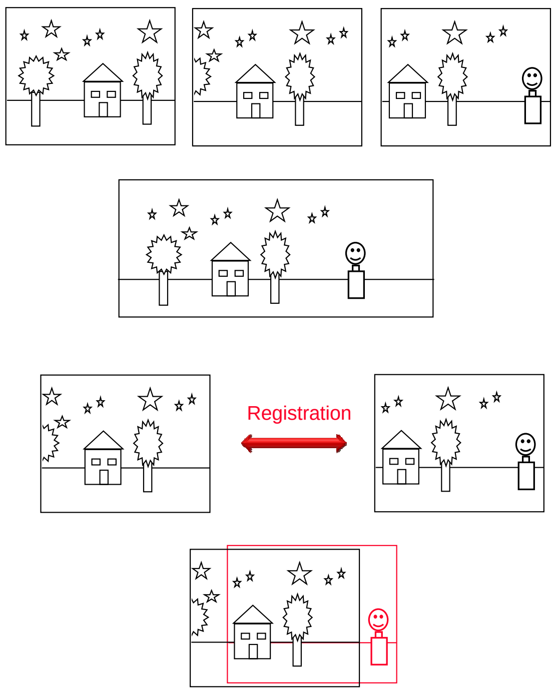
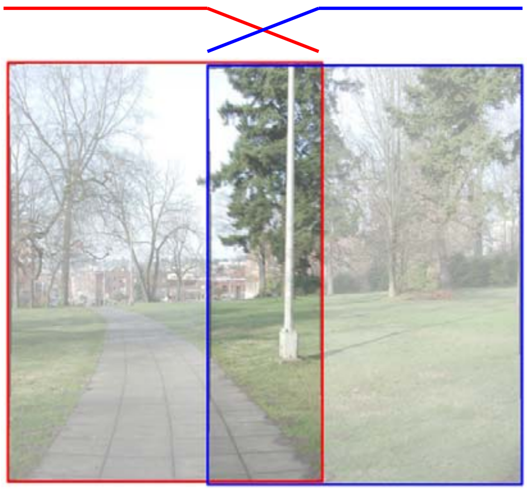


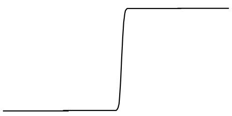
Image Blending



Blending

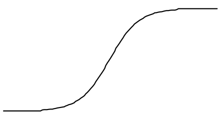
Multiresolution Spline

When splining two images, transition from one image to the other should behave:



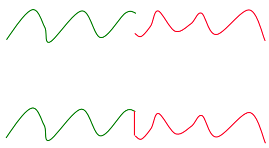
High Frequencies

Middle Frequencies

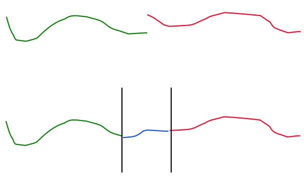


Low Frequencies

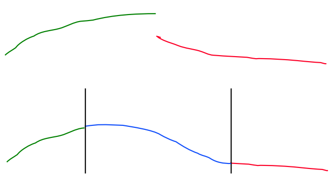
Multiresolution Spline



High Frequencies

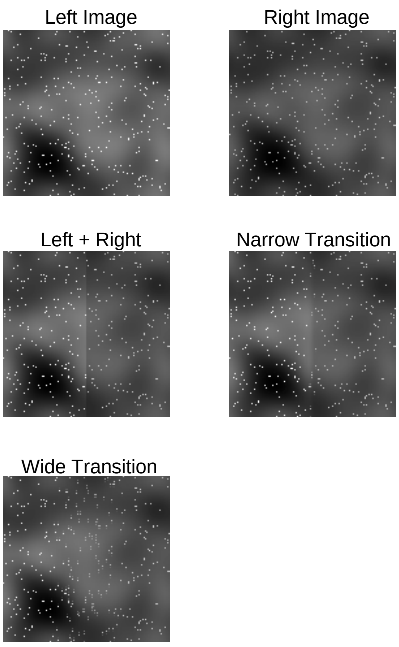


Middle Frequencies



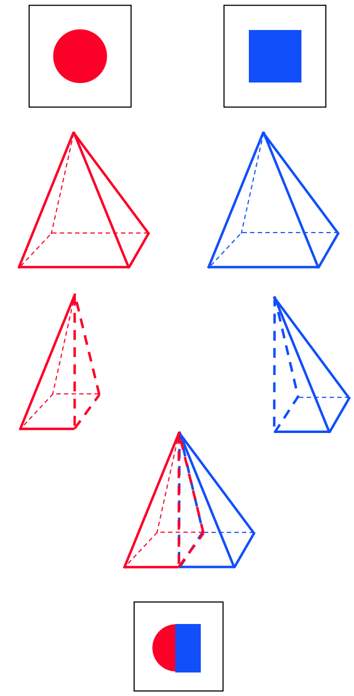
Low Frequencies

Multiresolution Spline - Example

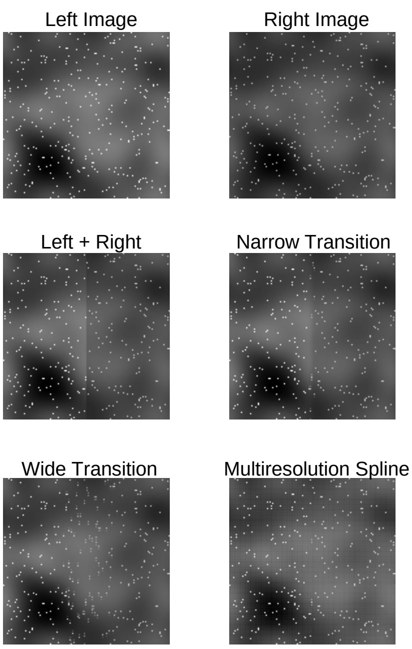


(Burt & Adelson)

Multiresolustion Spline - Using Laplacian Pyramid

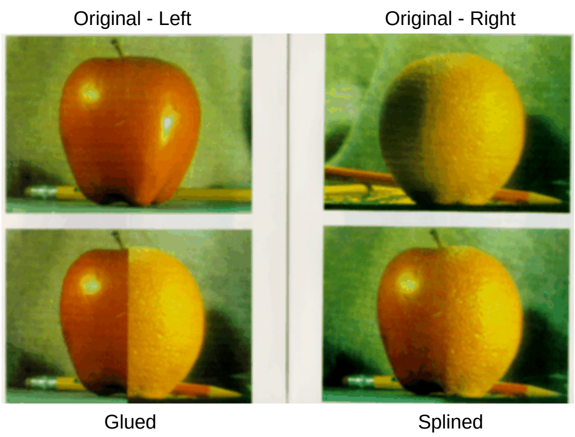


Multiresolution Spline - Example

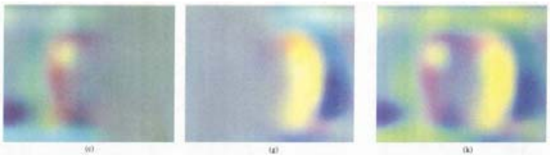


(Burt & Adelson)

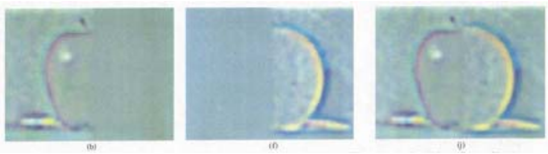
Multiresolution Spline - Example



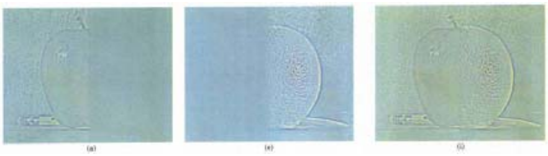
laplacian level 4



laplacian level 2



laplacian level 0

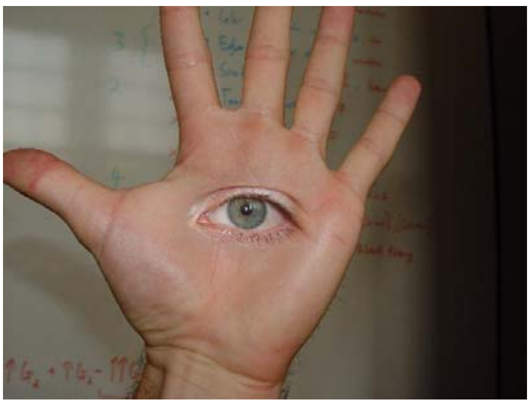


left pyramid right pyramid blended pyramid

Multiresolution Spline

Multiresolution Spline - Example

Multi-Res. Blending



© prof. dmartin

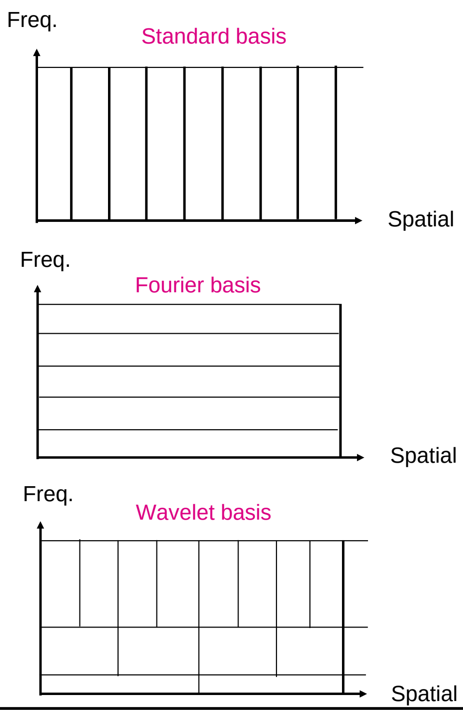
Image pyramids

- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

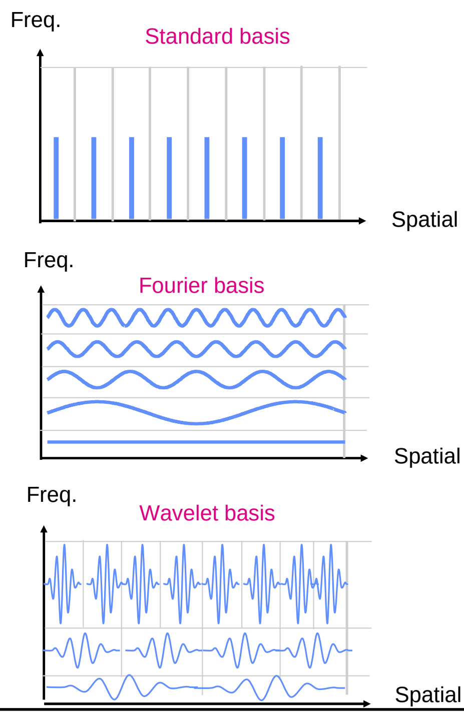
What is a good representation for image analysis?

- Pixel domain representation tells you “where” (pixel location), but not “what”.
  – In space, this representation is too localized
- Fourier transform domain tells you “what” (textural properties), but not “where”.
  – In space, this representation is too spread out.
- Want an image representation that gives you a local description of image events—what is happening where.
  – That representation might be “just right”.

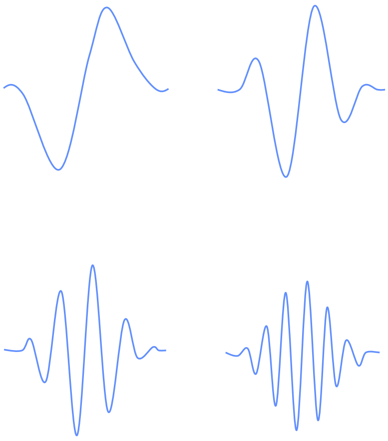
Space-Frequency Tiling



Space-Frequency Tiling

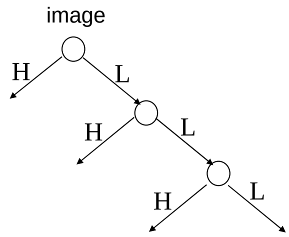


Various Wavelet basis



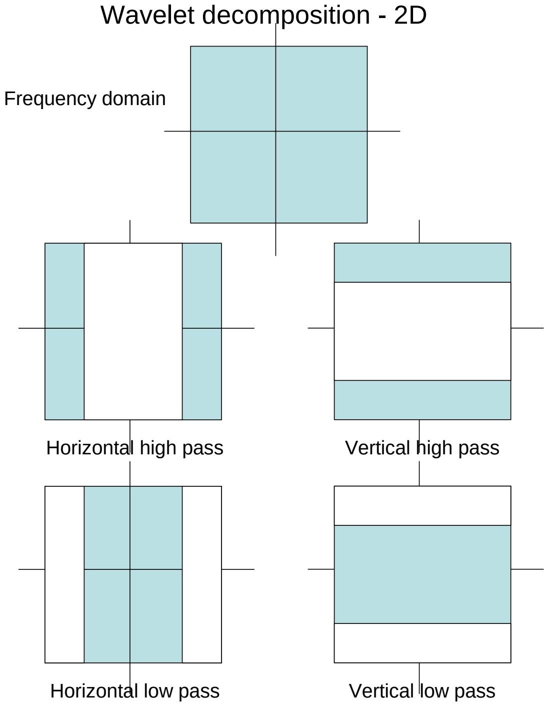
Wavelet - Frequency domain

Wavelet bands are split recursively



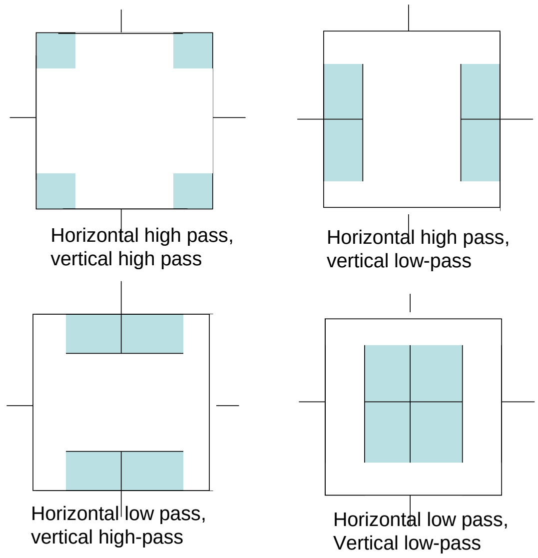
Wavelet - Frequency domain

Wavelet decomposition - 2D

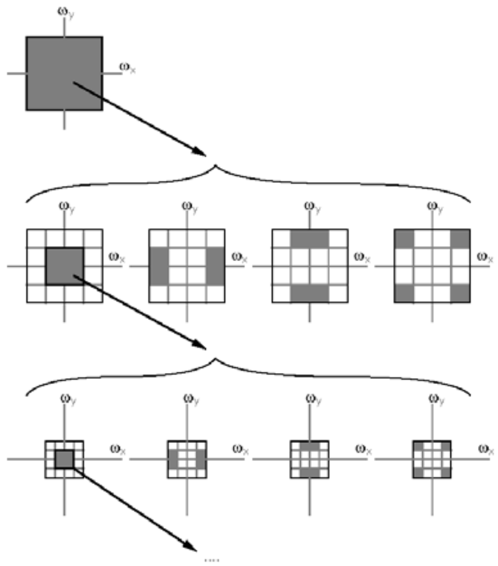


Wavelet - Frequency domain

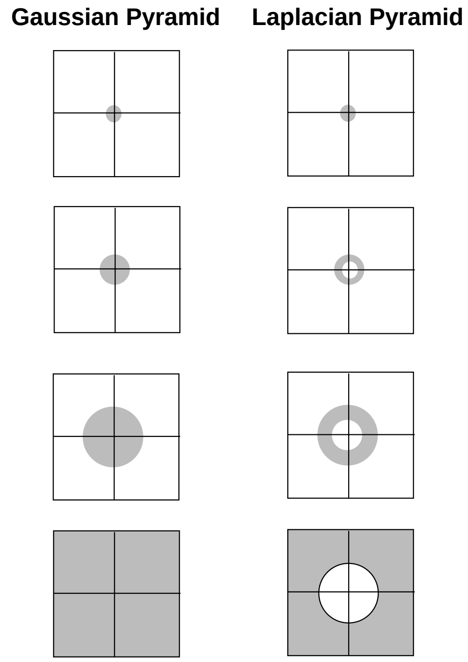
Apply the wavelet transform separably in both dimensions

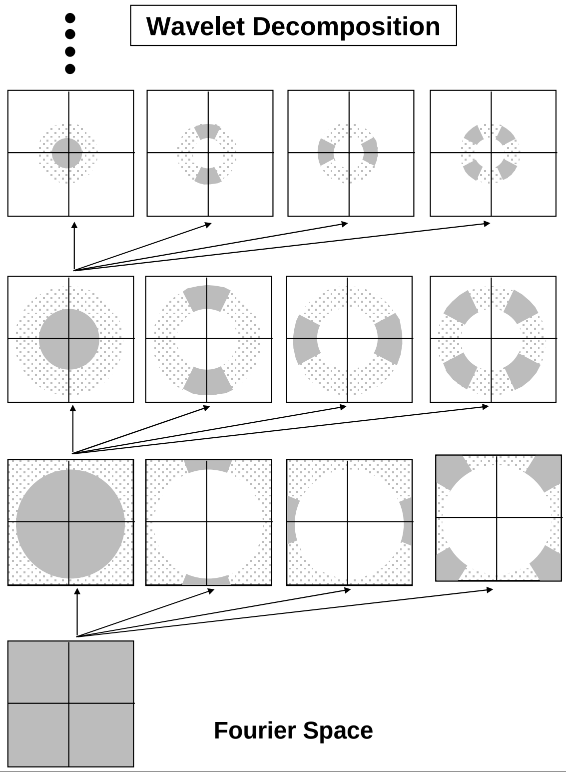


- Splitting can be applied recursively:

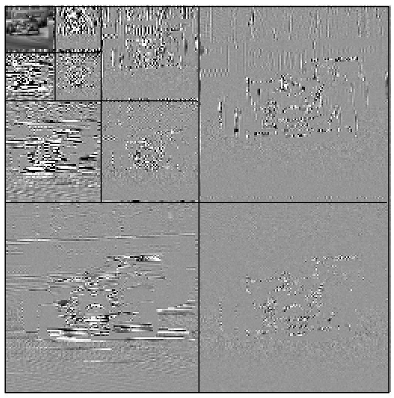


Pyramids in Frequency Domain

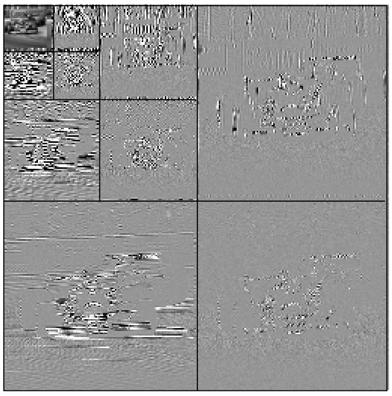




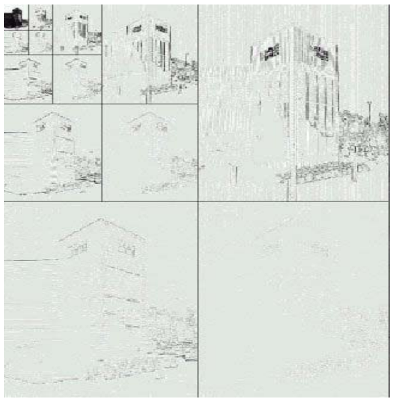
Wavelet Transform - Step By Step Example



Wavelet Transform - Example

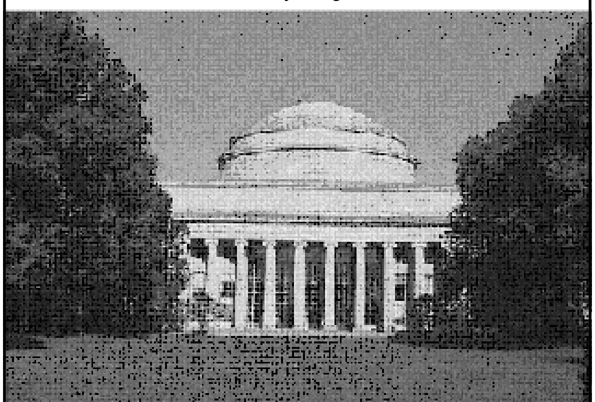


Wavelet Transform - Example



Application: Wavelet Shrinkage Denoising

Noisy image



From: B. Freeman

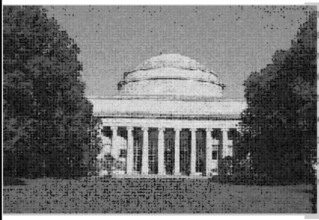
Clean image

Range [0, 255]
Dims [394, 599]

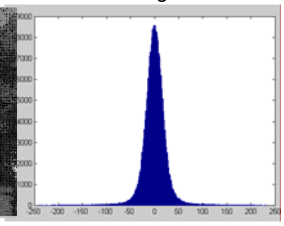
From: B. Freeman

Wavelet Shrinkage Denoising

Noisy image

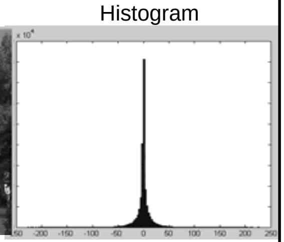


Wavelet coefficient Histogram



Clean image

Wavelet coefficient Histogram

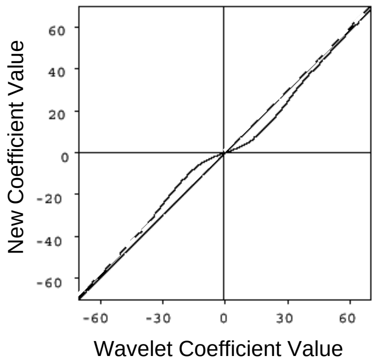


Get top histogram but want to get bottom histogram.

From: B. Freeman

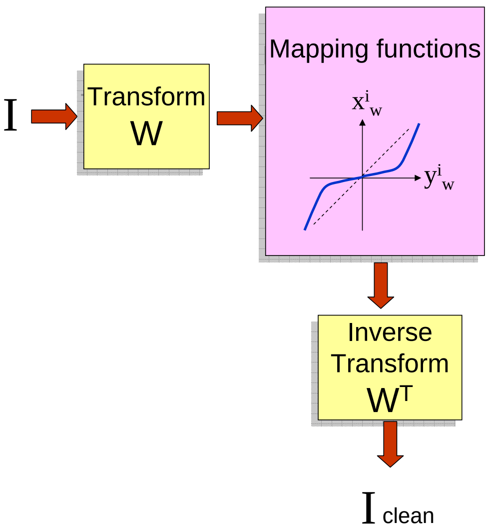
Wavelet Shrinkage Denoising

For every Wavelet Band define Shrinkage function:



From: B. Freeman

Wavelet Shrinkage Pipe-line



More results

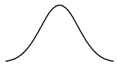
More results

Image Pyramids - Comparison

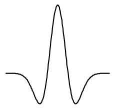
Image pyramid levels = Filter then sample.

Filters:

Gaussian Pyramid



Laplacian Pyramid



Wavelet Pyramid

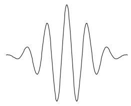
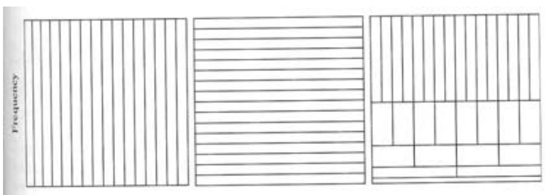


Image Linear Transforms

Transform	Basis	Characteristics
Delta	Standard	Localized in space Not localized in Frequency
Fourier	Sines+Cosines	Not localized in space Localized in Frequency
Wavelet Pyramid	Wavelet Filters	Localized in space Localized in Frequency

Delta Fourier Wavelet



space space space

Convolution and Transforms in matrix notation (1D case)

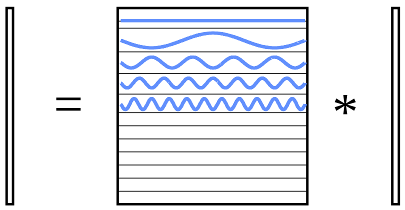
$$\vec{F} = U\vec{f}$$

transformed image

Vectorized image

Basis vectors (Fourier, Wavelet, etc)

Fourier Transform



Fourier transform | Fourier bases | pixel domain image

Fourier bases are global: each transform coefficient depends on all pixel locations.

From: B. Freeman

Transform in matrix notation (1D case)

Forward Transform:

$$\vec{F} = U\vec{f}$$

transformed image

Vectorized image

Basis vectors (Fourier, Wavelet, etc)

Inverse Transform:

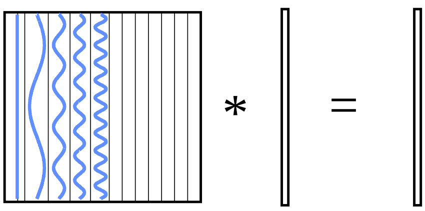
$$U^{-1}\vec{F} = \vec{f}$$

Basis vectors

Vectorized image

transformed image

Inverse Fourier Transform

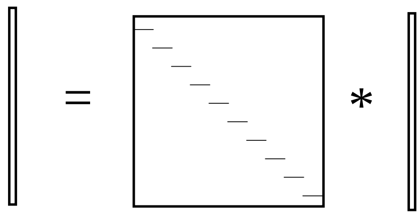


Fourier bases | Fourier transform | pixel domain image

Every image pixel is a linear combination of the Fourier basis weighted by the coefficient.

Note that if U is orthonormal basis then $U^{-1} = U^{t}$

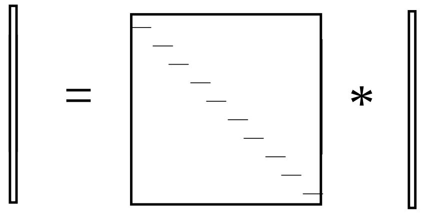
Convolution



Convolution Result | Circular Matrix of Filter Kernels | pixel image

From: B. Freeman

Pyramid = Convolution + Sampling



Convolution Result

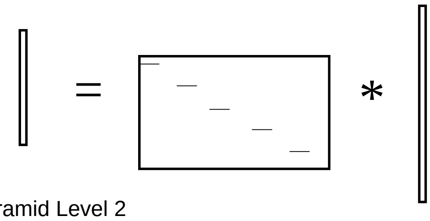
Matrix of Filter Kernels

pixel image

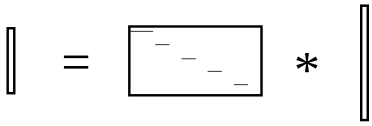
From: B. Freeman

Pyramid = Convolution + Sampling

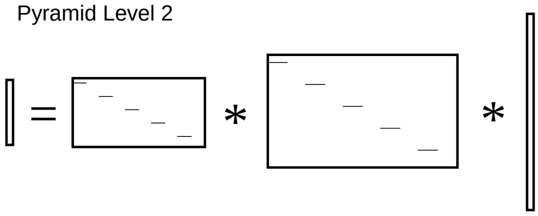
Pyramid Level 1



Pyramid Level 2

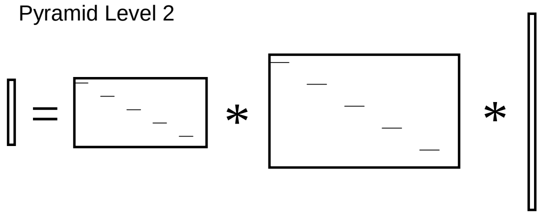


Pyramid Level 2

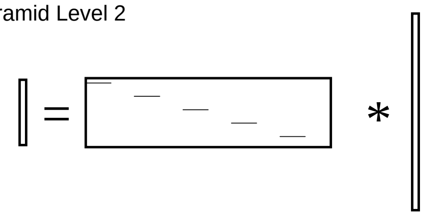


Pyramid = Convolution + Sampling

Pyramid Level 2



Pyramid Level 2



Pyramid as Matrix Computation - Example

U1 =

1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0

- Next pyramid level

U2 =

1	4	6	4	1	0	0	0
0	0	1	4	6	4	1	0
0	0	0	0	1	4	6	4
0	0	0	0	0	0	1	4

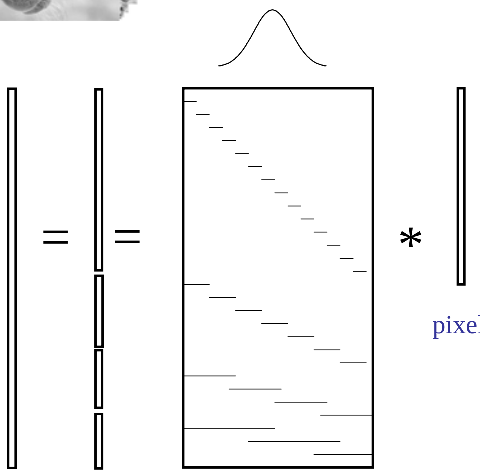
- The combined effect of the two pyramid levels

U2 * U1 =

1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0	0	0	0	0
0	0	0	0	1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0
0	0	0	0	0	0	0	0	1	4	10	20	31	40	44	40	30	16	4	0
0	0	0	0	0	0	0	0	0	0	0	0	1	4	10	20	25	16	4	0

from: B.Freeman

Gaussian Pyramid



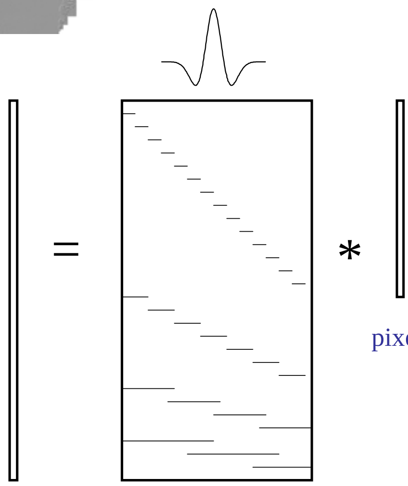
pixel image

Gaussian pyramid

Overcomplete representation.
Low-pass filters, sampled appropriately for their blur.

From: B. Freeman

Laplacian Pyramid

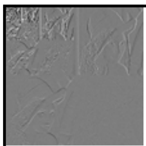


pixel image

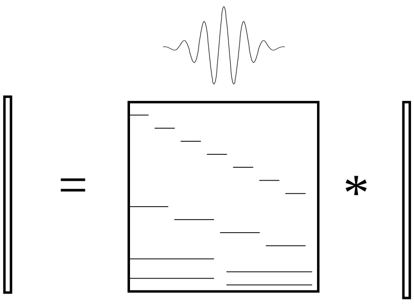
Laplacian pyramid

Overcomplete representation.
Transformed pixels represent bandpassed image information.

From: B. Freeman



Wavelet Transform



Wavelet pyramid

Ortho-normal transform (like Fourier transform), but with localized basis functions.

pixel image

From: B. Freeman

The End