Image Representation

- Gaussian pyramids
- Laplacian Pyramids
- Wavelet Pyramids
- Applications

Image Pyramids

Image features at different resolutions require filters at different scales.

Edges (derivatives):

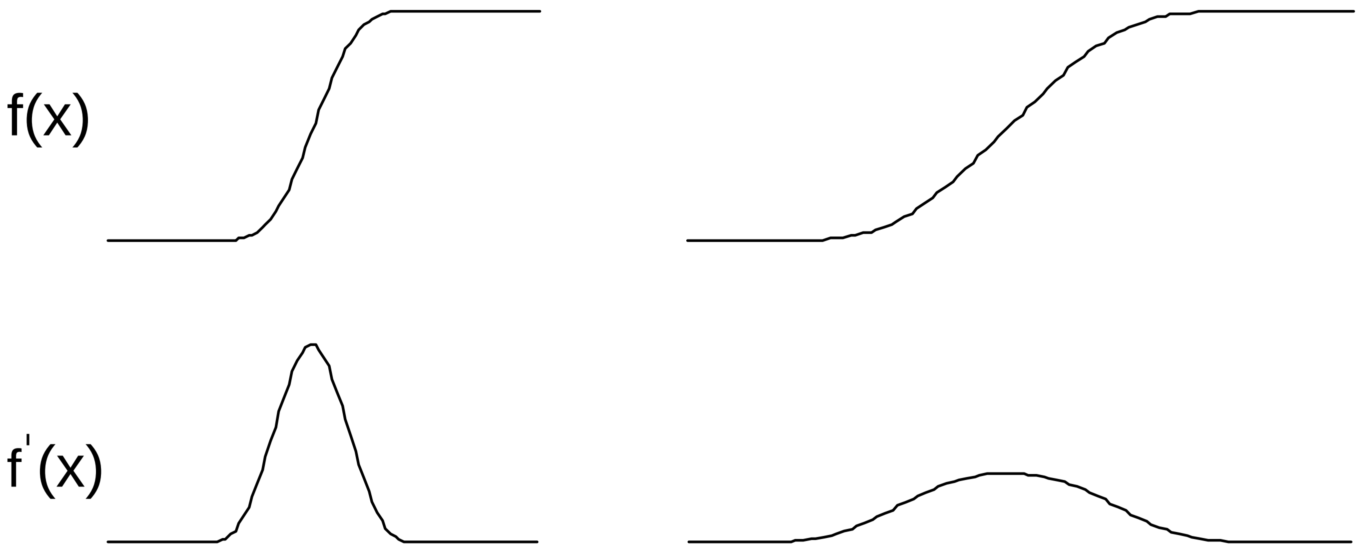
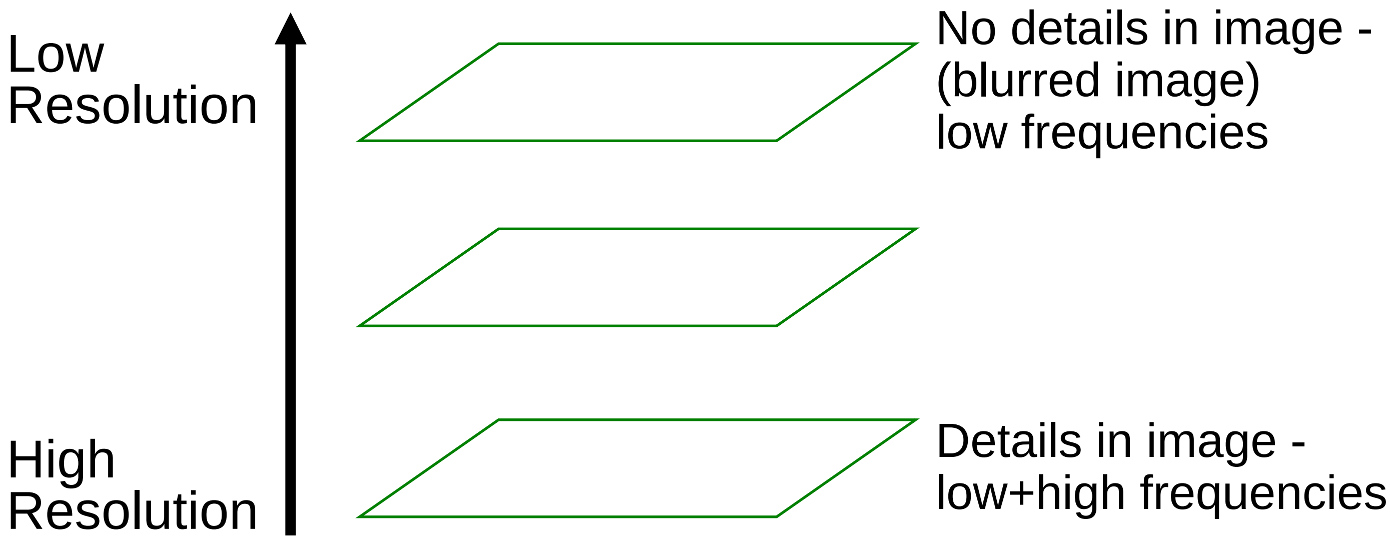


Image Pyramids

Image Pyramid = Hierarchical representation of an image



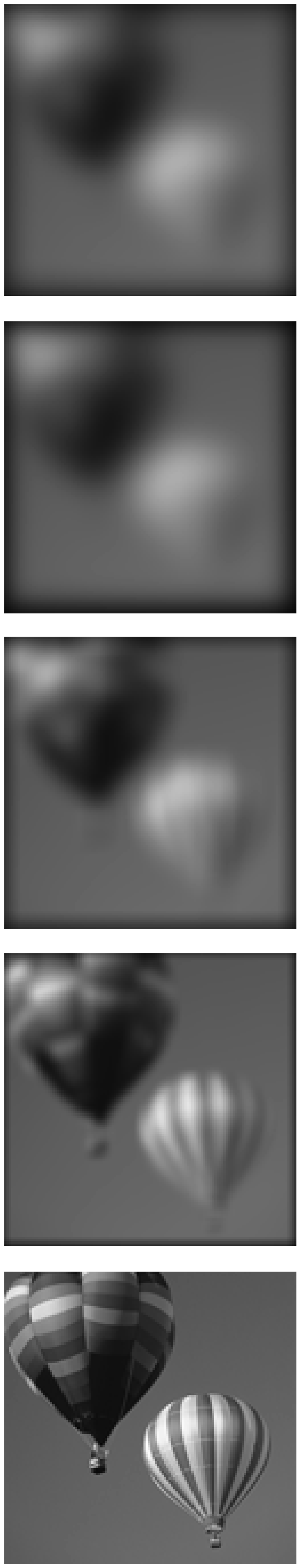
A collection of images at different resolutions.

Image pyramids

- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

Image Pyramid

Low resolution



High resolution

Image Pyramid
Frequency Domain

Low resolution

High resolution

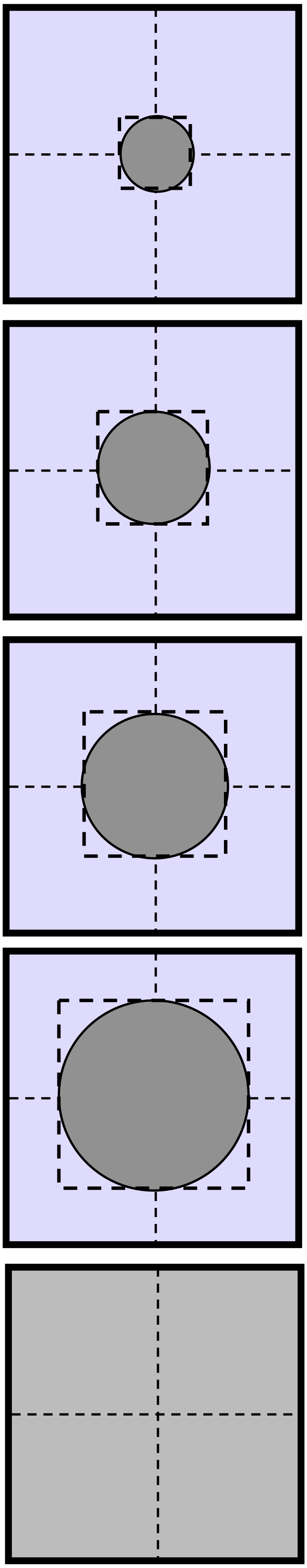
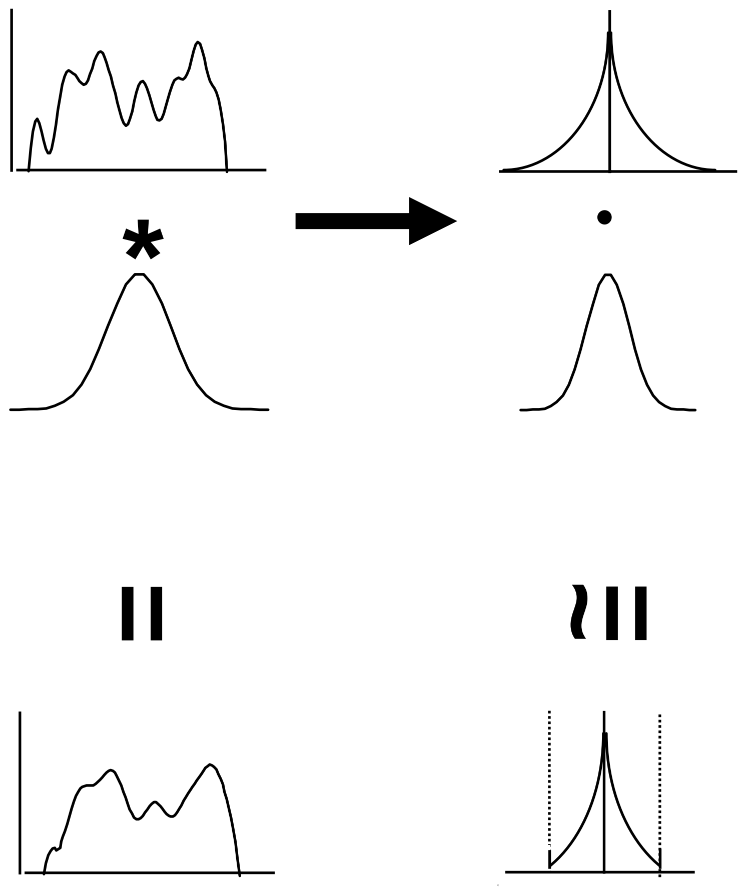


Image Blurring = low pass filtering



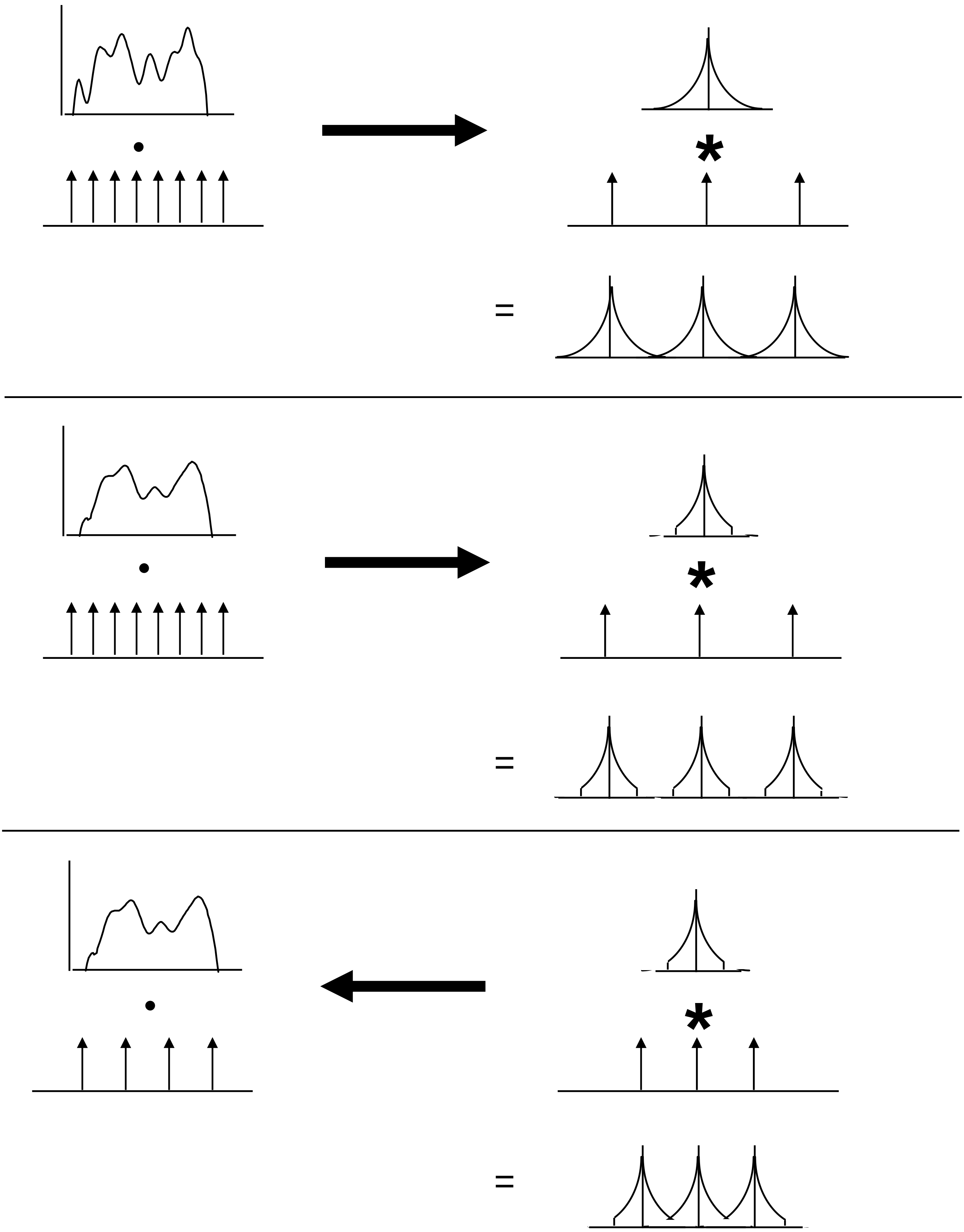
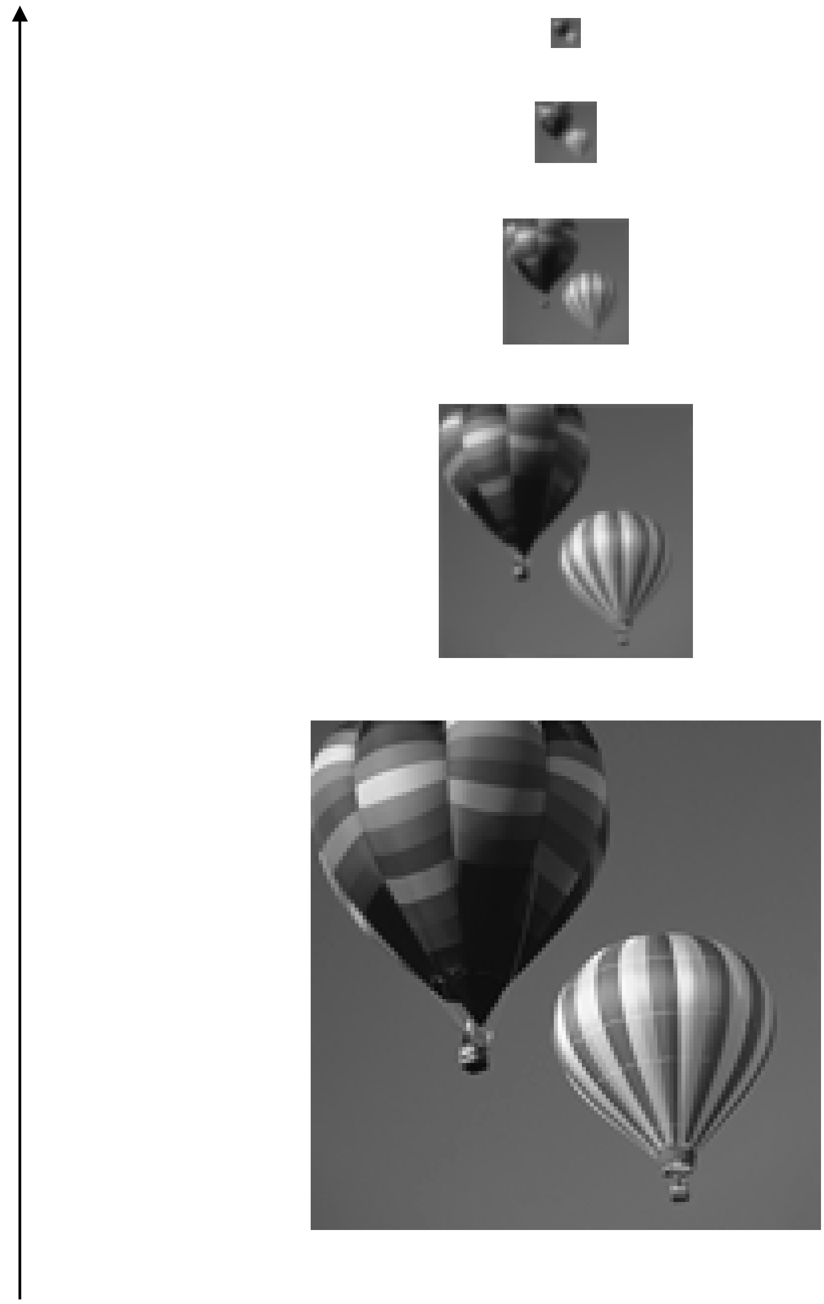


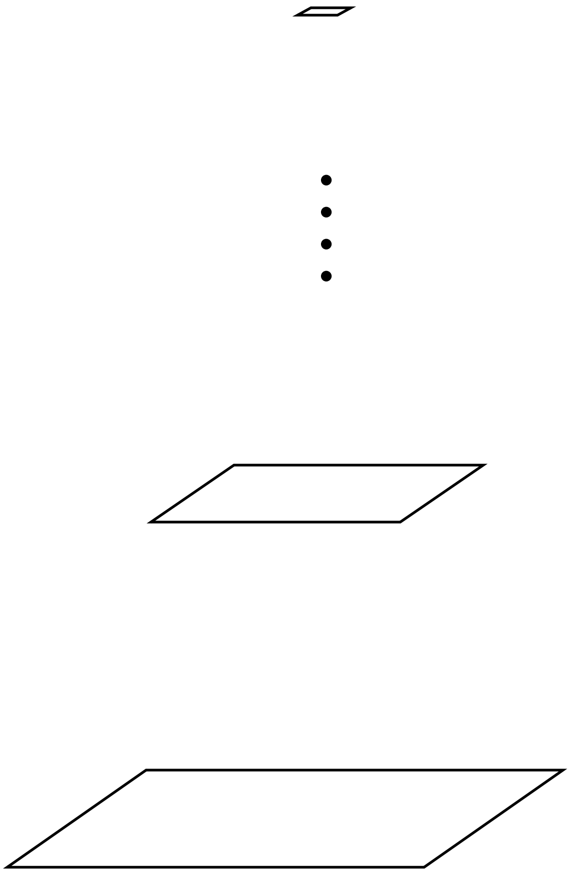
Image Pyramid

Low resolution



High resolution

Gaussian Pyramid



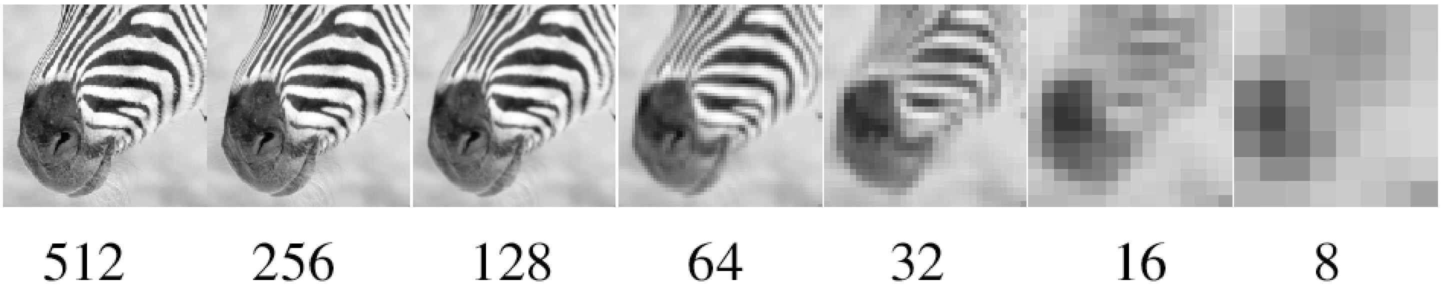
Level n
1×1

⋮

Level 1
$2^{n-1} \times 2^{n-1}$

Level 0
$2^n \times 2^n$

Gaussian Pyramid



Gaussian Pyramid

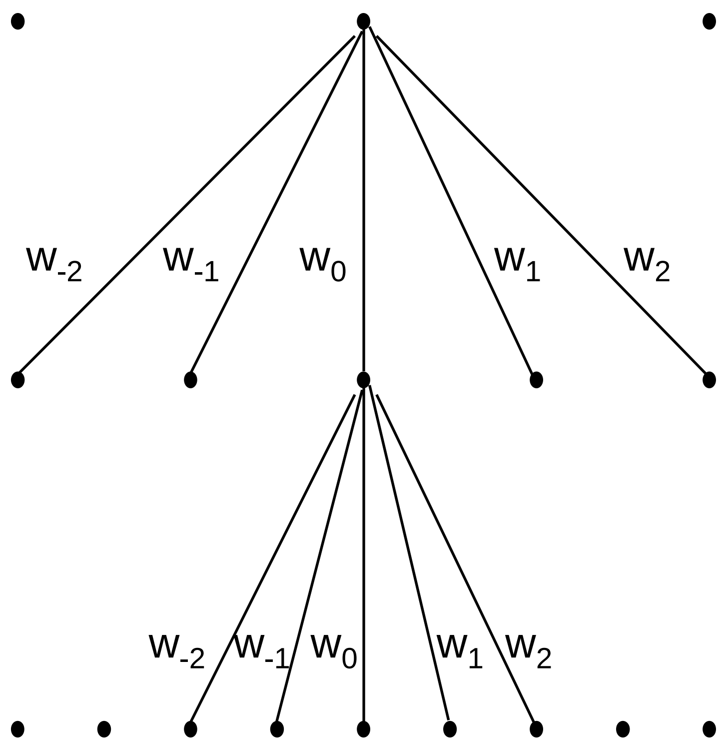
Burt & Adelson (1981)

Normalized: $\Sigma w_i = 1$

Symmetry: $w_i = w_{-i}$

Unimodal: $w_i \geq w_j$ for $0 < i < j$

Equal Contribution: for all j Σw_{j+2i} = constant



Gaussian Pyramid

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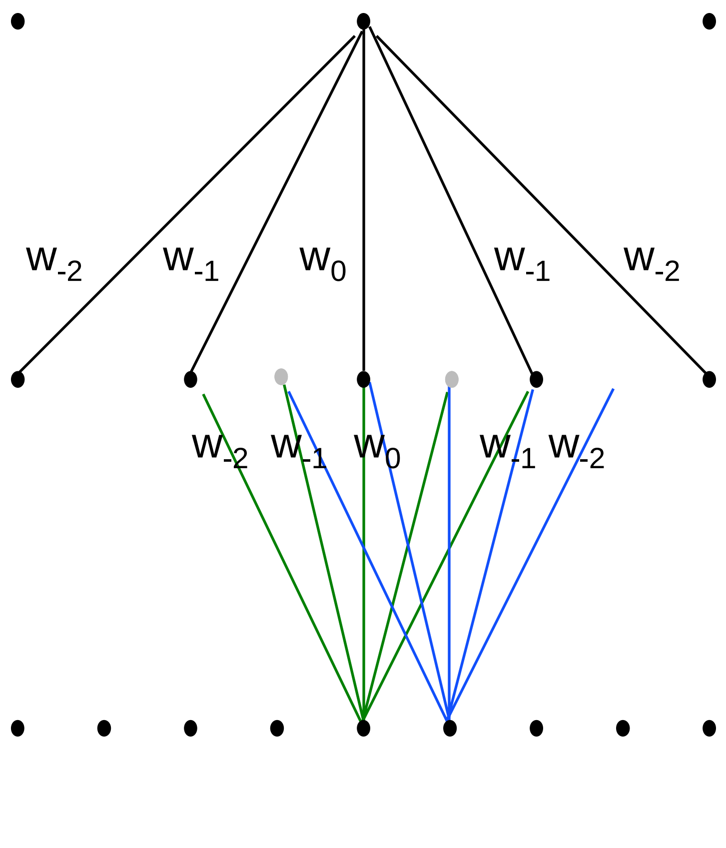


Gaussian Pyramid

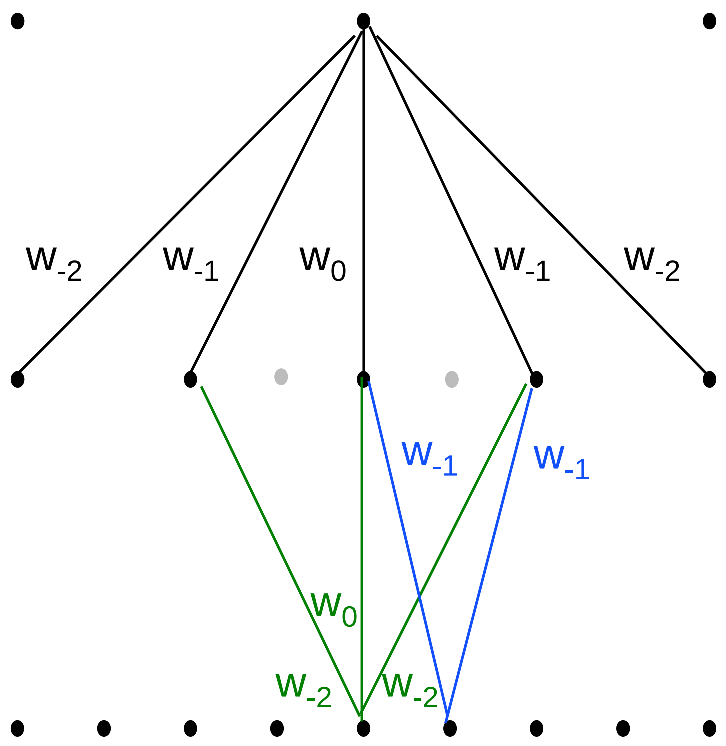
Burt & Adelson (1981)

Normalized: $\Sigma w_i = 1$

Symmetry: $w_i = w_{-i}$

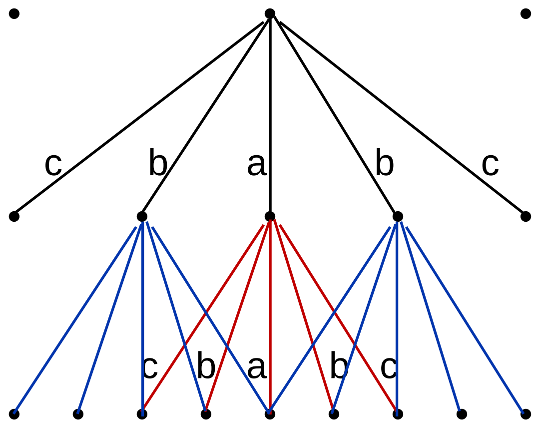
Unimodal: $w_i \geq w_j$ for $0 < i < j$

Equal Contribution: for all j Σw_{j+2i} = constant



Gaussian Pyramid

Burt & Adelson (1981)



$$a + 2b + 2c = 1$$
$$a + 2c = 2b$$

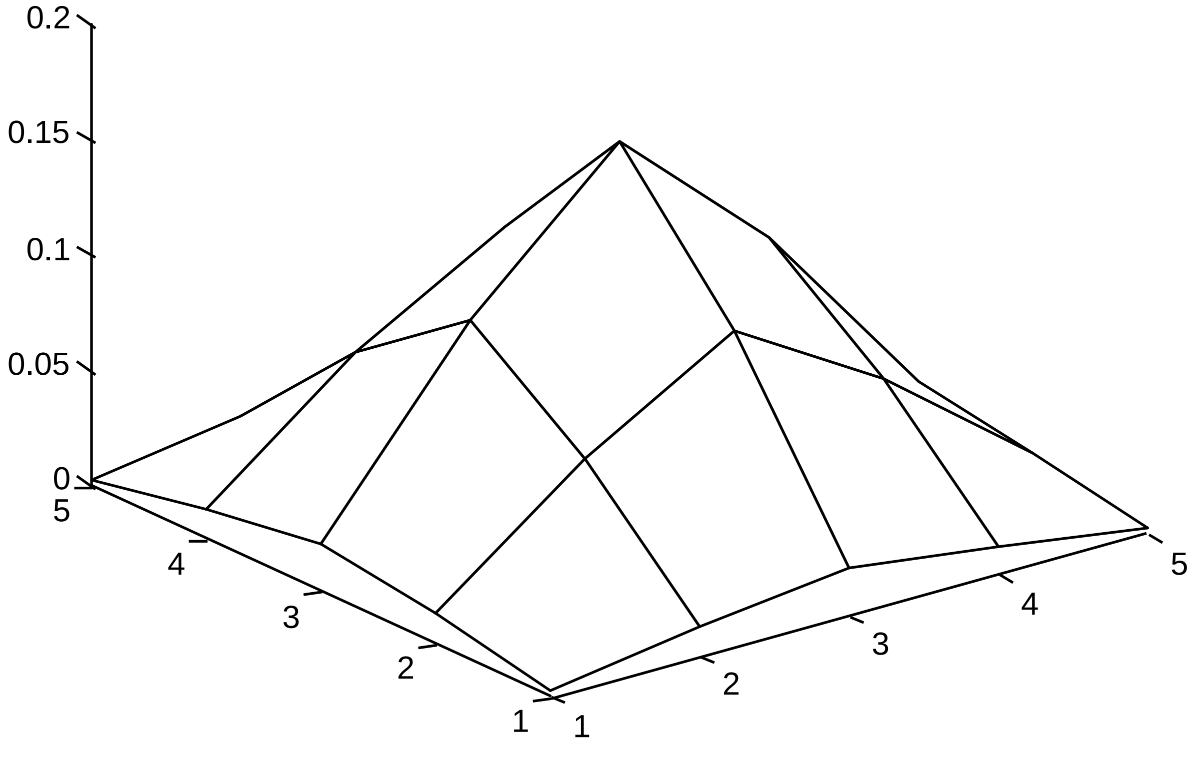
$$\boxed{\begin{array}{l} a > 0.25 \\ b = 0.25 \\ c = 0.25 - a/2 \end{array}}$$

For a = 0.4 most similar to a Gauusian filter

g = [0.05 0.25 0.4 0.25 0.05]

low_pass_filter = g' * g =

$$\begin{bmatrix} 0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \\ 0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\ 0.0200 & 0.1000 & 0.1600 & 0.1000 & 0.0200 \\ 0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\ 0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \end{bmatrix}$$



Gaussian Pyramid - Computational Aspects

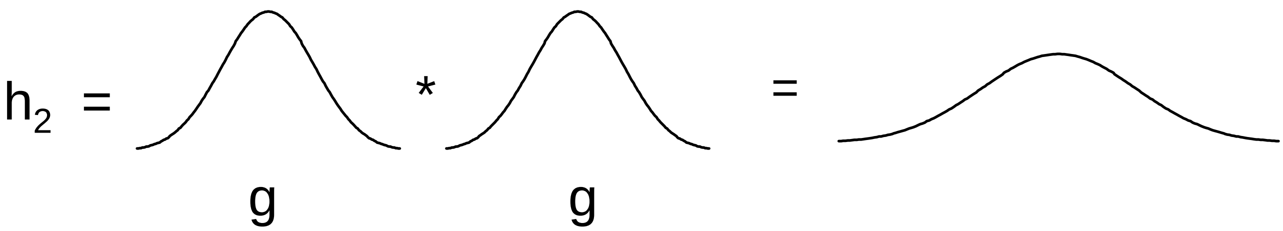
Memory:

$$2^N X 2^N (1 + 1/4 + 1/16 + \dots) = 2^N X 2^N * 4/3$$

Computation:

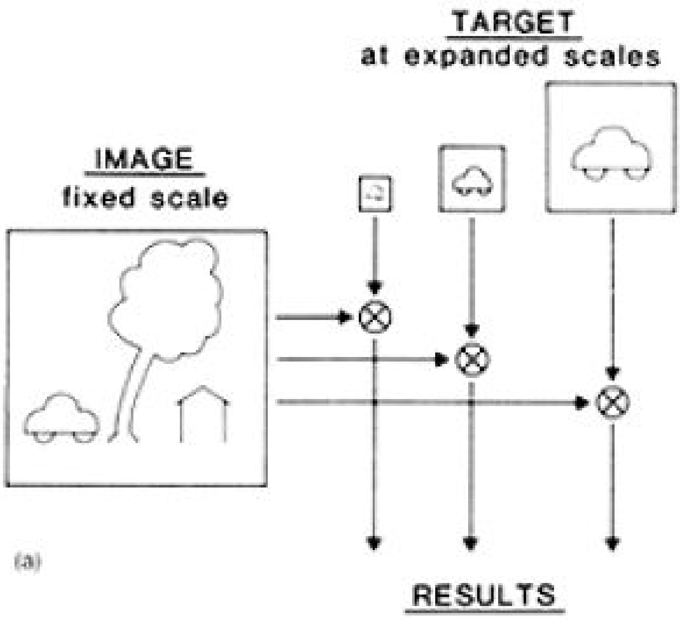
Level i can be computed with a single convolution with filter: $h_i = \underbrace{g * g * g *}_{\text{i times}} .....$

Example:

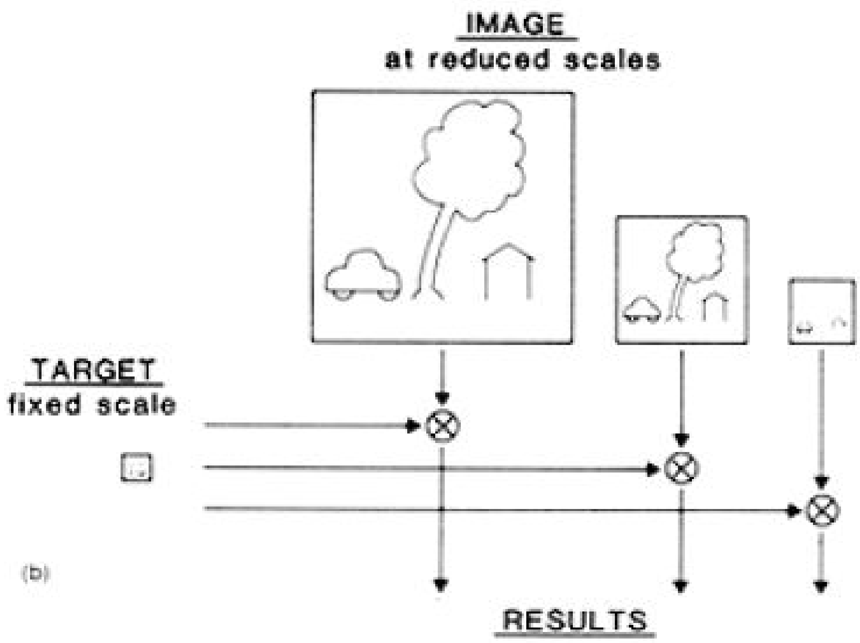


MultiScale Pattern Matching

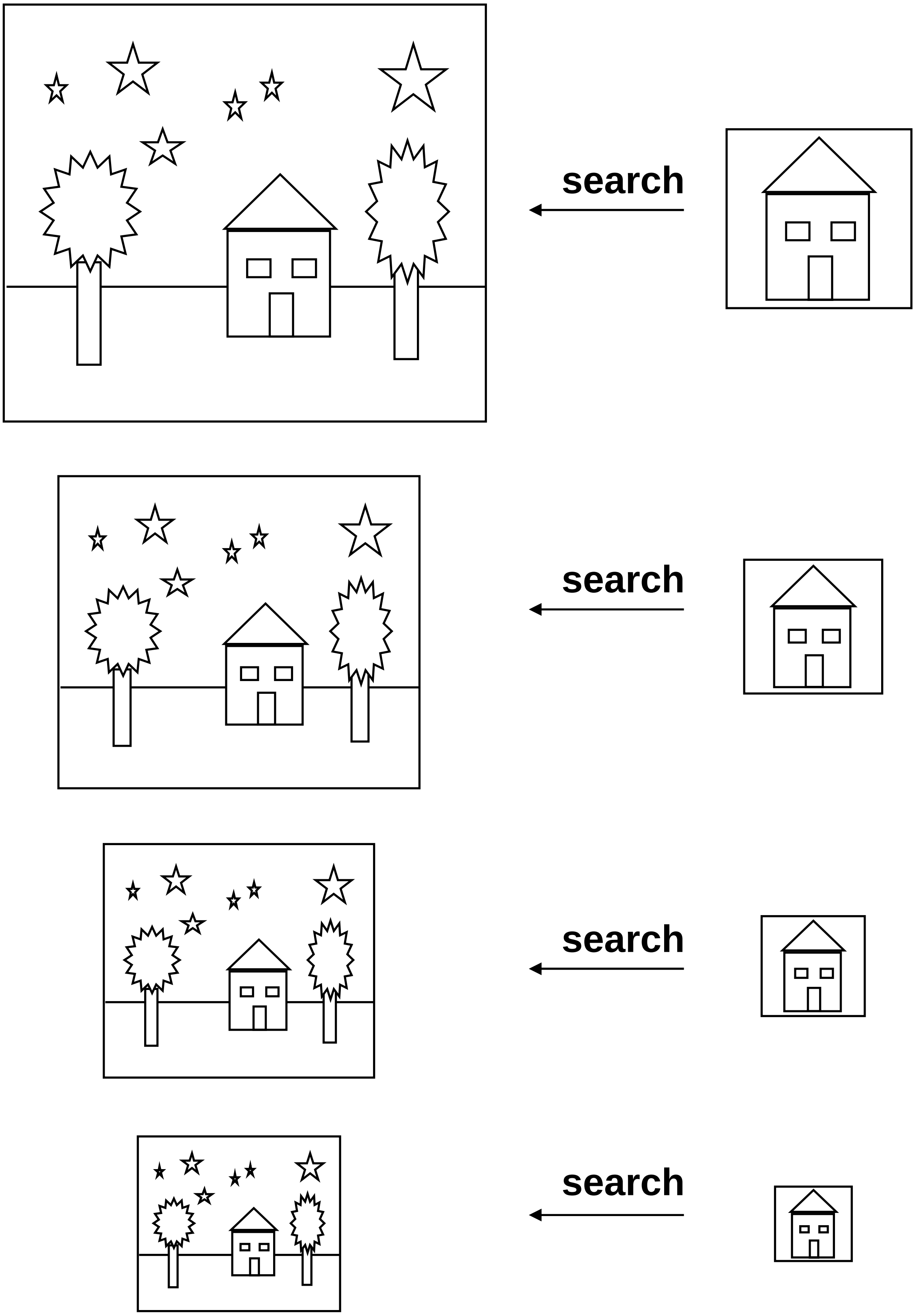
Option 1:
Scale target and search for each in image.



Option 2:
Search for original target in image pyramid.



Hierarchical Pattern Matching



Pattern matching using Pyramids - Example

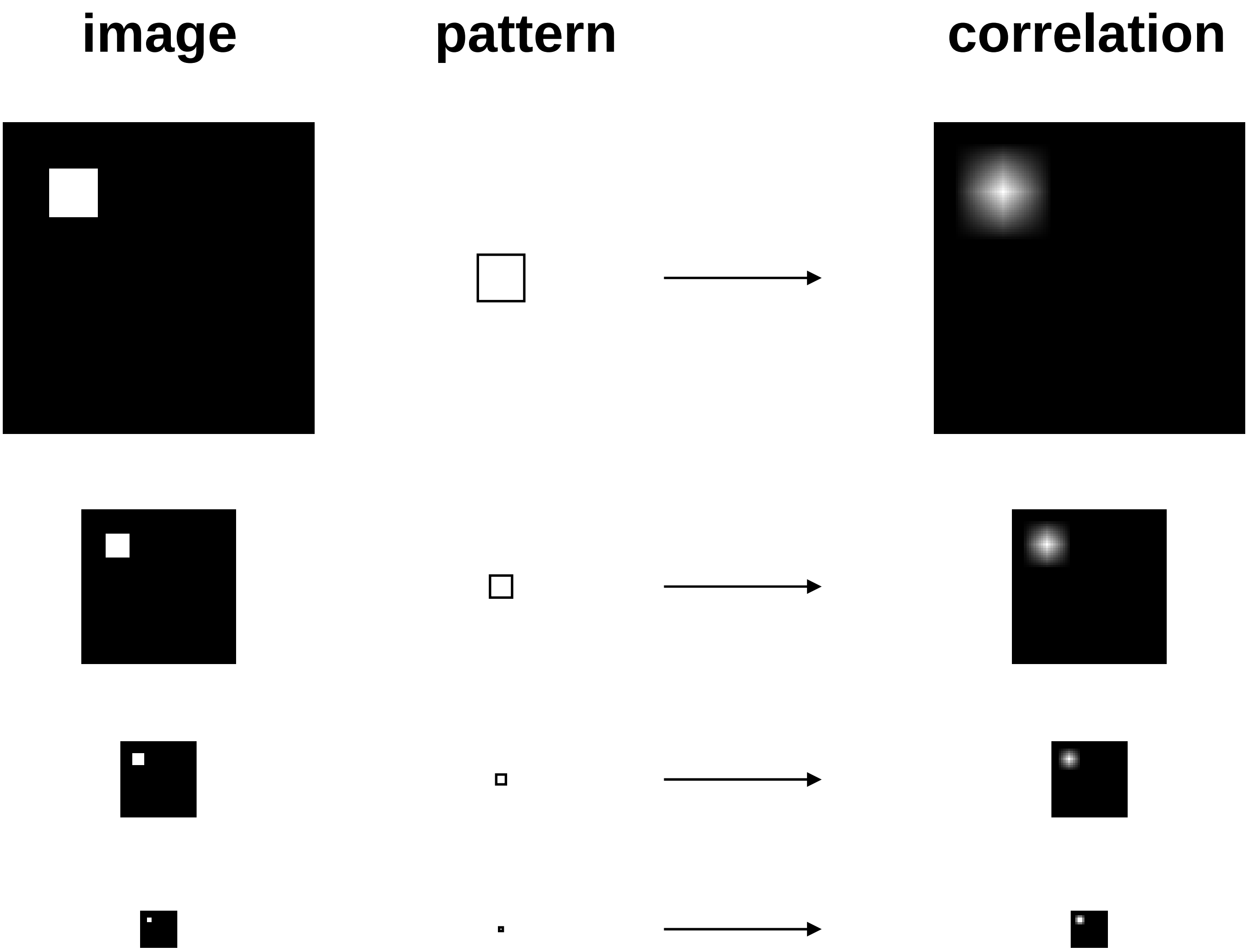


Image pyramids

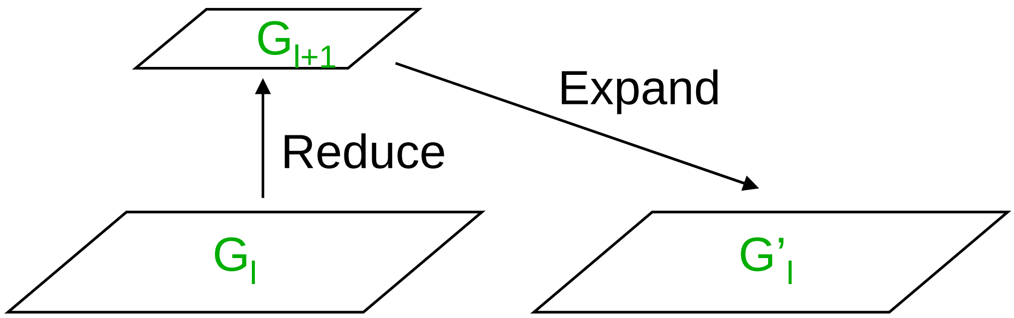
- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

Laplacian Pyramid

Motivation = Compression, redundancy removal.
compression rates are higher for predictable values.
e.g. values around 0.

$G_0, G_1, \ldots.$ = the levels of a Gaussian Pyramid.

Predict level G_l from level G_{l+1} by Expanding G_{l+1} to G'_l



Denote by L_l the error in prediction:

$$\boxed{L_l = G_l - G'_l}$$

$L_0, L_1, \ldots.$ = the levels of a Laplacian Pyramid.

What does blurring take away?

original

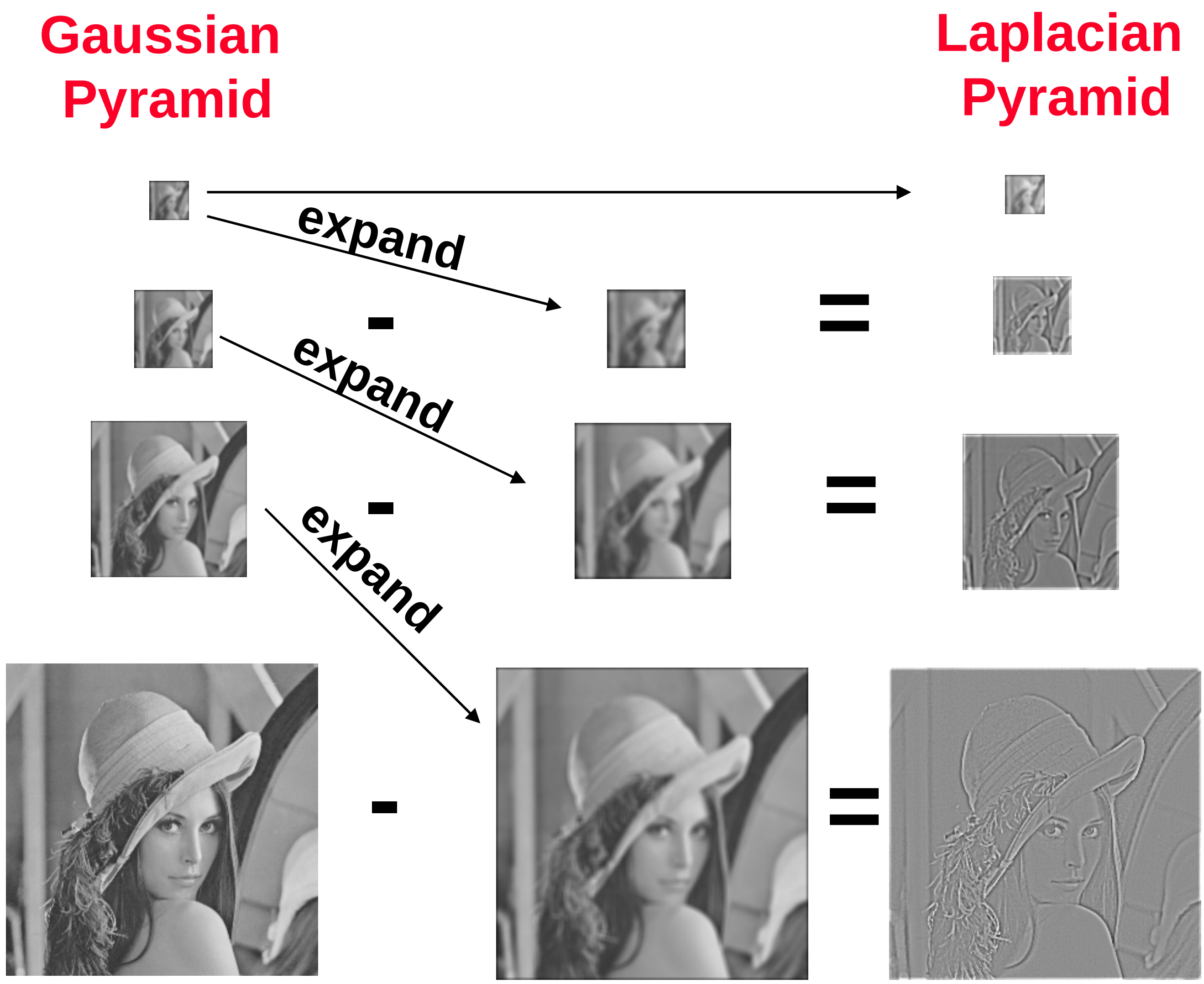
What does blurring take away?

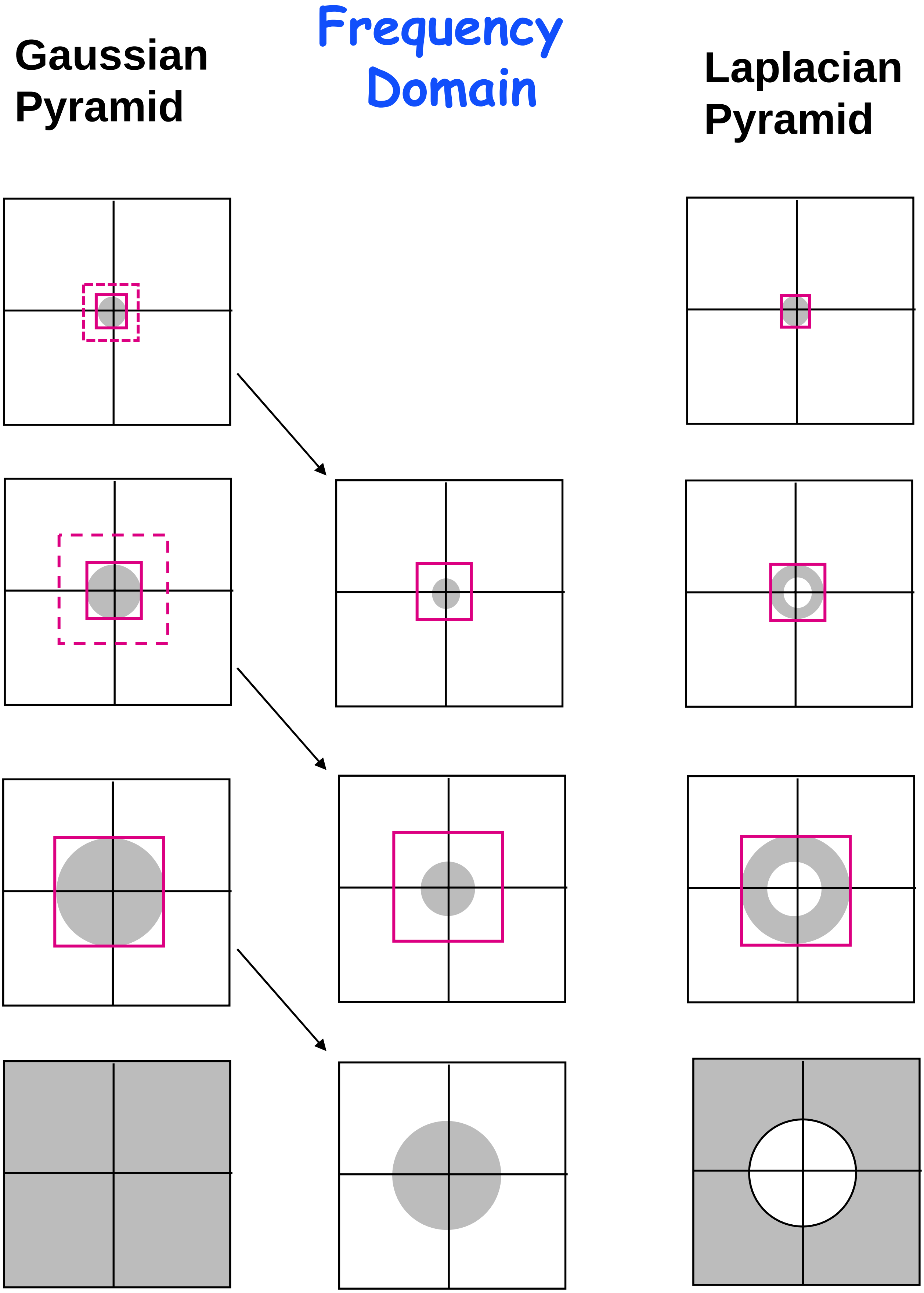
smoothed (5x5 Gaussian)

What does blurring take away?

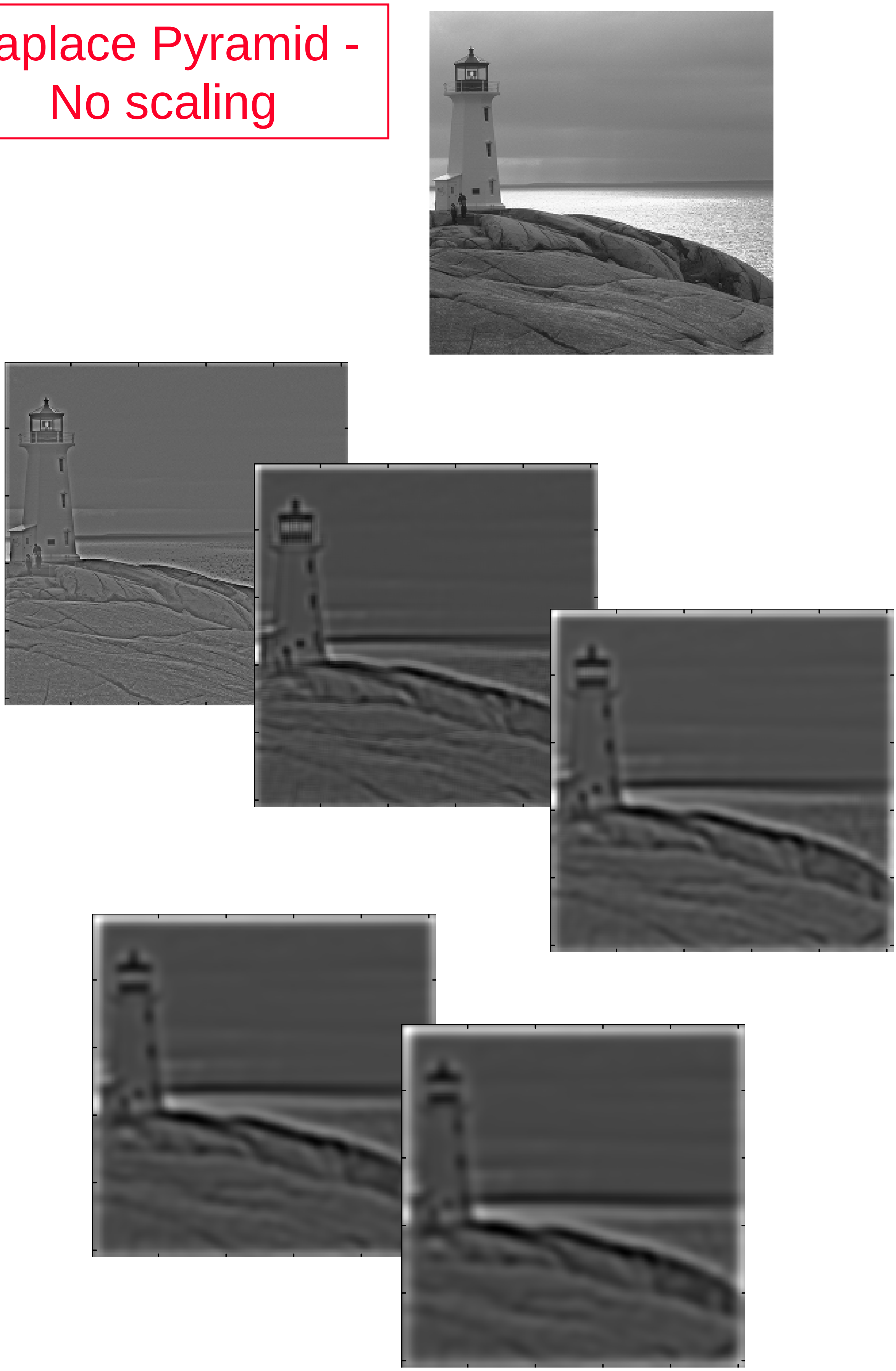
smoothed – original

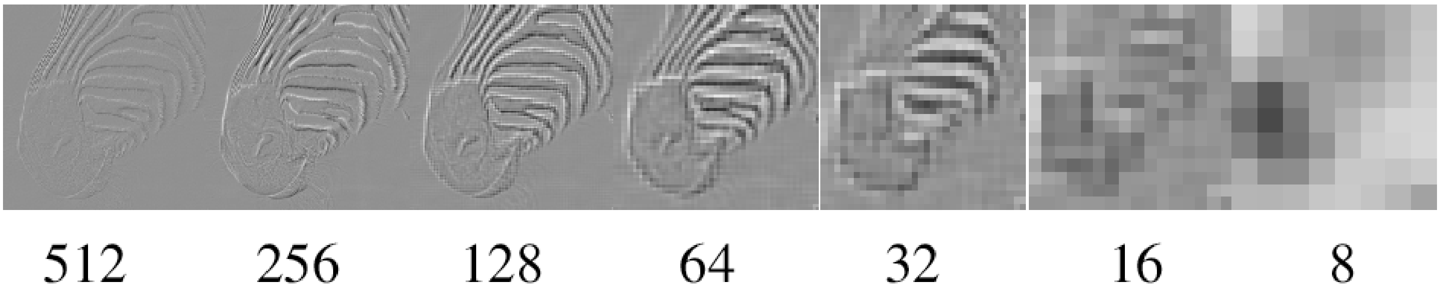
Laplacian Pyramid

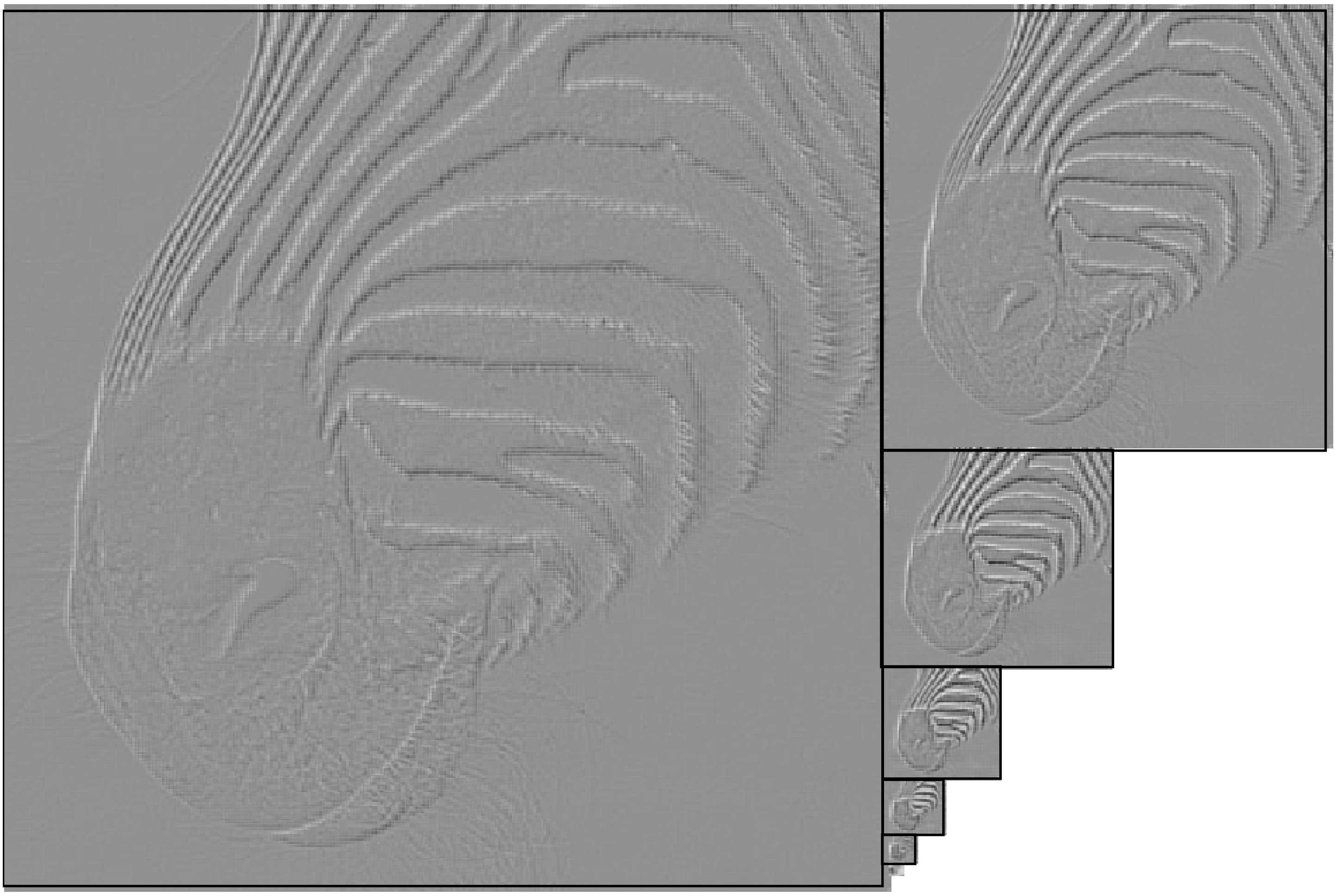



Gaussian Pyramid
Frequency Domain
Laplacian Pyramid

Laplace Pyramid - No scaling





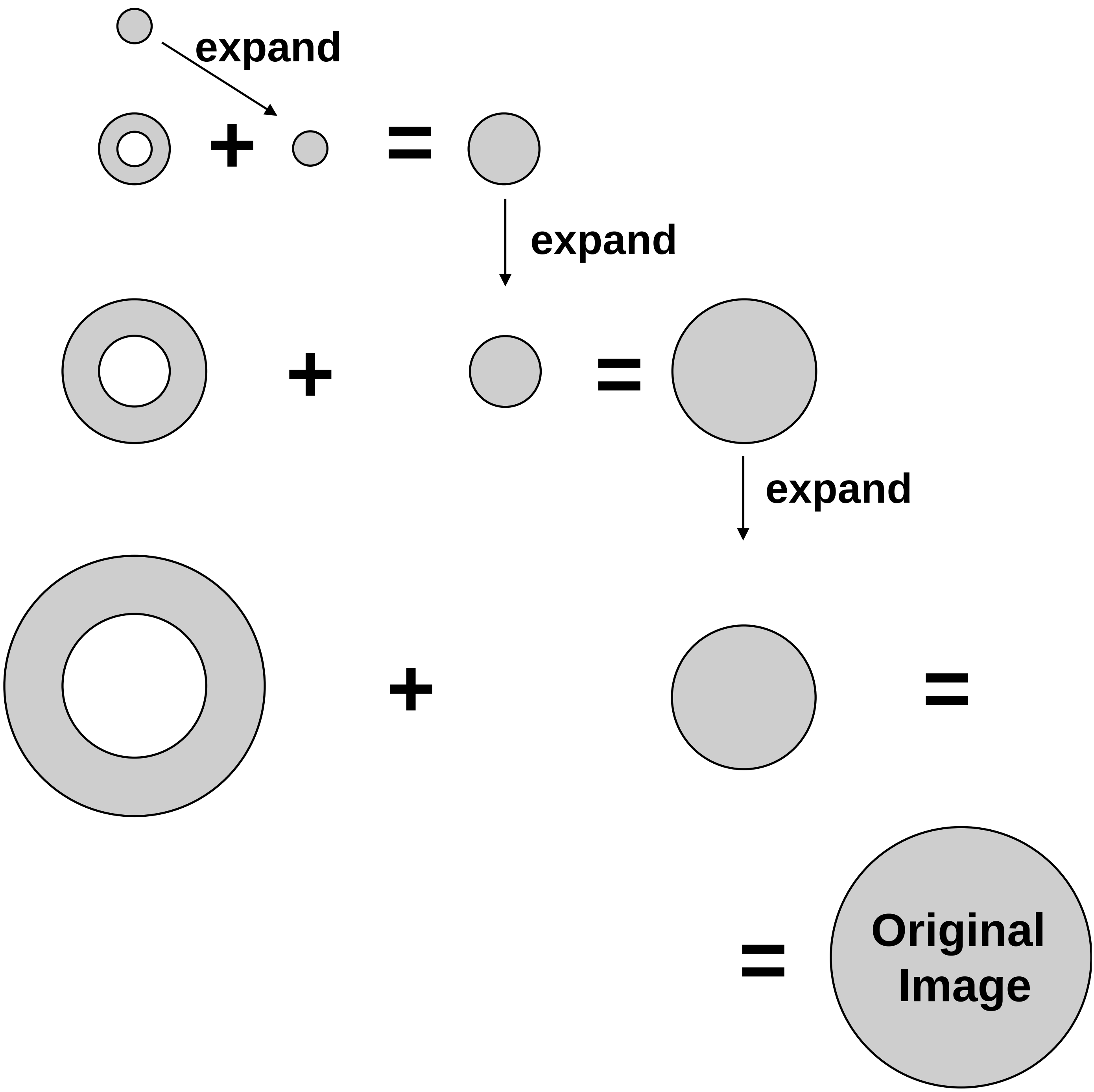


from: B.Freeman

Reconstruction of the original image from the Laplacian Pyramid

$G_l = L_l + G'_l$

Laplacian Pyramid



Laplacian Pyramid - Computational Aspects

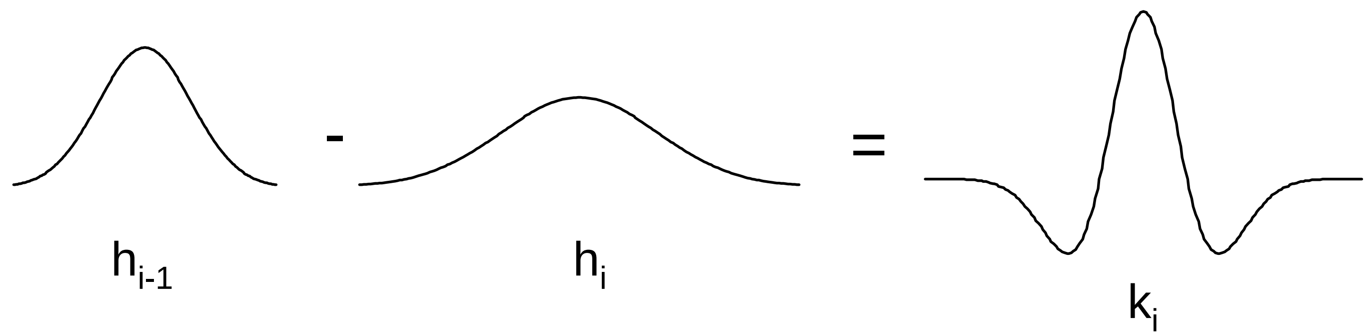
Memory:

$2^N X 2^N (1 + 1/4 + 1/16 + \ldots) = 2^N X 2^N * 4/3$

However coefficients are highly compressible.

Computation:

L_i can be computed from G_0 with a single convolution with filter: $k_i = h_{i-1} - h_i$



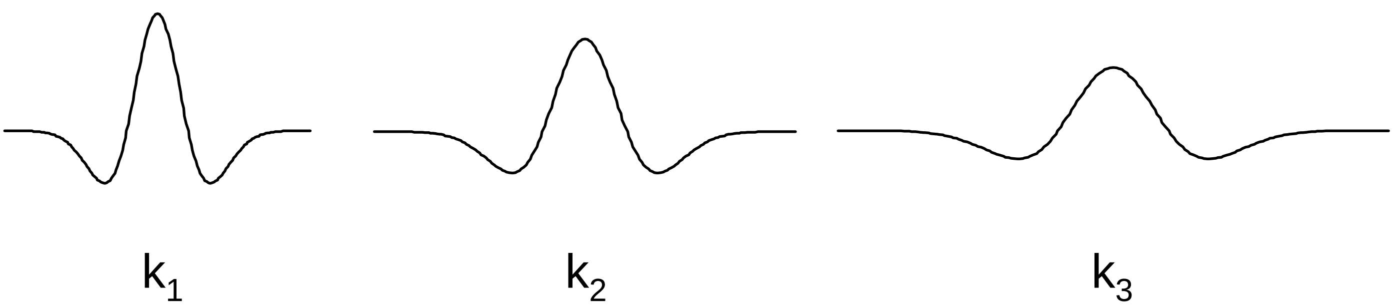
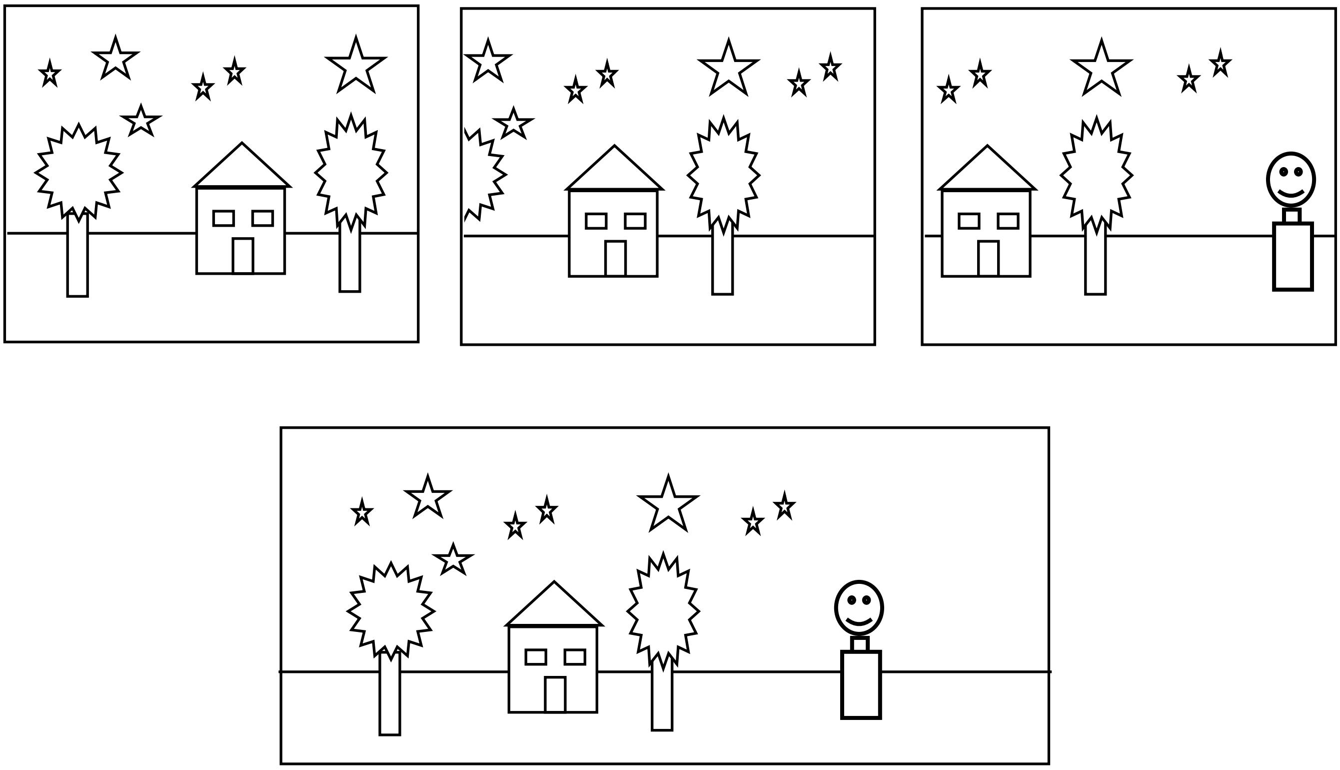


Image Mosaicing



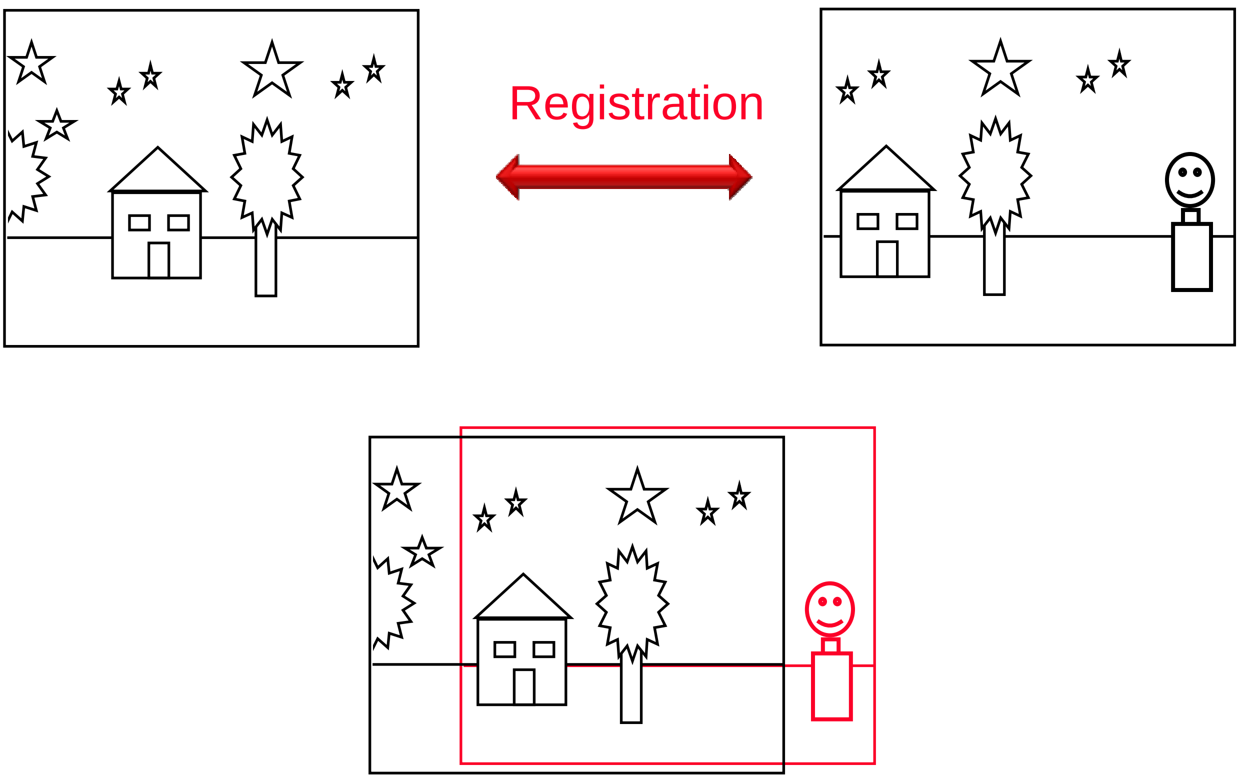
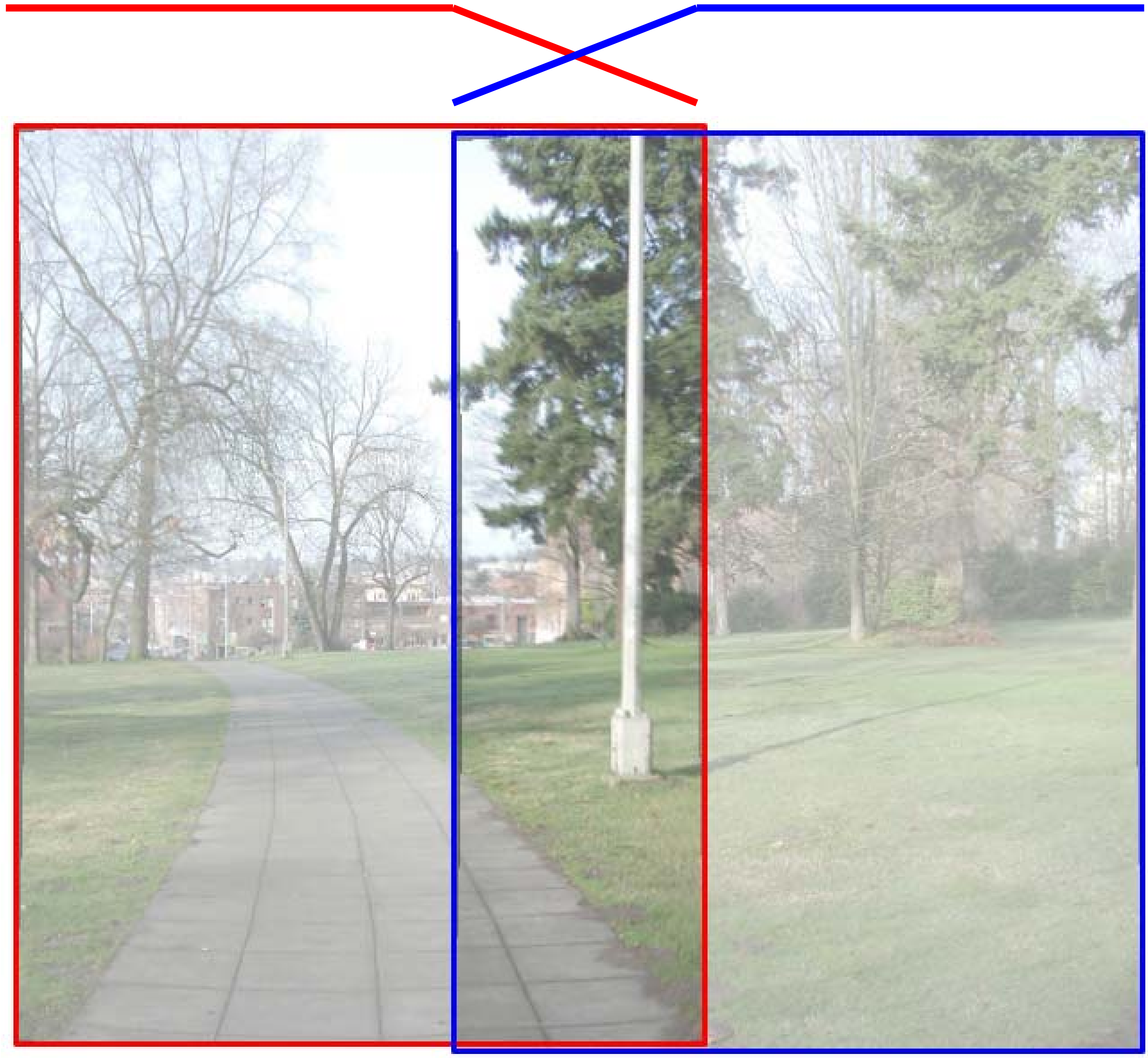


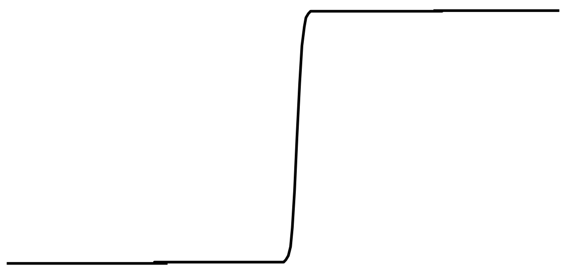
Image Blending



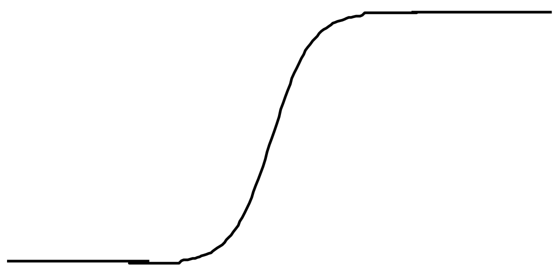
Blending

Multiresolution Spline

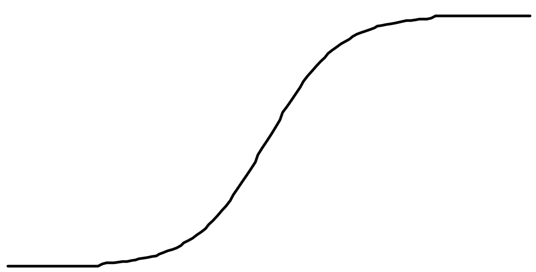
When splining two images, transition from one image to the other should behave:



High Frequencies

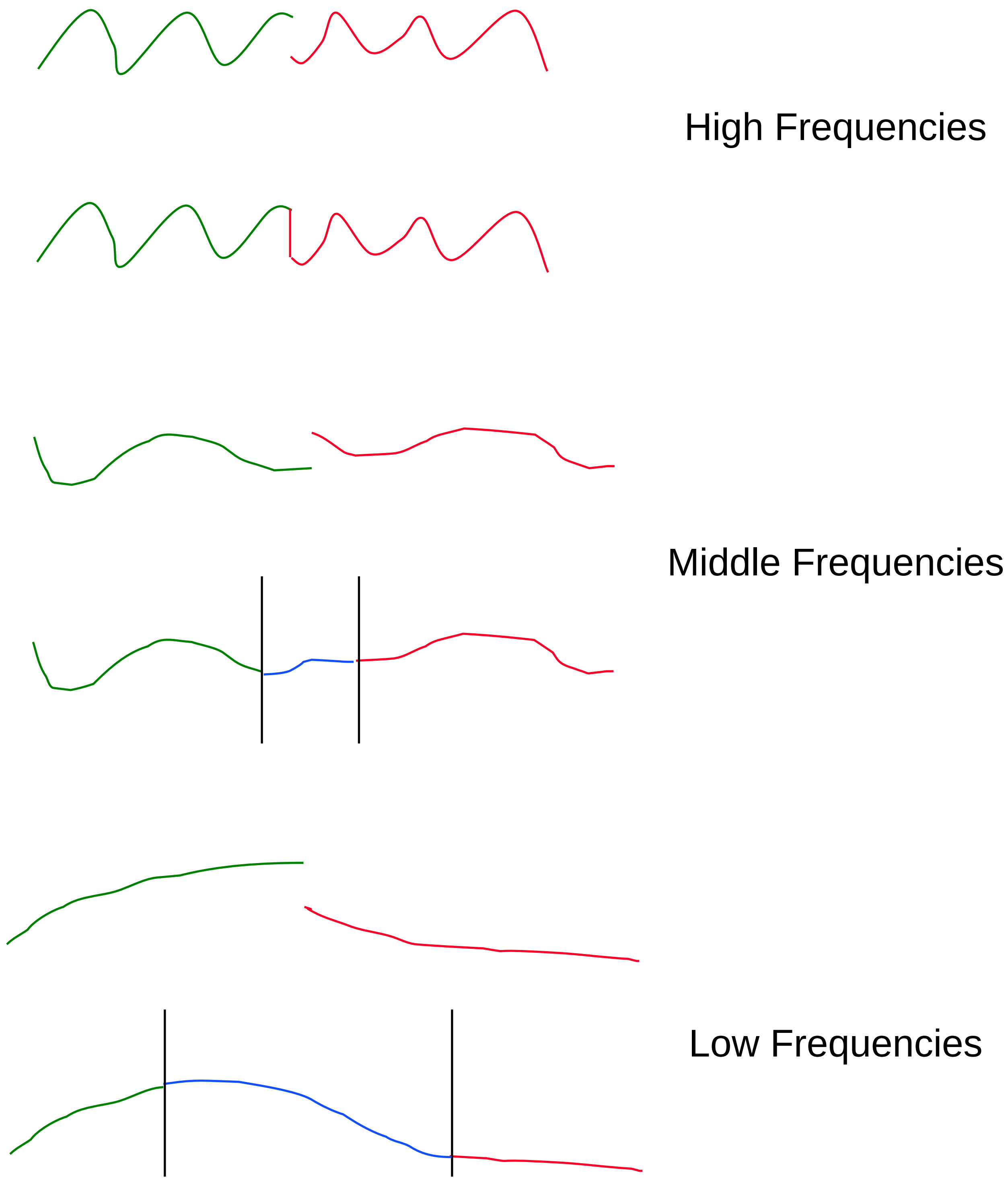


Middle Frequencies



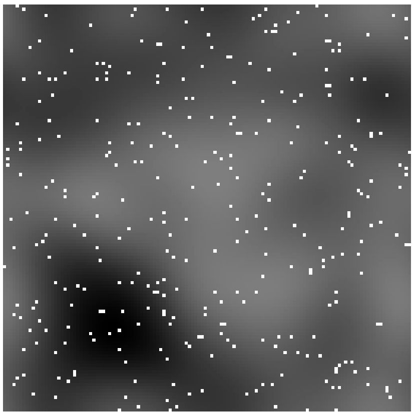
Low Frequencies

Multiresolution Spline

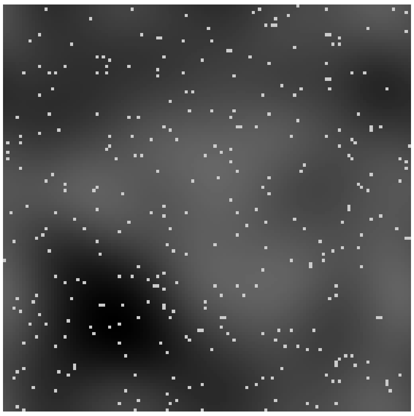


Multiresolution Spline - Example

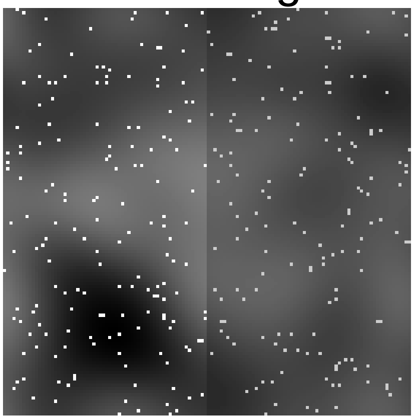
Left Image



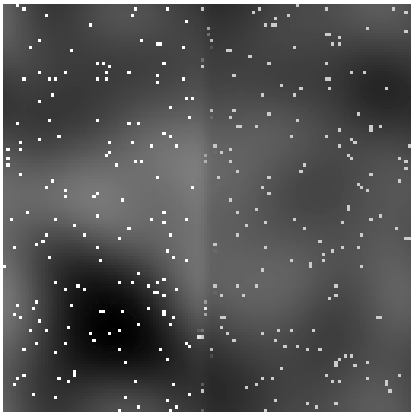
Right Image



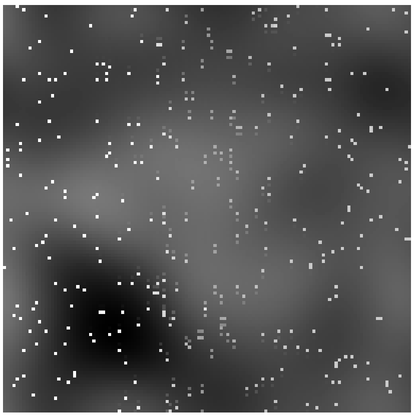
Left + Right



Narrow Transition

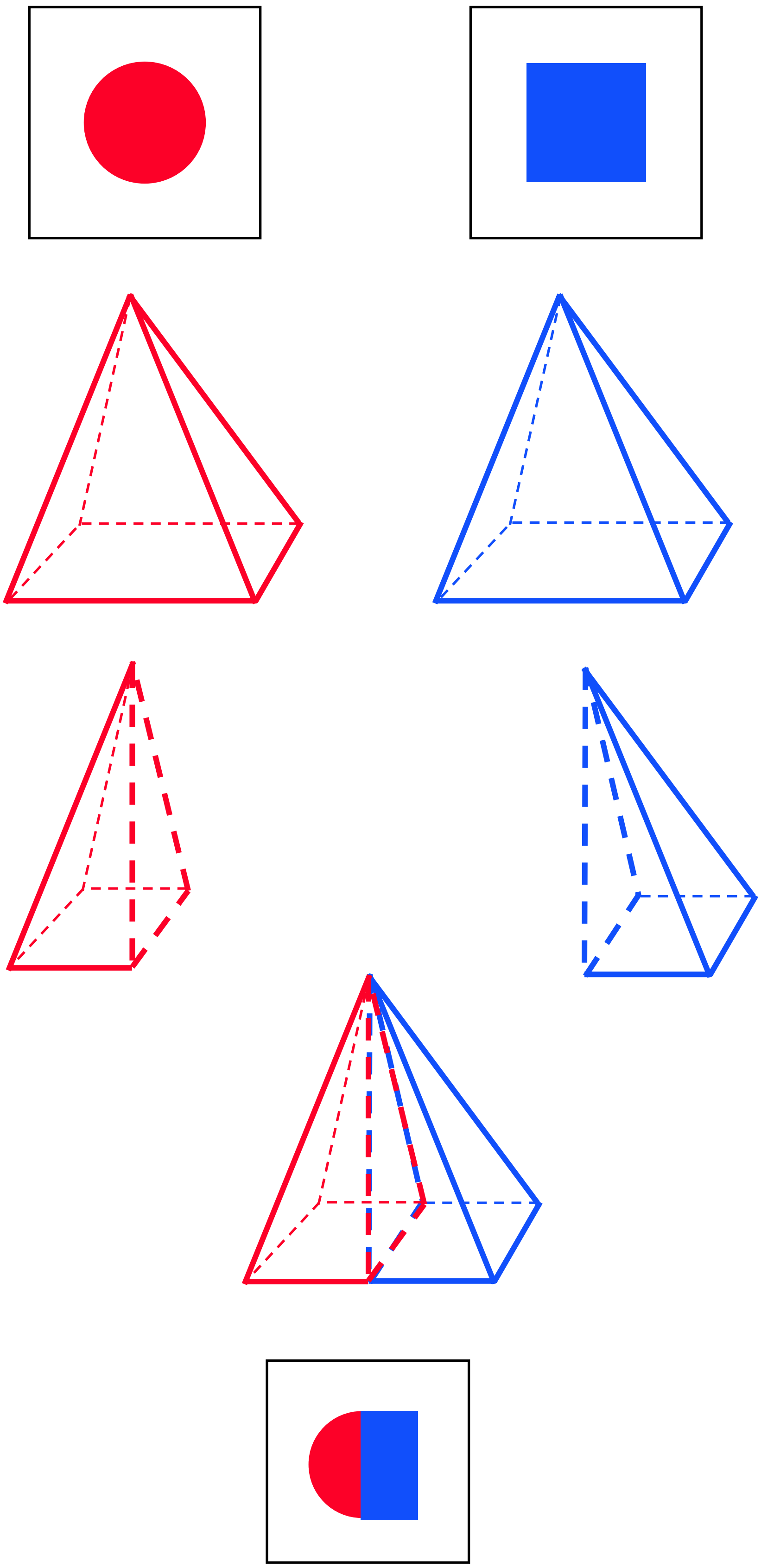


Wide Transition



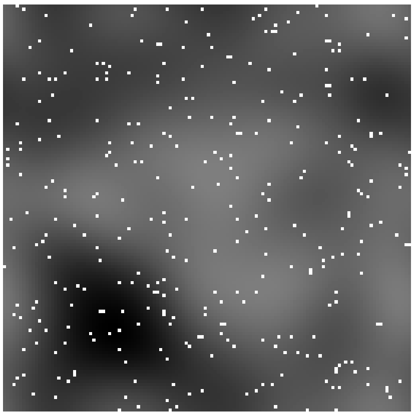
(Burt & Adelson)

Multiresolustion Spline - Using Laplacian Pyramid

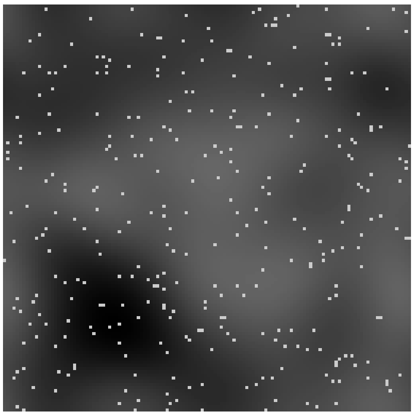


Multiresolution Spline - Example

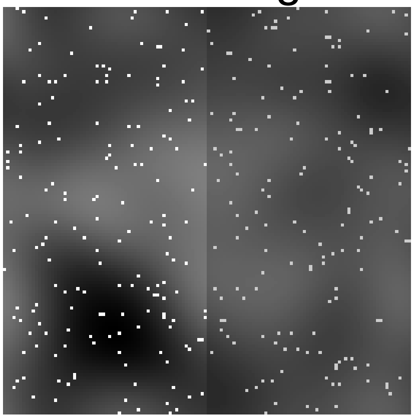
Left Image



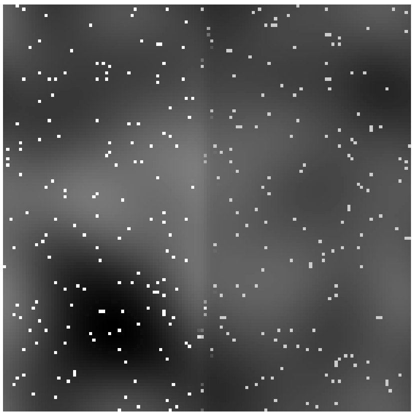
Right Image



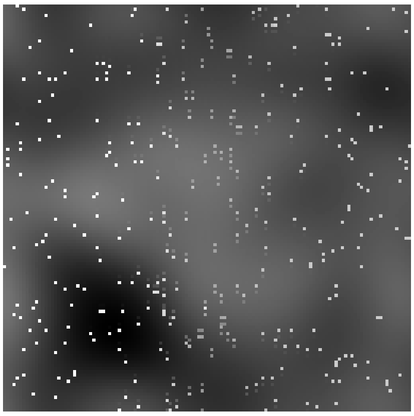
Left + Right



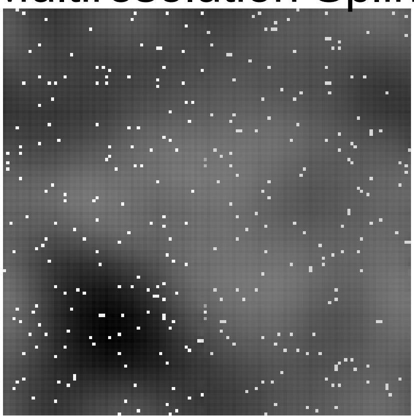
Narrow Transition



Wide Transition



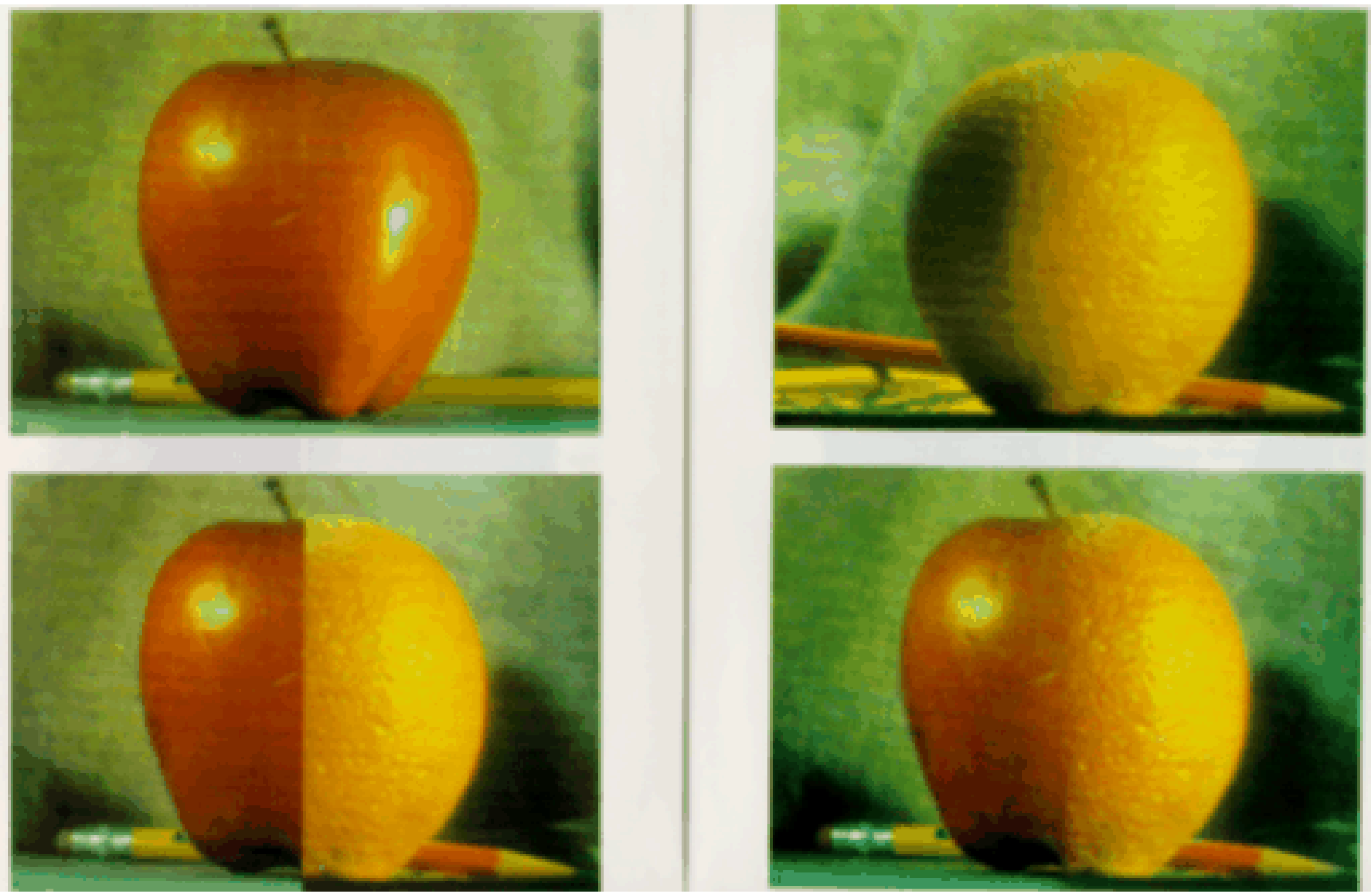
Multiresolution Spline



(Burt & Adelson)

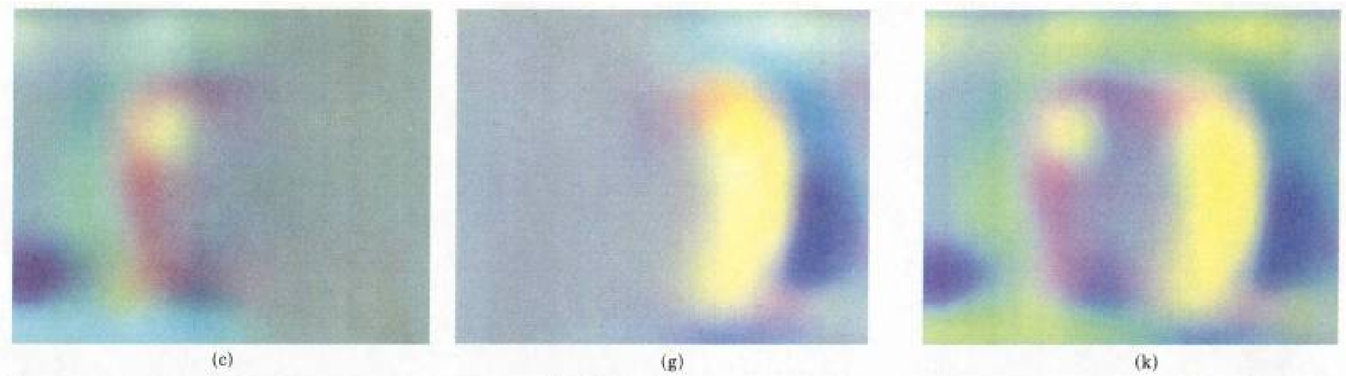
Multiresolution Spline - Example

Original - Left   Original - Right

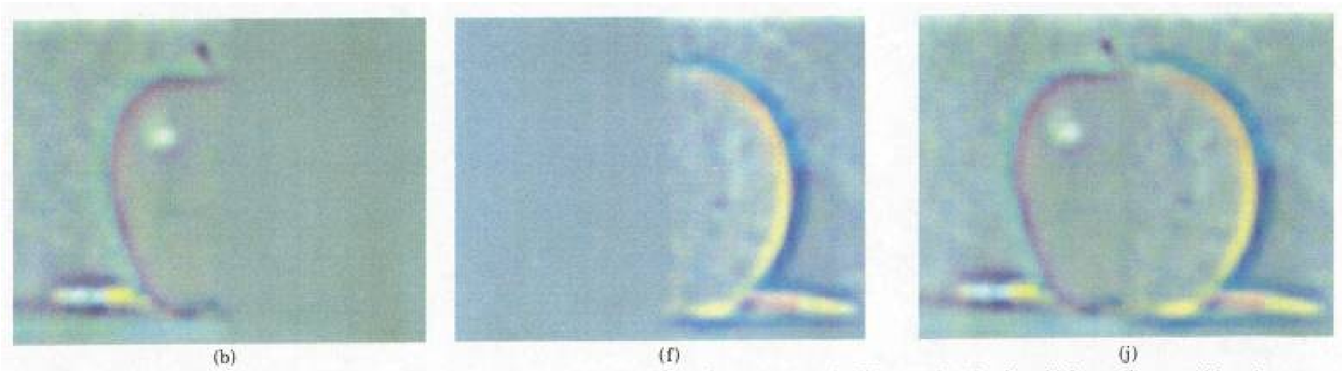


Glued   Splined

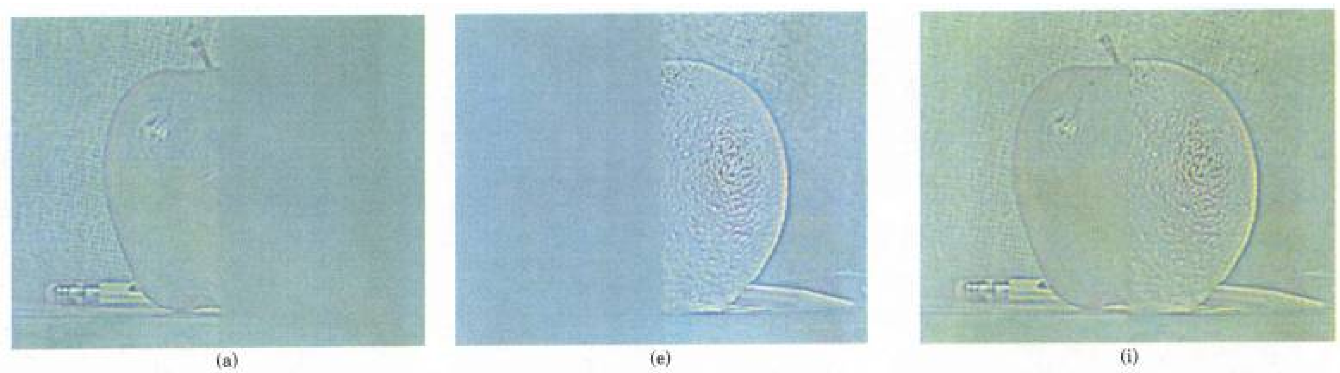
laplacian level 4



laplacian level 2

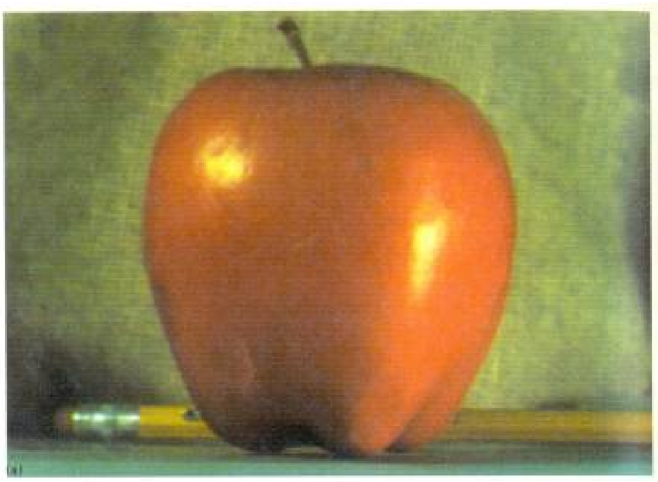


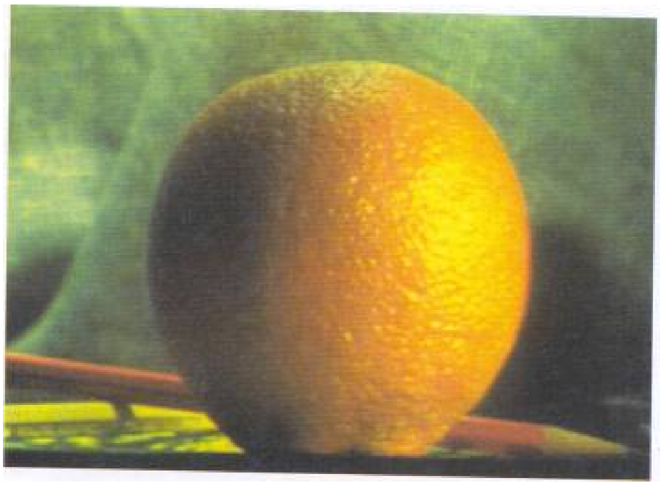
laplacian level 0

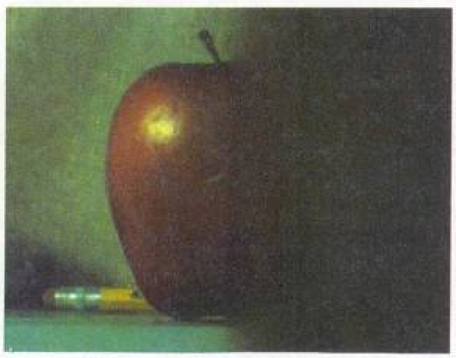


left pyramid right pyramid blended pyramid

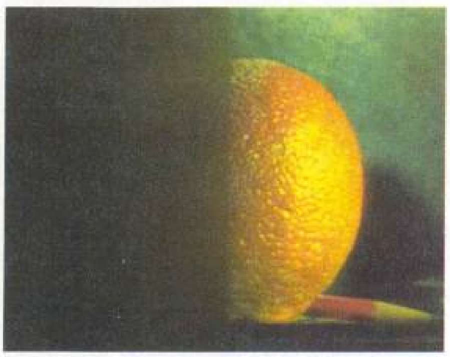
Multiresolution Spline



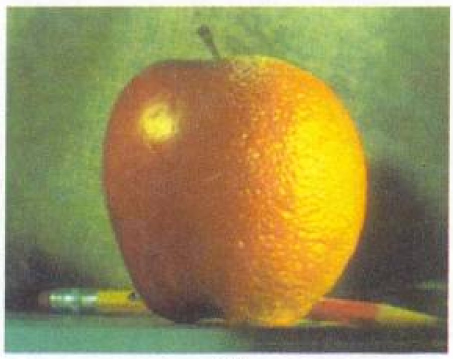




(d)

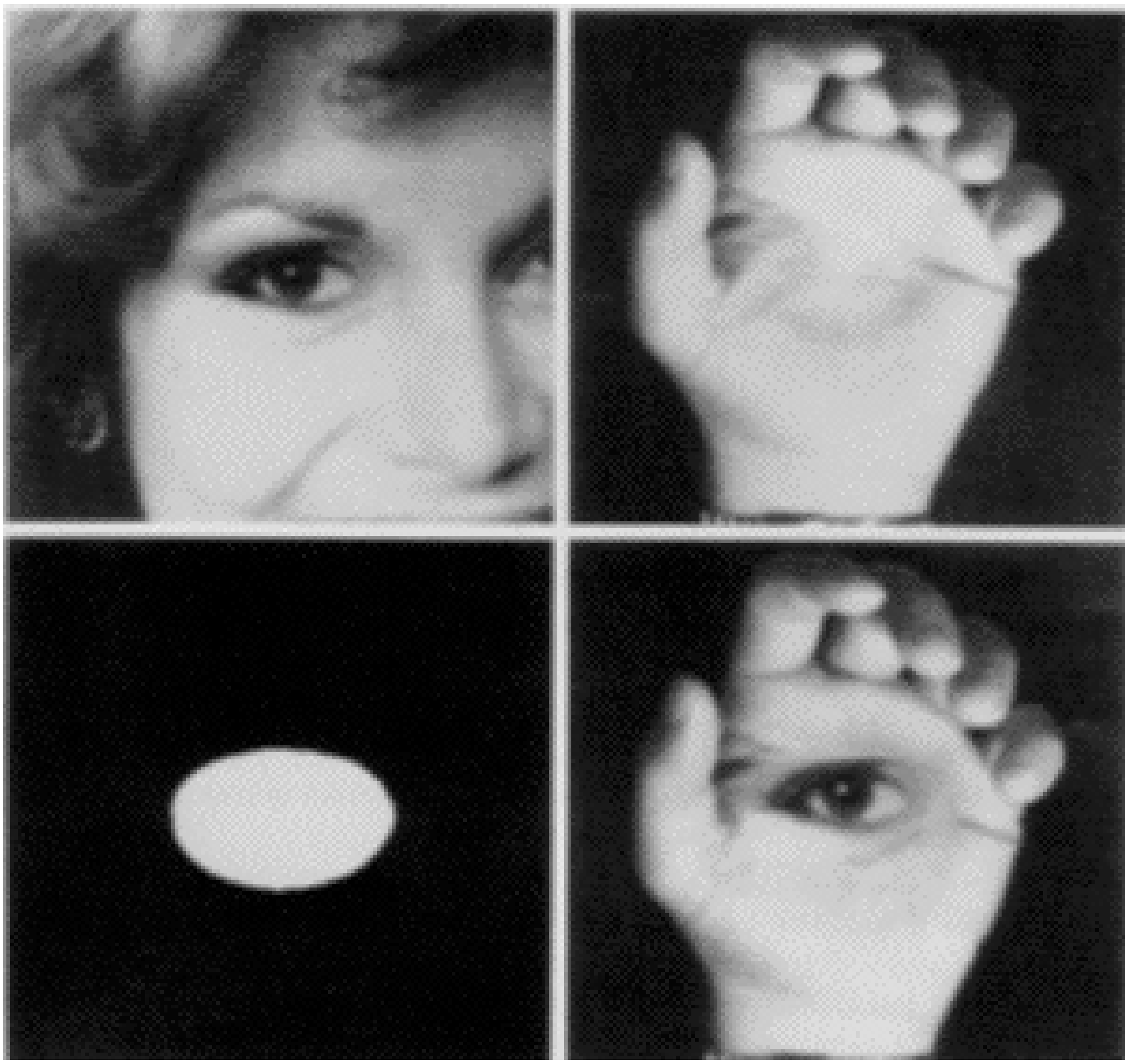


(h)



(l)

Multiresolution Spline - Example



Multi-Res. Blending

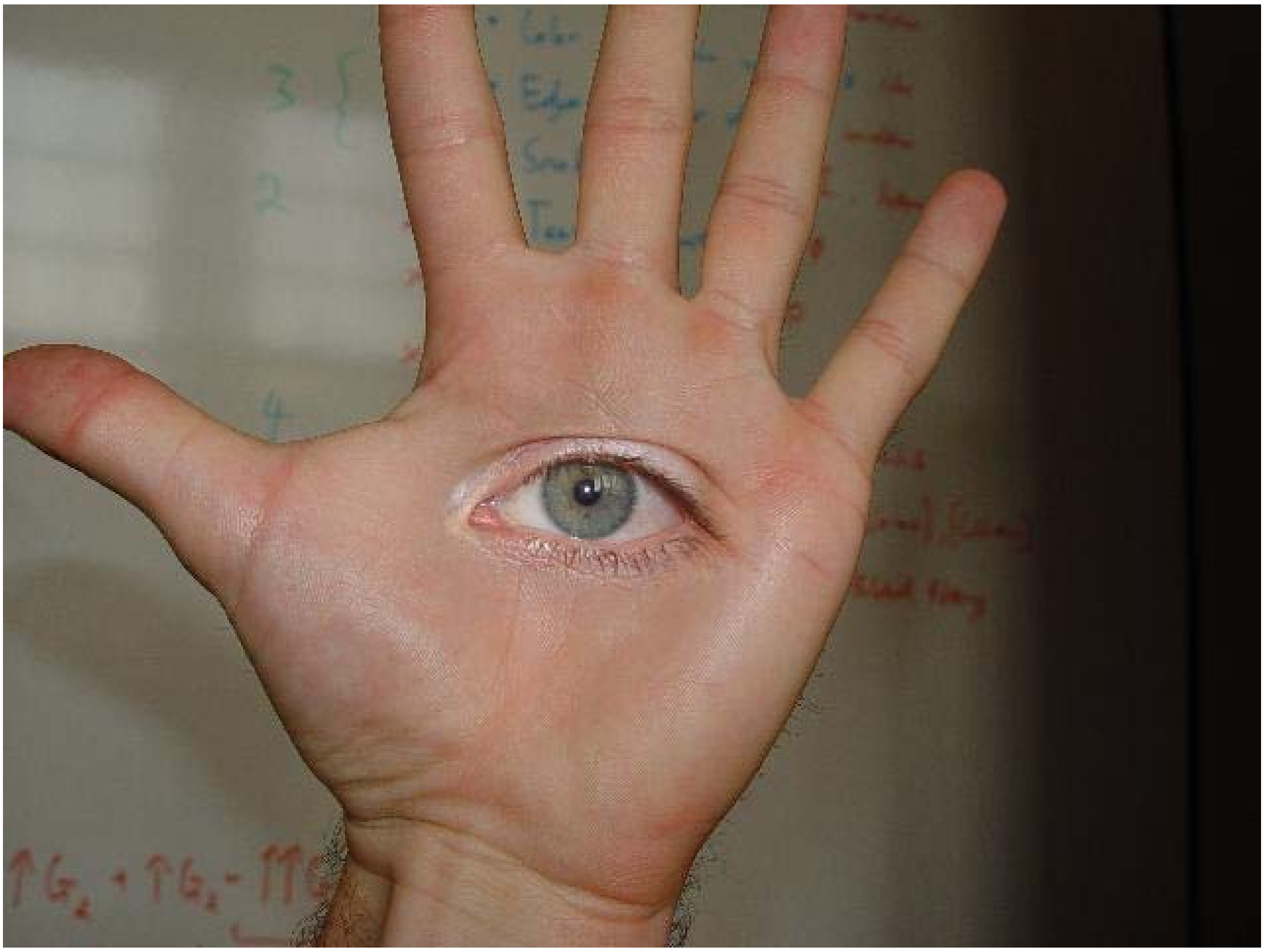


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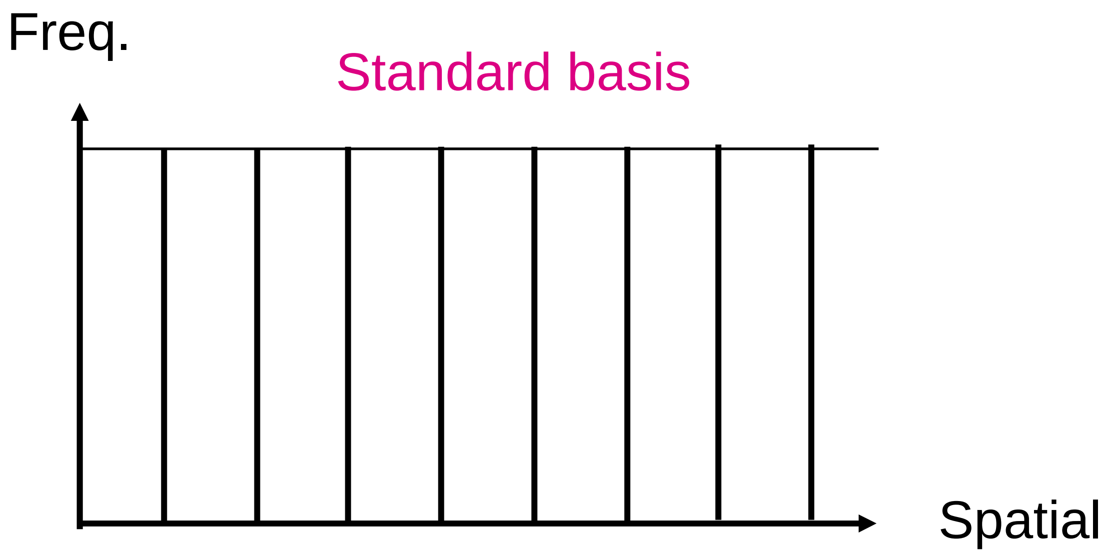
Image pyramids

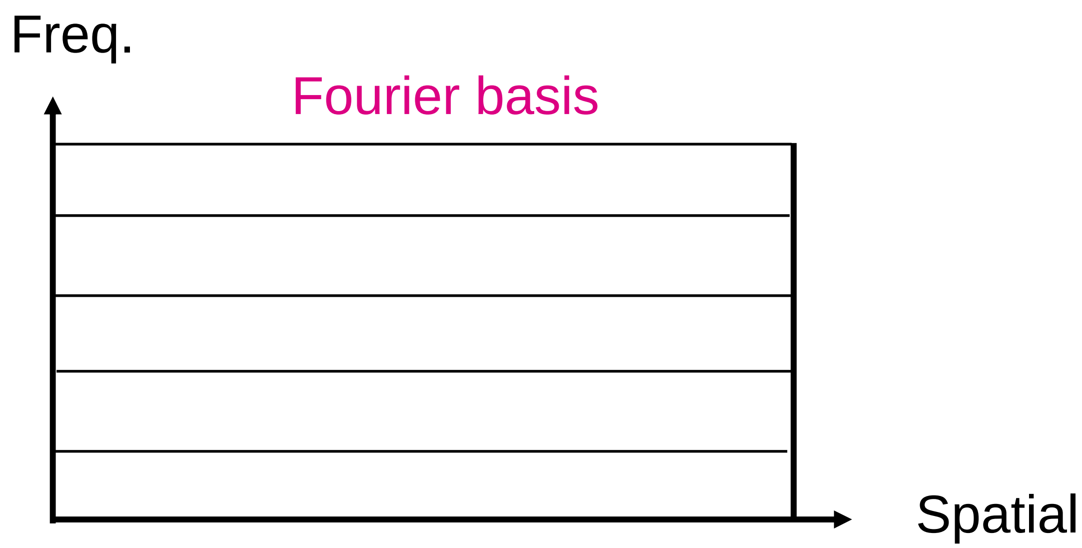
- Gaussian Pyramids
- Laplacian Pyramids
- Wavelet/QMF

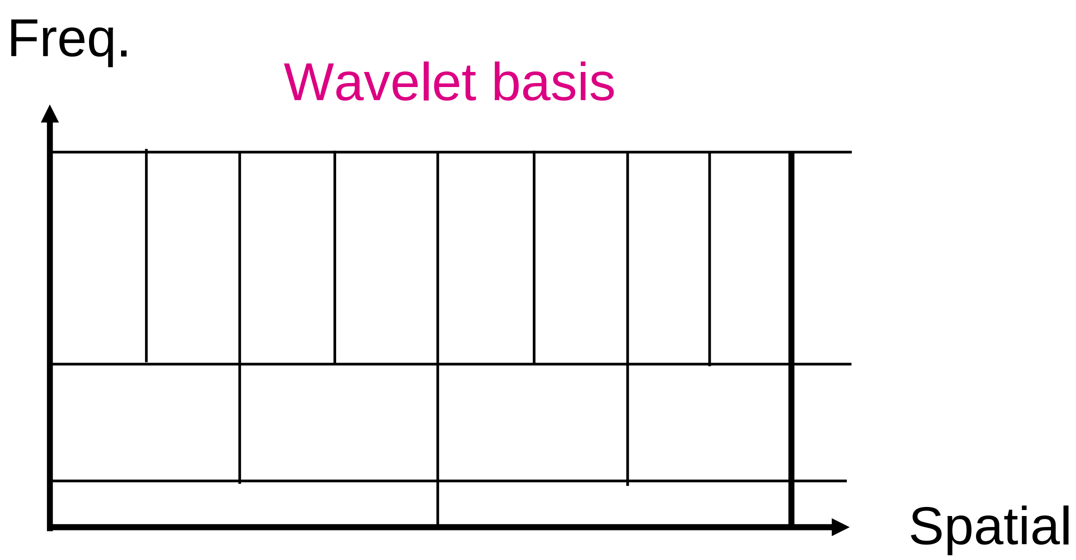
What is a good representation for image analysis?

- Pixel domain representation tells you “where” (pixel location), but not “what”.
  - In space, this representation is too localized
- Fourier transform domain tells you “what” (textural properties), but not “where”.
  - In space, this representation is too spread out.
- Want an image representation that gives you a local description of image events—what is happening where.
  - That representation might be “just right”.

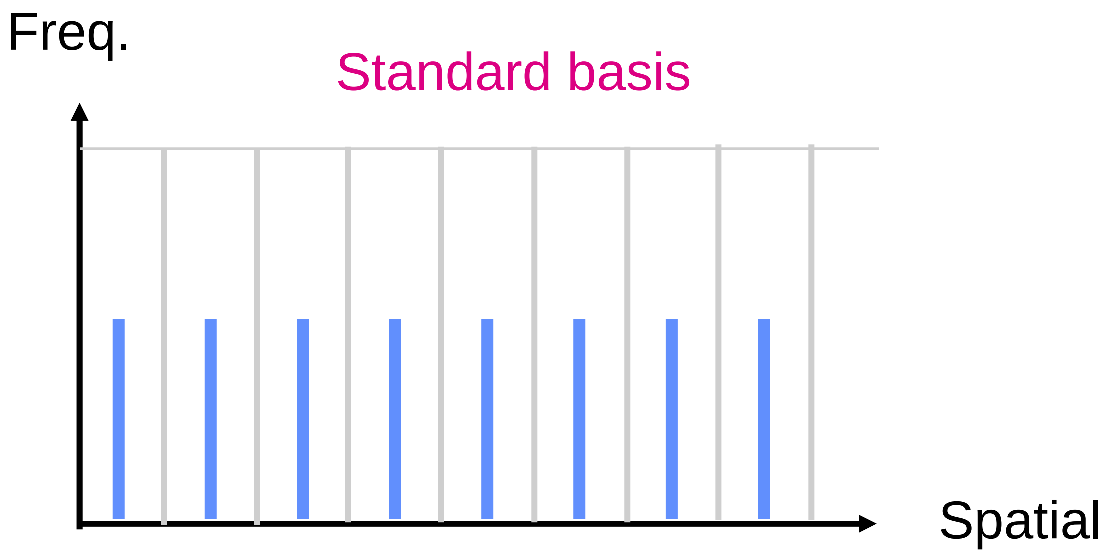
Space-Frequency Tiling

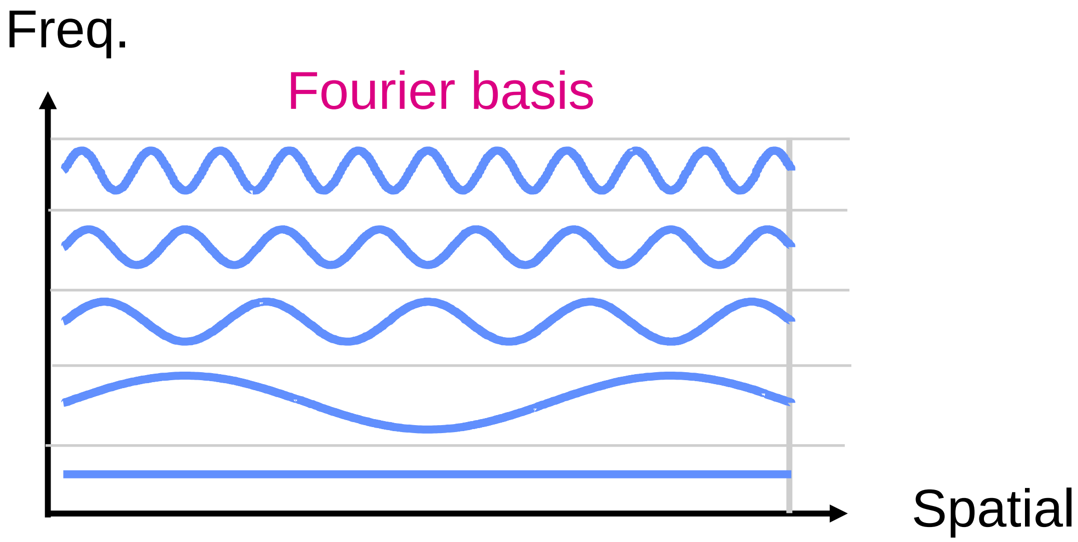


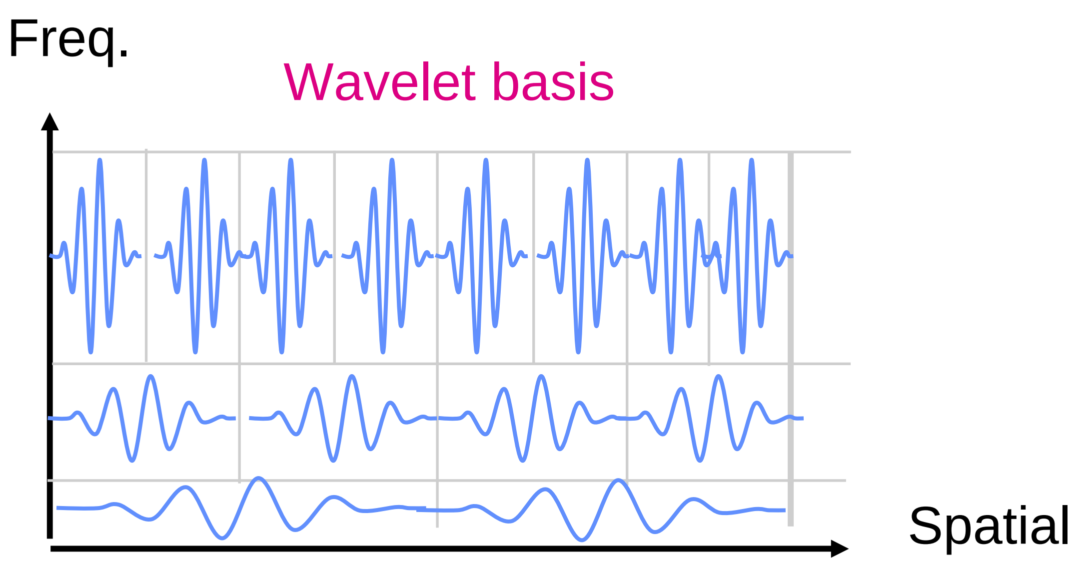




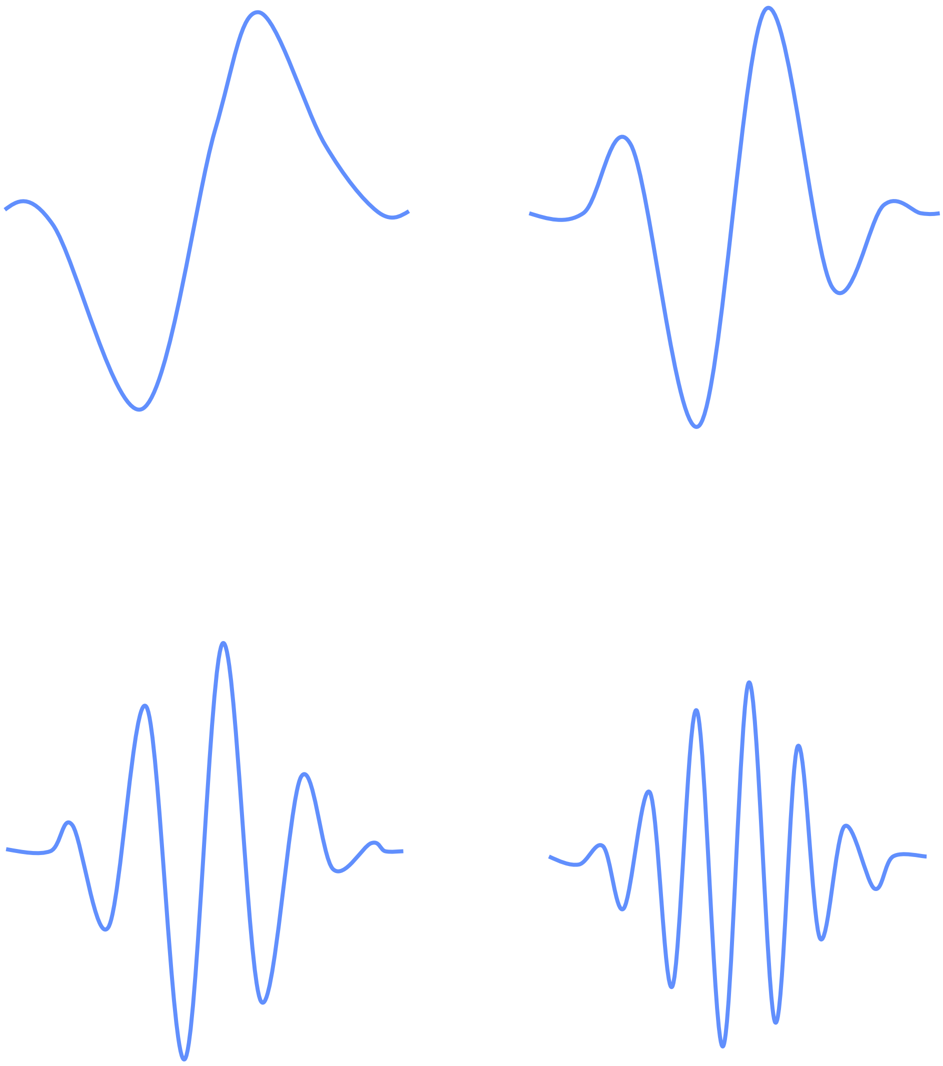
Space-Frequency Tiling





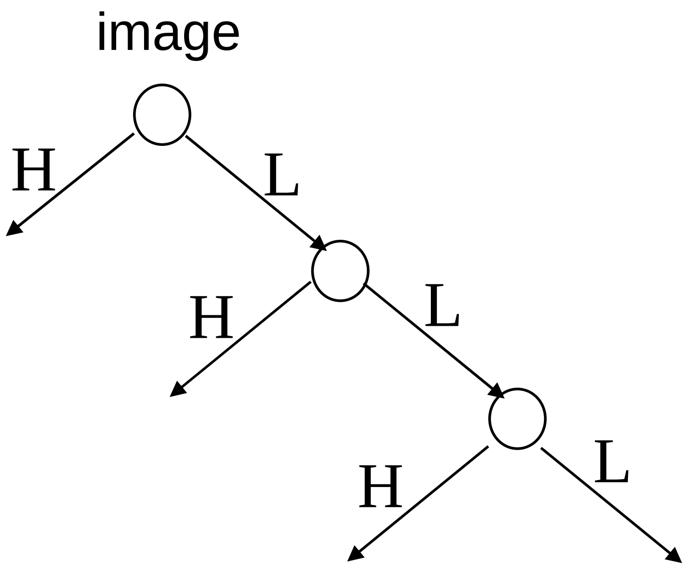


Various Wavelet basis



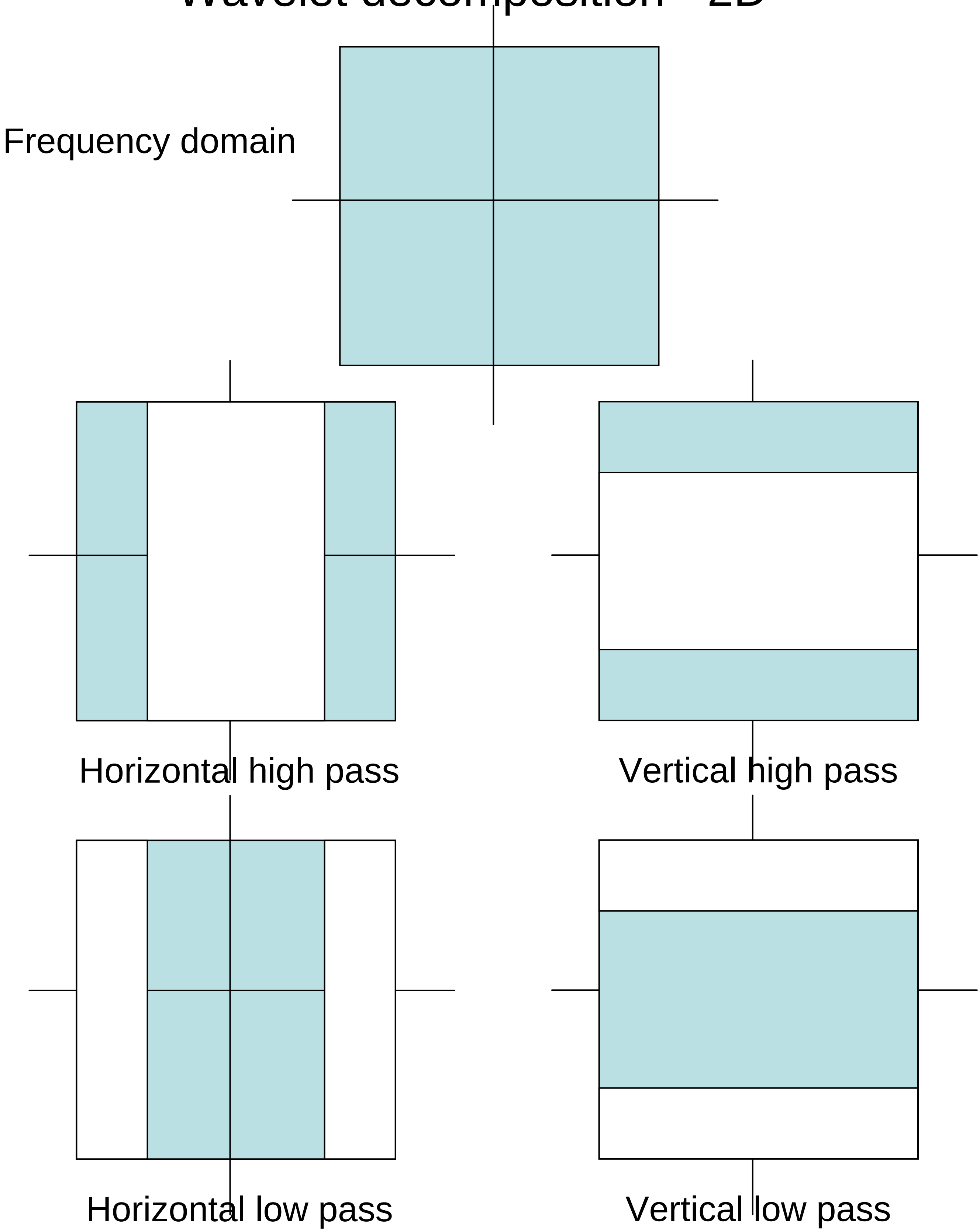
Wavelet - Frequency domain

Wavelet bands are split recursively



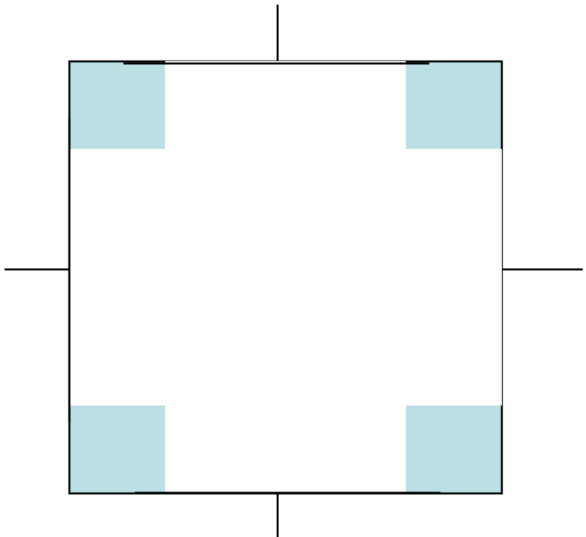
Wavelet - Frequency domain

Wavelet decomposition - 2D

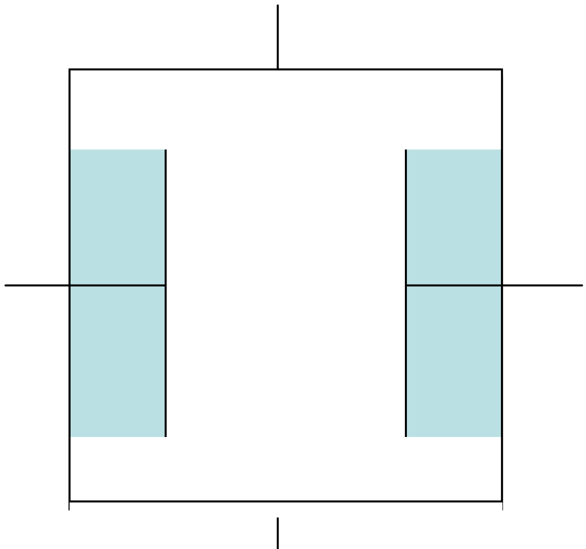


Wavelet - Frequency domain

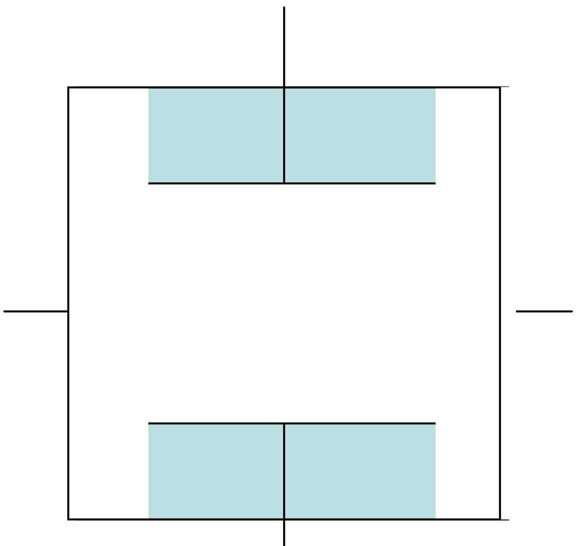
Apply the wavelet transform separably in both dimensions



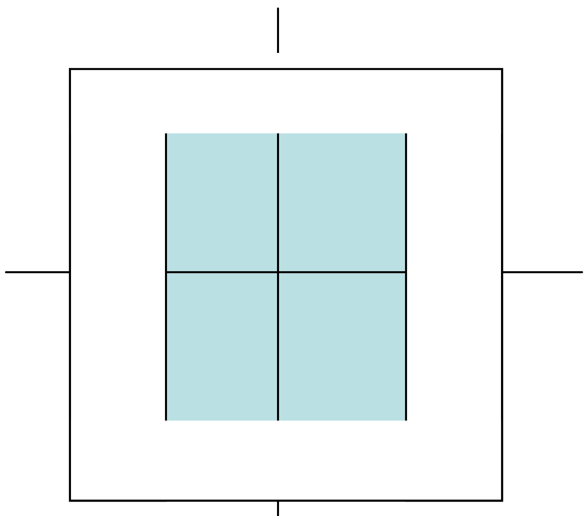
Horizontal high pass,
vertical high pass



Horizontal high pass,
vertical low-pass

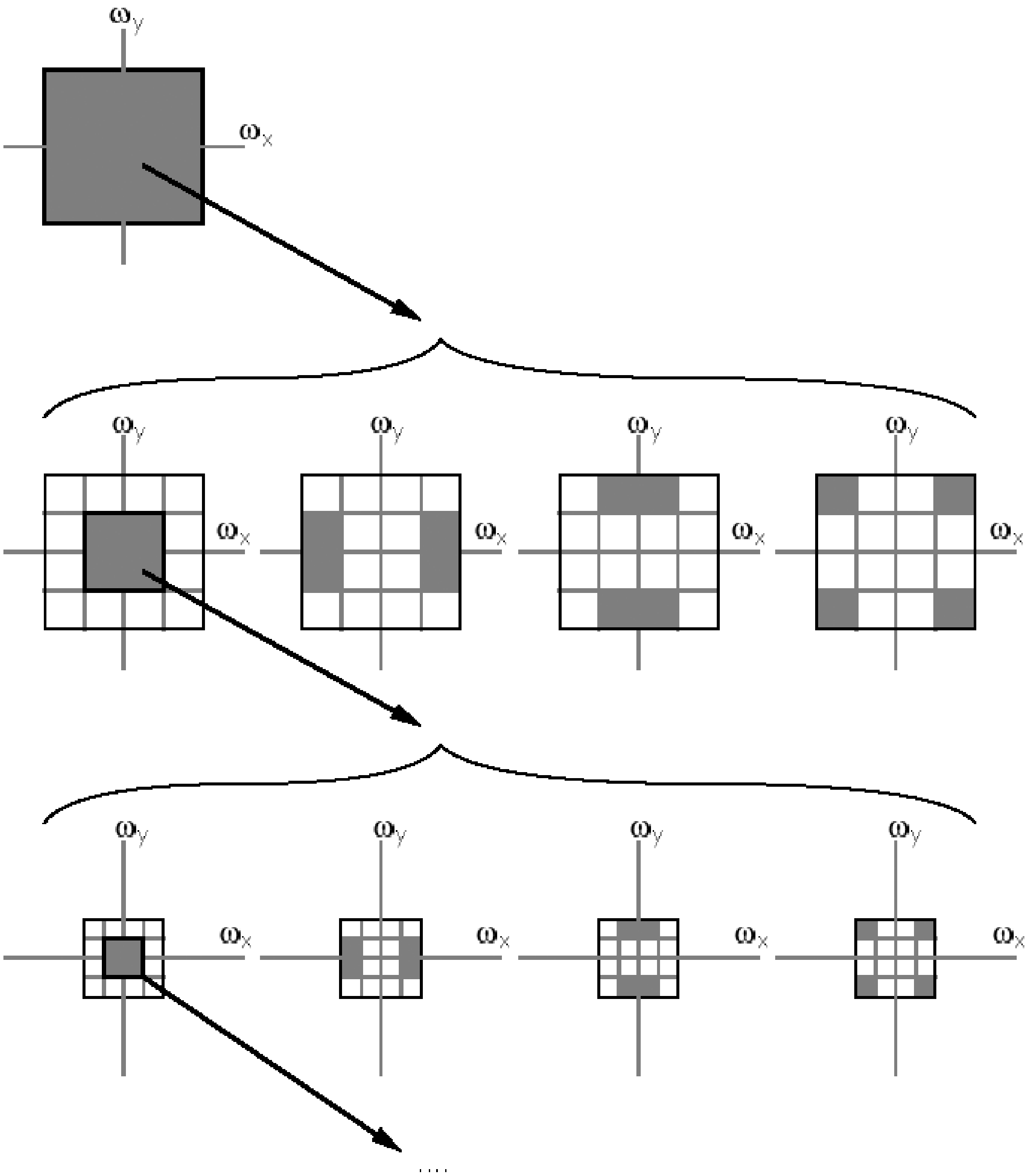


Horizontal low pass,
vertical high-pass



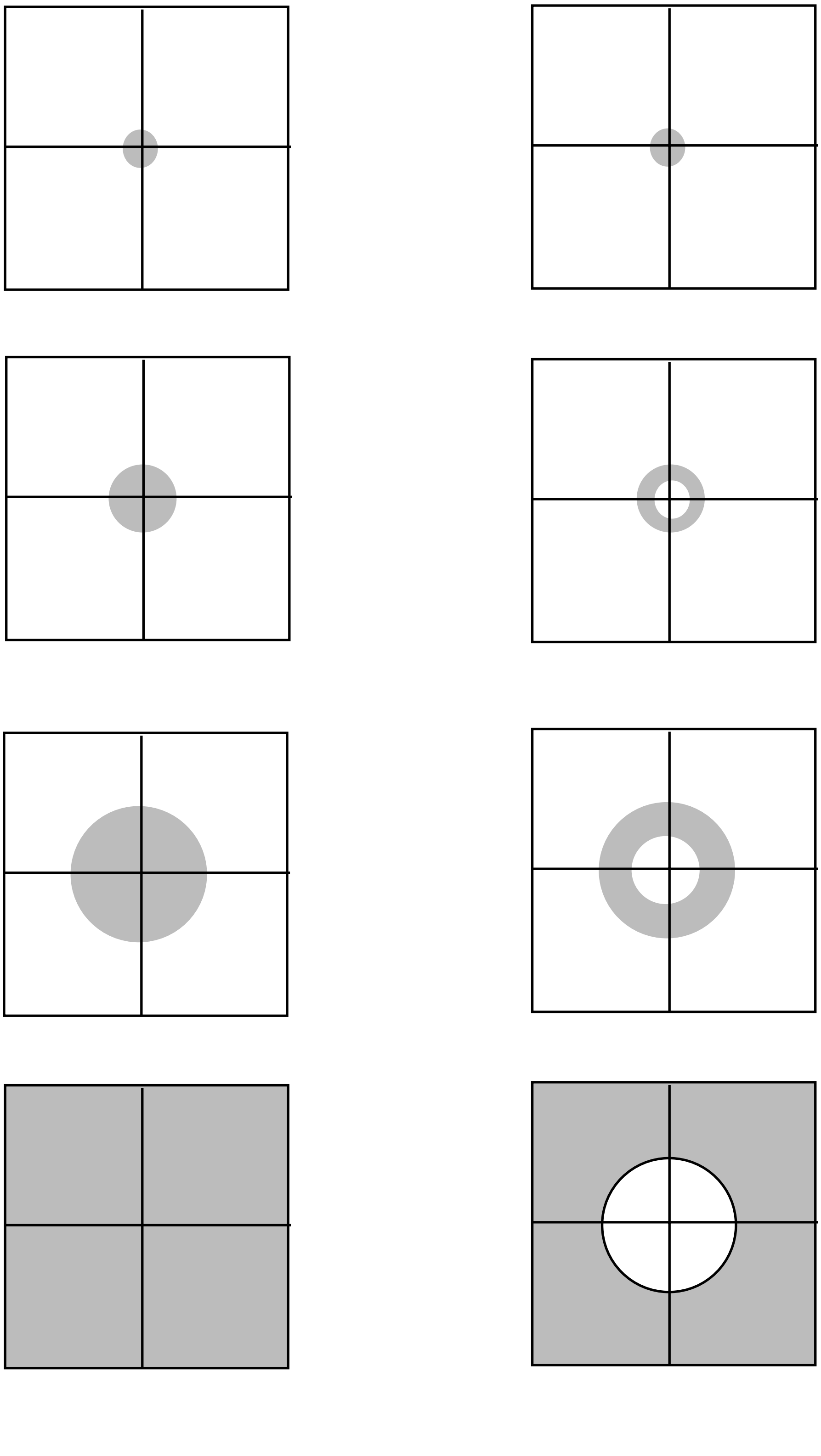
Horizontal low pass,
Vertical low-pass

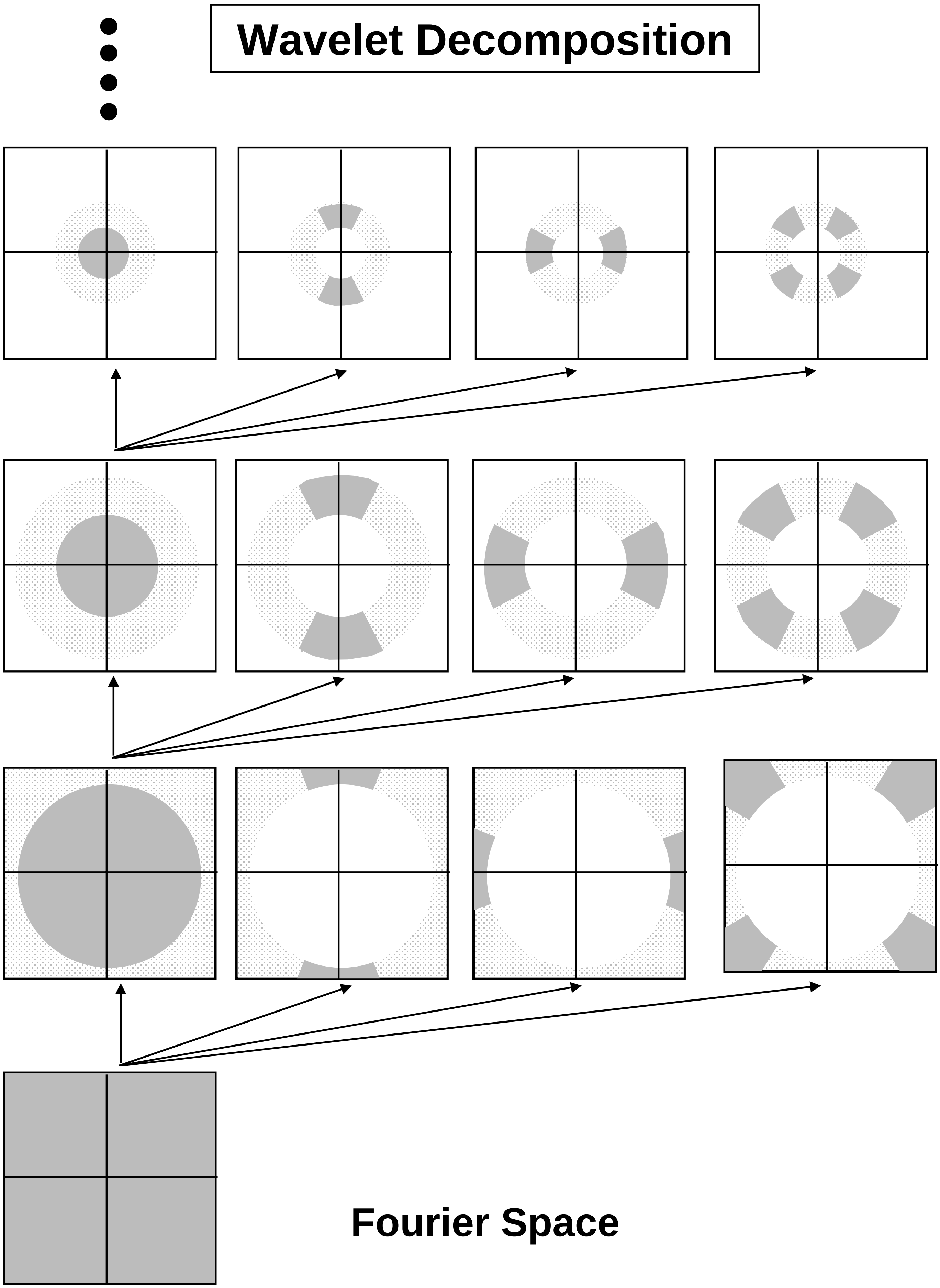
- Splitting can be applied recursively:



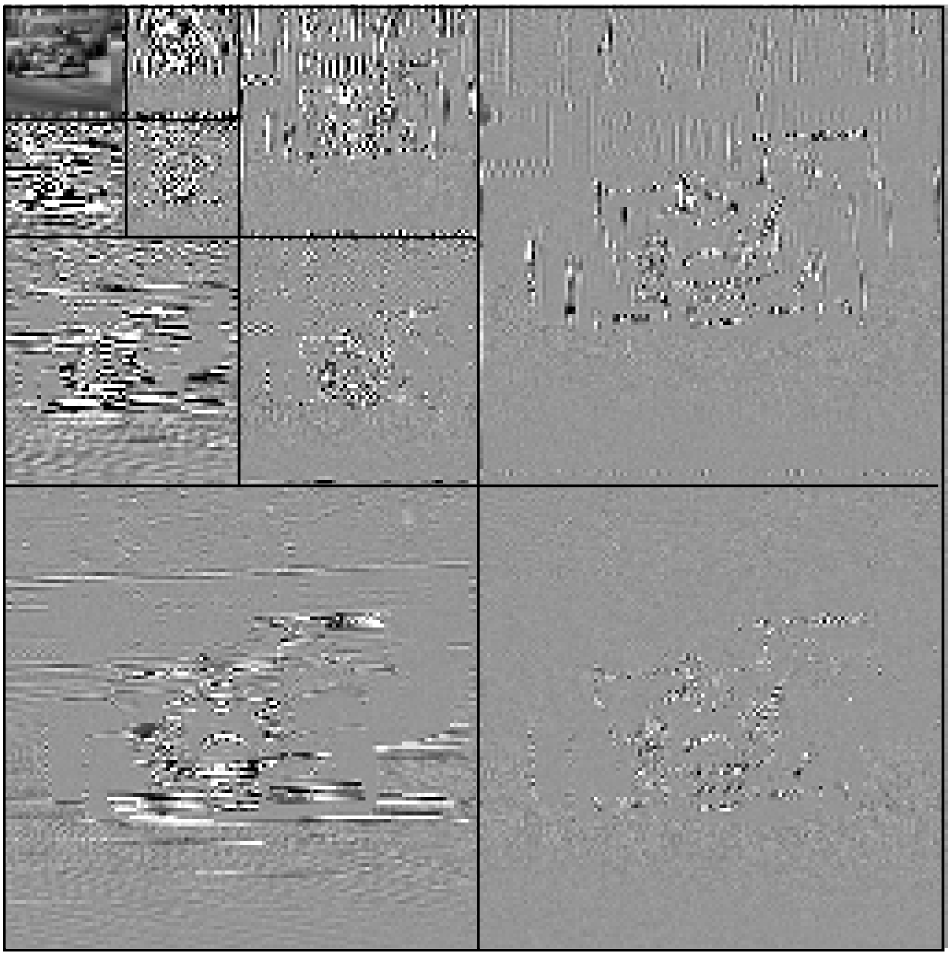
Pyramids in Frequency Domain

Gaussian Pyramid **Laplacian Pyramid**

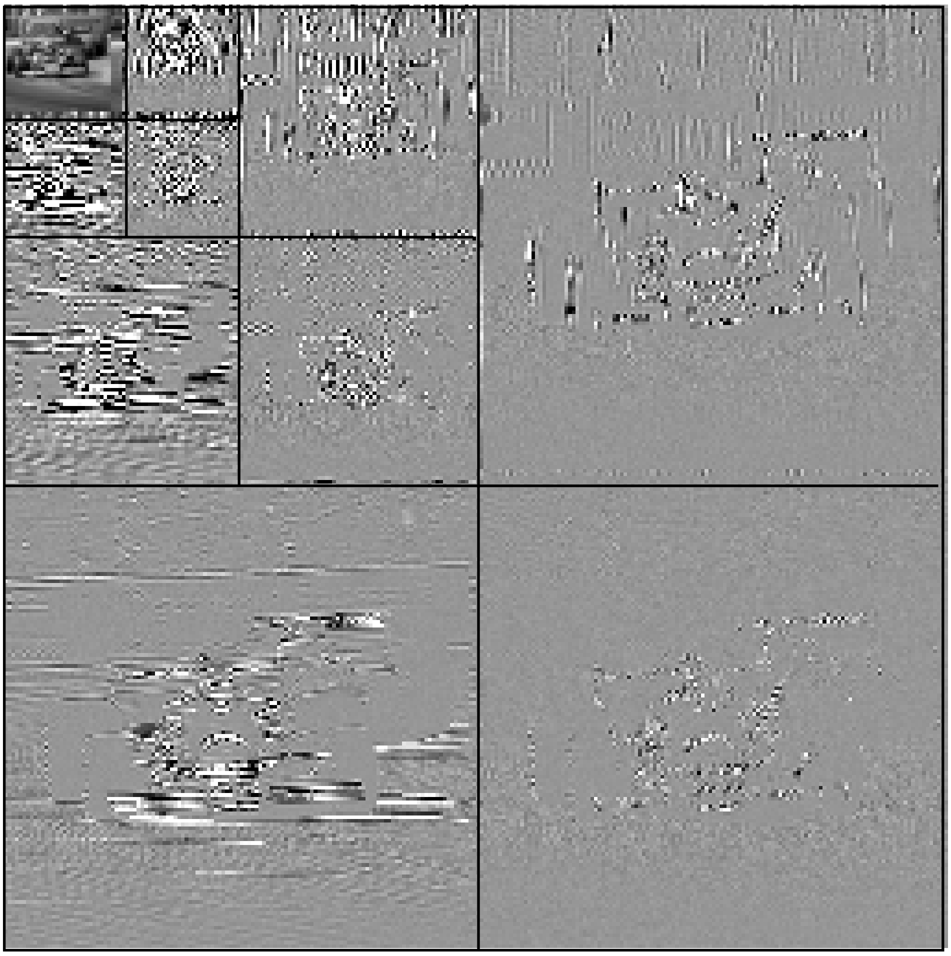



Wavelet Decomposition
Fourier Space

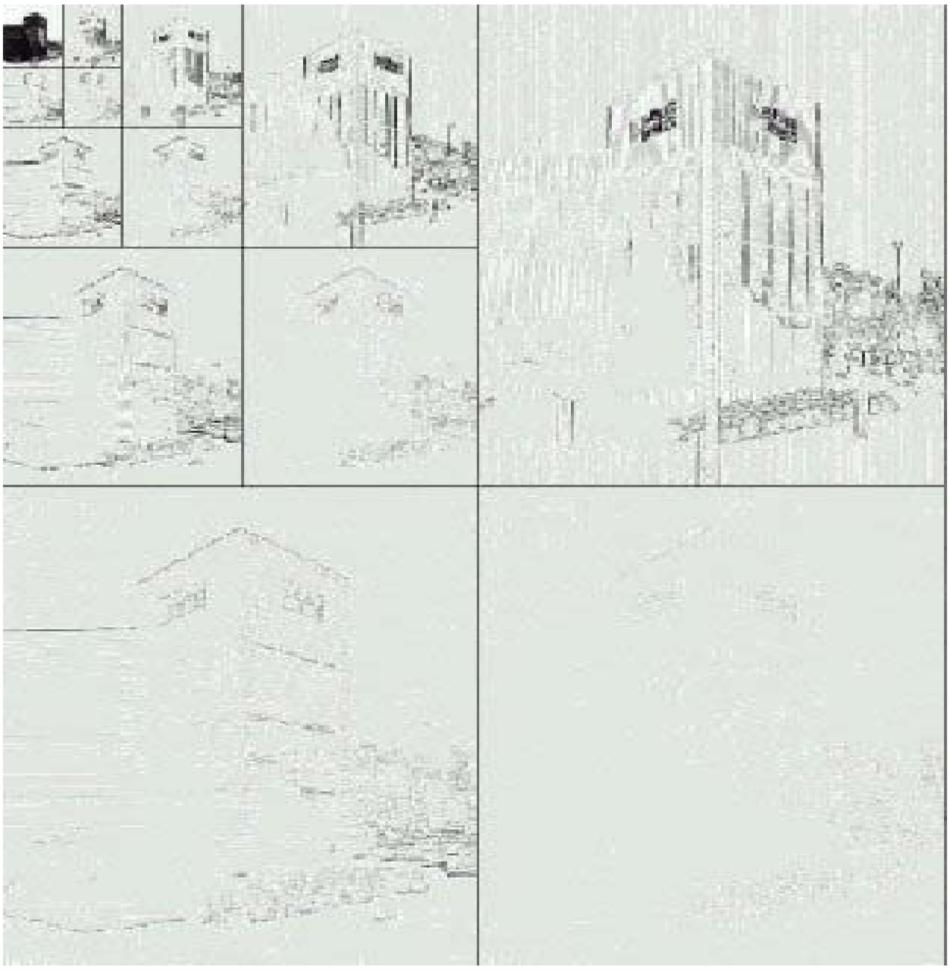
Wavelet Transform - Step By Step Example



Wavelet Transform - Example

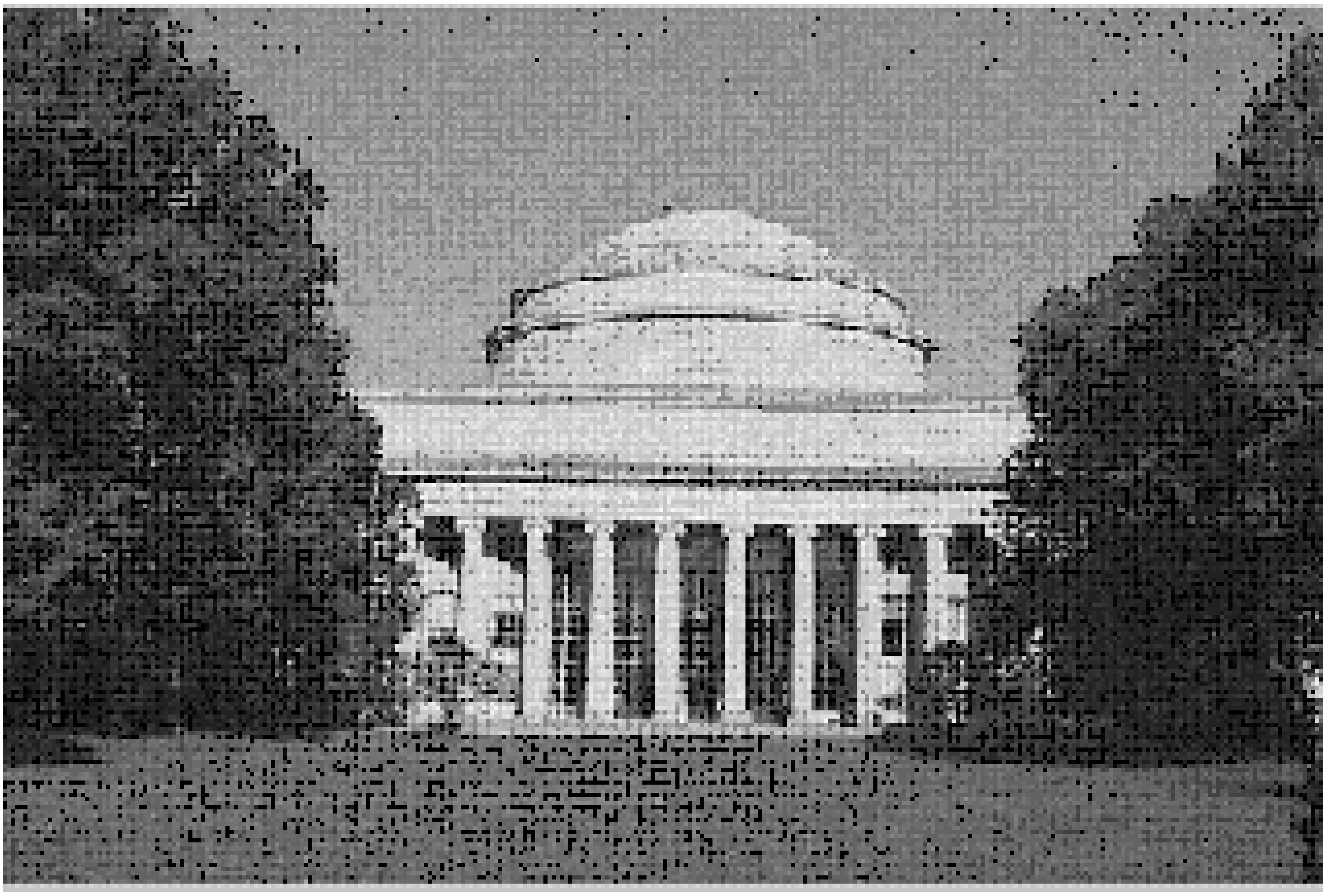


Wavelet Transform - Example



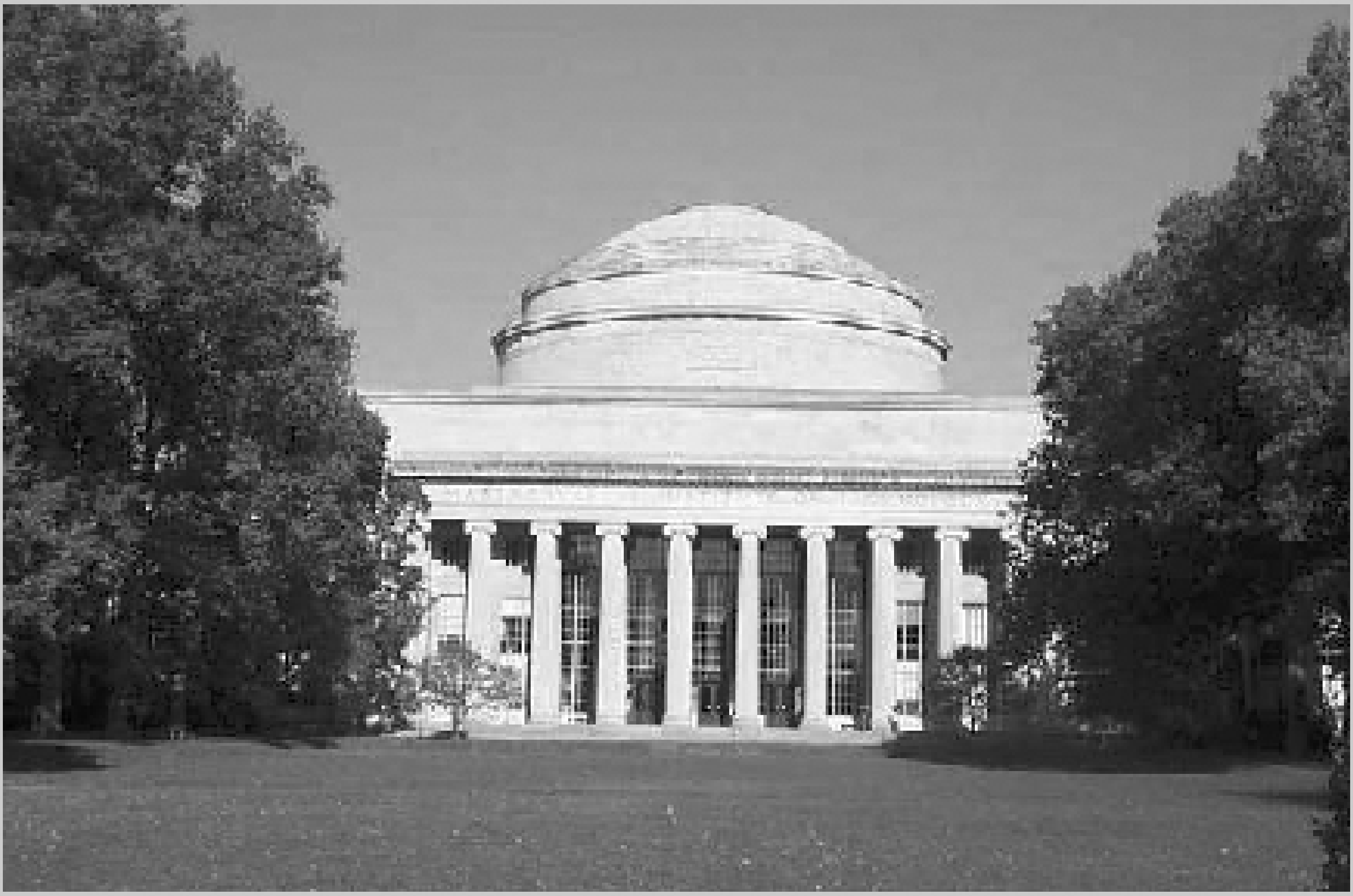
Application: Wavelet Shrinkage Denoising

Noisy image



From: B. Freeman

Clean image

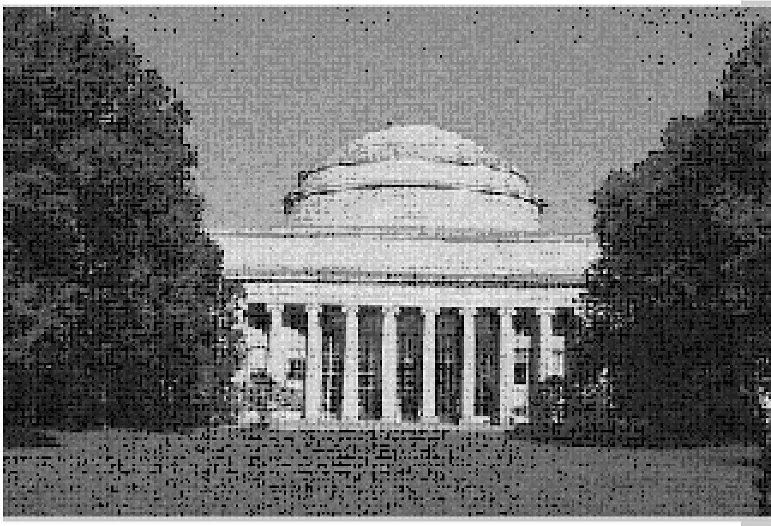


Range [0, 255]
Dims [394, 599]

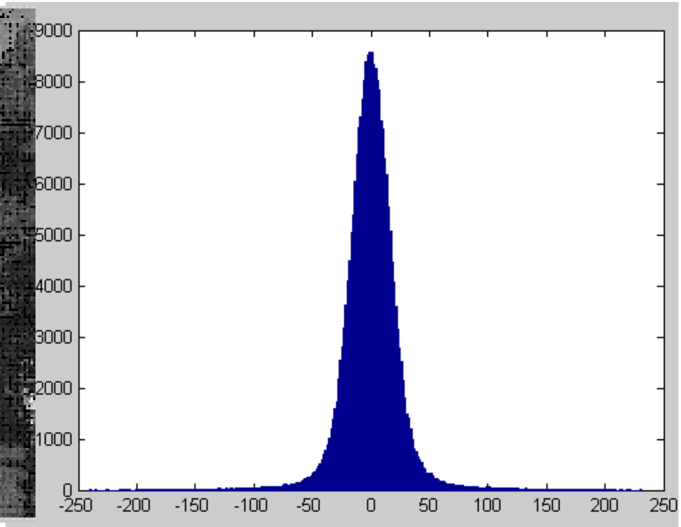
From: B. Freeman

Wavelet Shrinkage Denoising

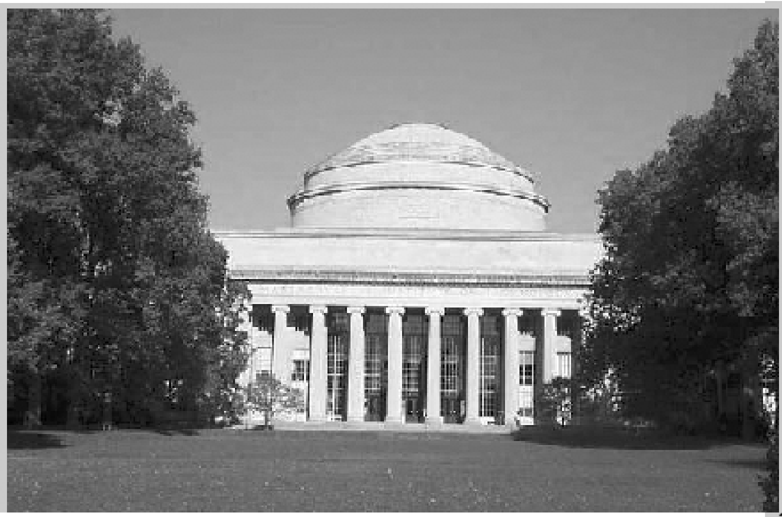
Noisy image



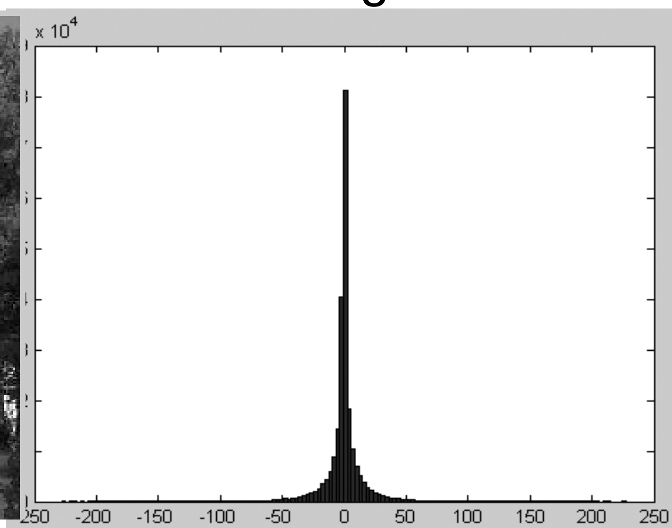
Wavelet coefficient
Histogram



Clean image



Wavelet coefficient
Histogram

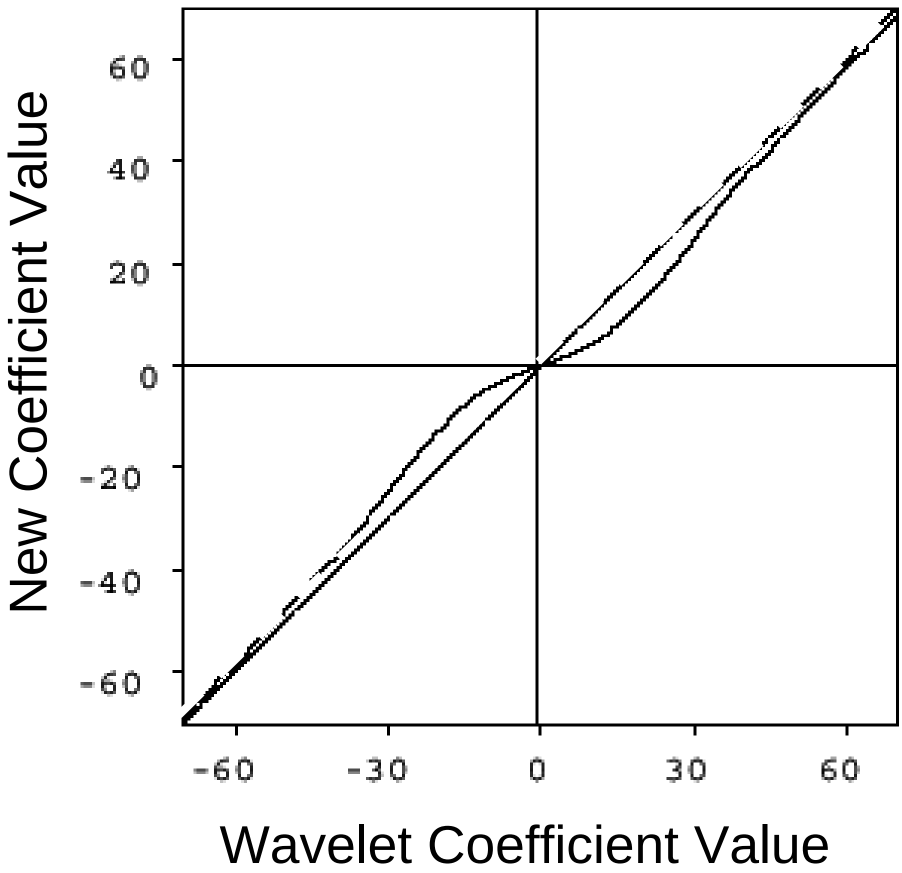


Get top histogram but want to get bottom histogram.

From: B. Freeman

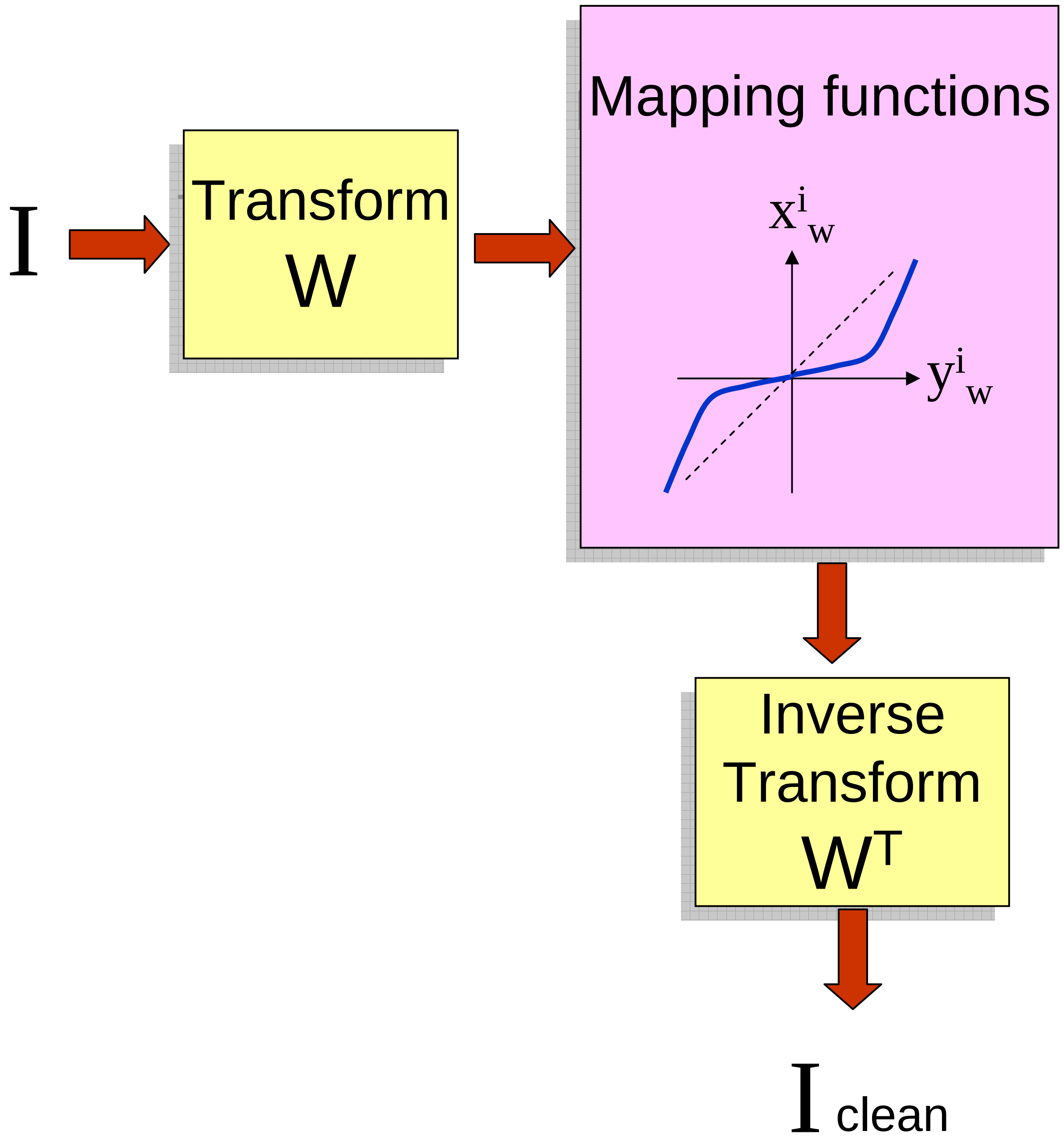
Wavelet Shrinkage Denoising

For every Wavelet Band define
Shrinkage function:



From: B. Freeman

Wavelet Shrinkage Pipe-line



More results

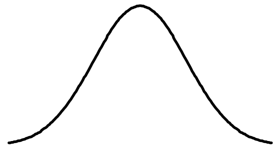
More results

Image Pyramids - Comparison

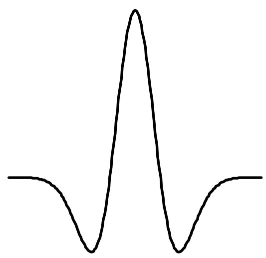
Image pyramid levels = Filter then sample.

Filters:

Gaussian Pyramid



Laplacian Pyramid



Wavelet Pyramid

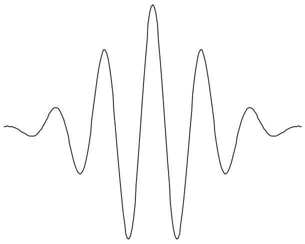
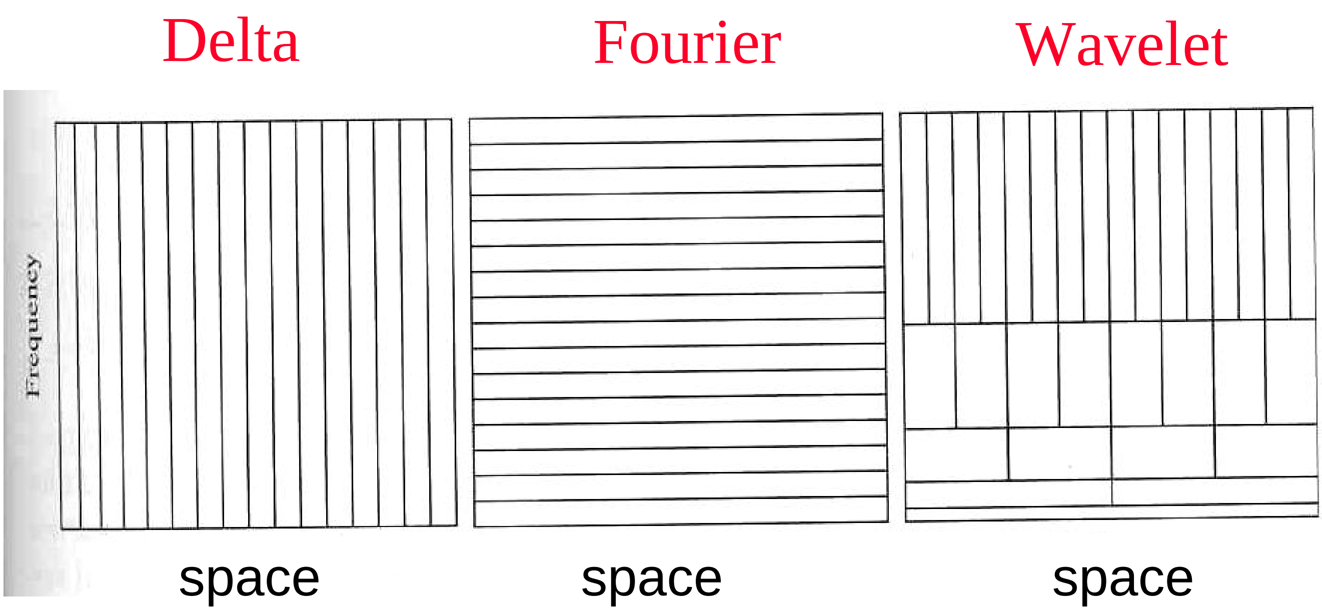


Image Linear Transforms

Transform	Basis	Characteristics
Delta	Standard	Localized in space Not localized in Frequency
Fourier	Sines+Cosines	Not localized in space Localized in Frequency
Wavelet Pyramid	Wavelet Filters	Localized in space Localized in Frequency



Convolution and Transforms in matrix notation (1D case)

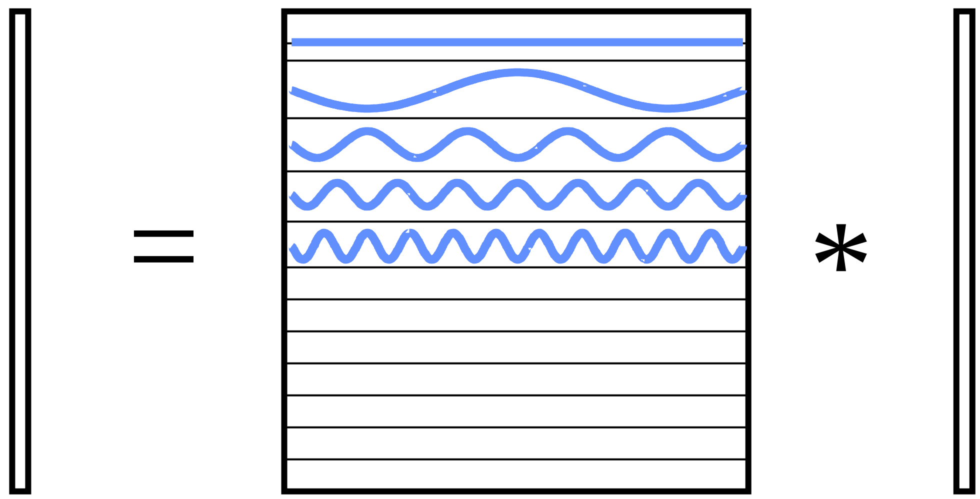
$$\vec{F} = U\vec{f}$$

transformed image

Vectorized image

Basis vectors (Fourier, Wavelet, etc)

Fourier Transform



Fourier bases are global: each transform coefficient depends on all pixel locations.

From: B. Freeman

Transform in matrix notation (1D case)

Forward Transform:

$$\vec{F} = U\vec{f}$$

transformed image

Vectorized image

Basis vectors (Fourier, Wavelet, etc)

Inverse Transform:

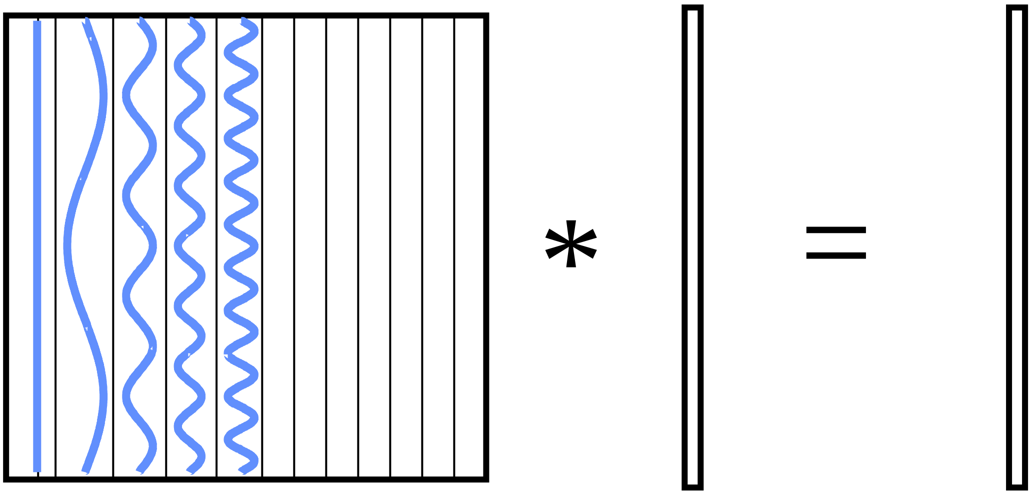
$$U^{-1}\vec{F} = \vec{f}$$

Basis vectors

Vectorized image

transformed image

Inverse Fourier Transform



Fourier bases

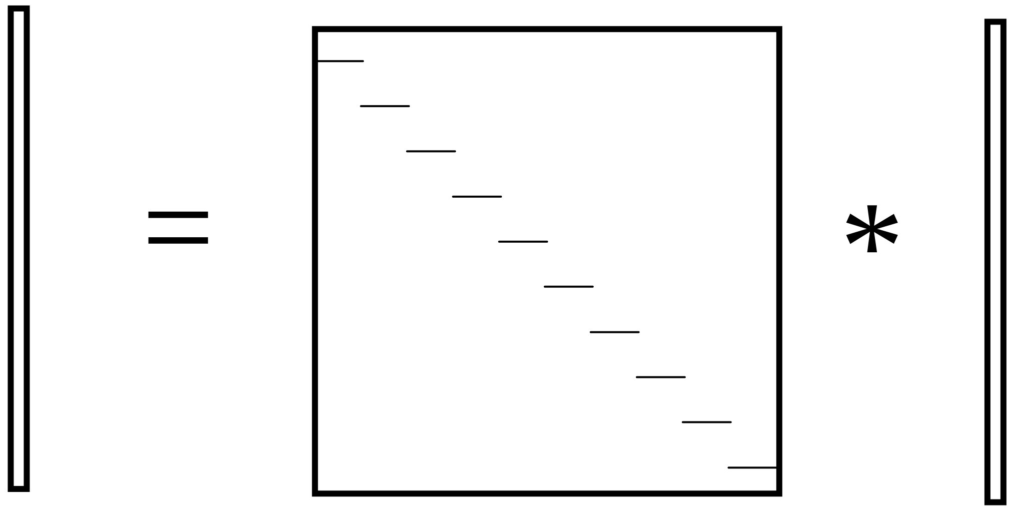
Fourier transform

pixel domain image

Every image pixel is a linear combination of the Fourier basis weighted by the coefficient.

Note that if U is orthonormal basis then $U^{-1} = U^{t}$

Convolution



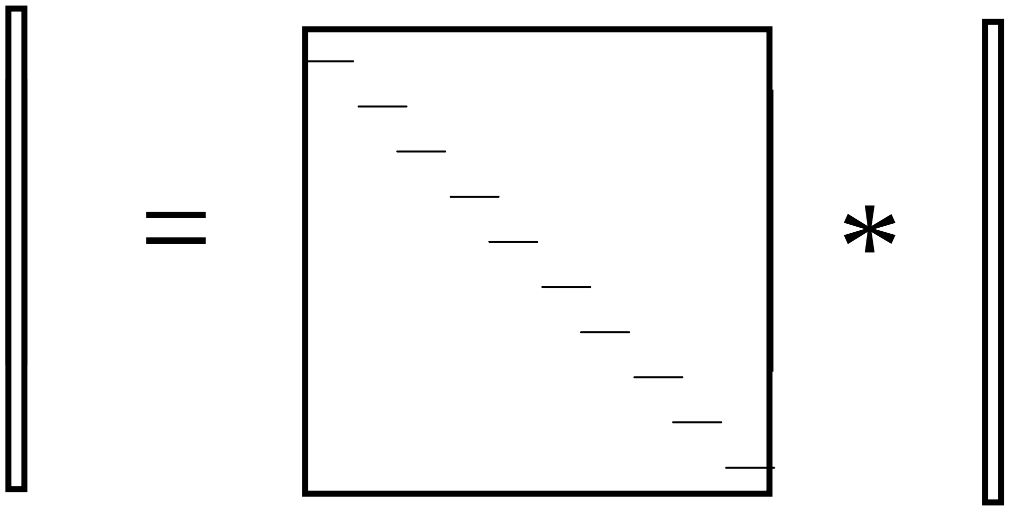
Convolution Result

Circular Matrix of Filter Kernels

pixel image

From: B. Freeman

Pyramid =
Convolution + Sampling



Convolution Result

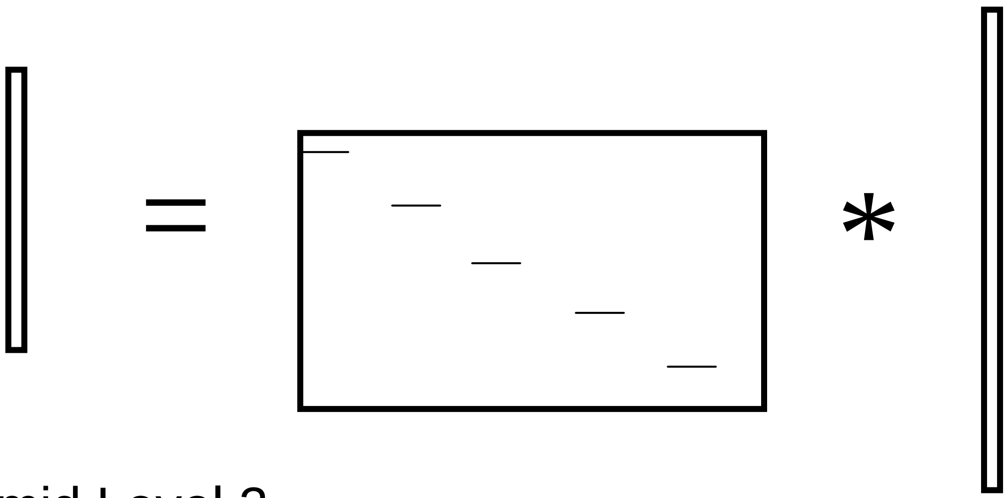
Matrix of Filter Kernels

pixel image

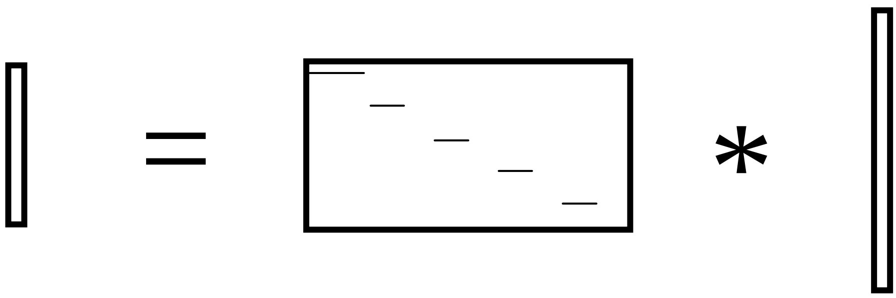
From: B. Freeman

Pyramid =
Convolution + Sampling

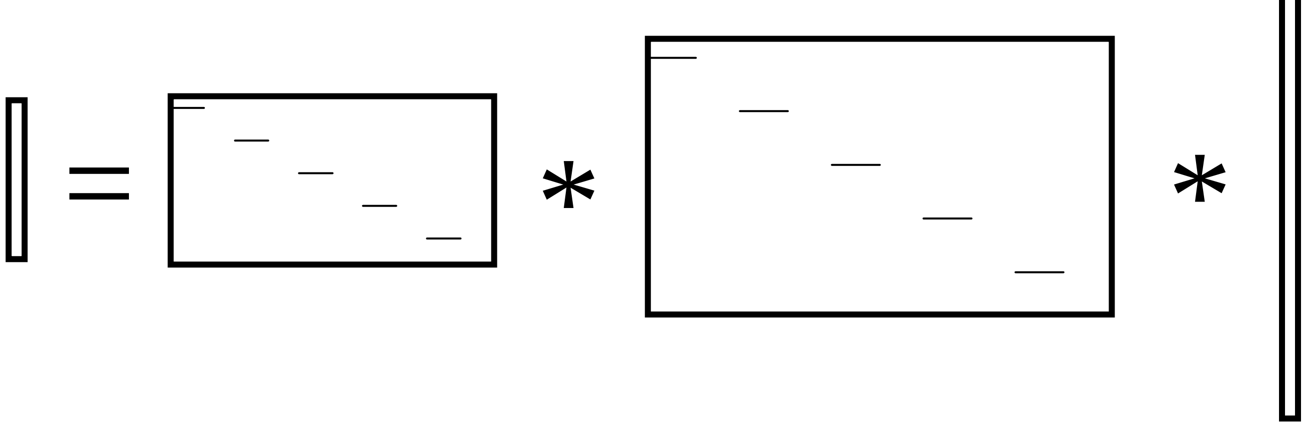
Pyramid Level 1



Pyramid Level 2

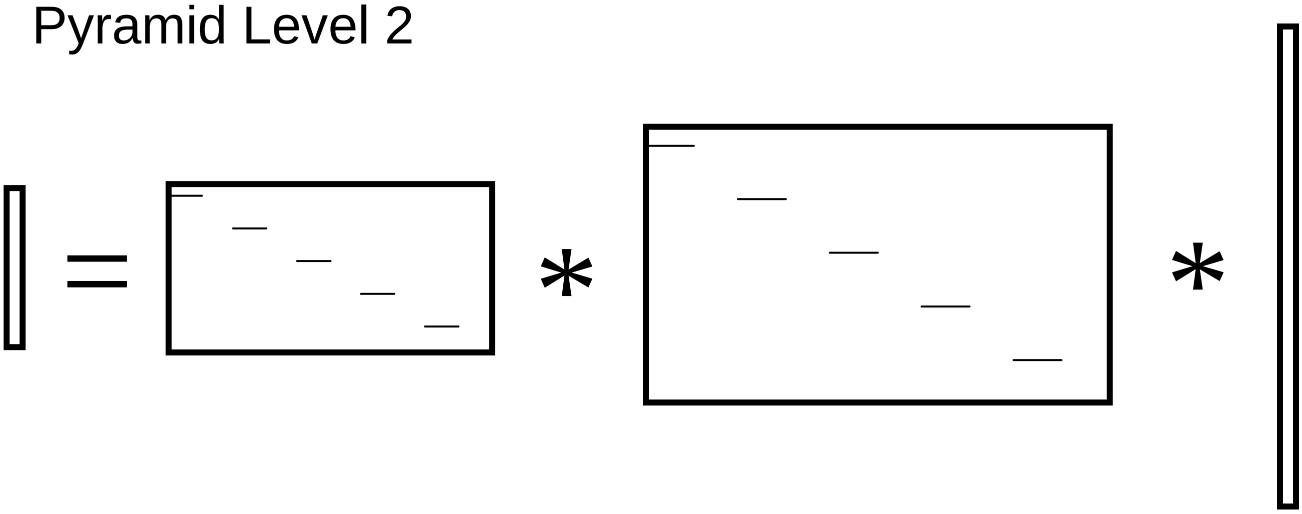


Pyramid Level 2

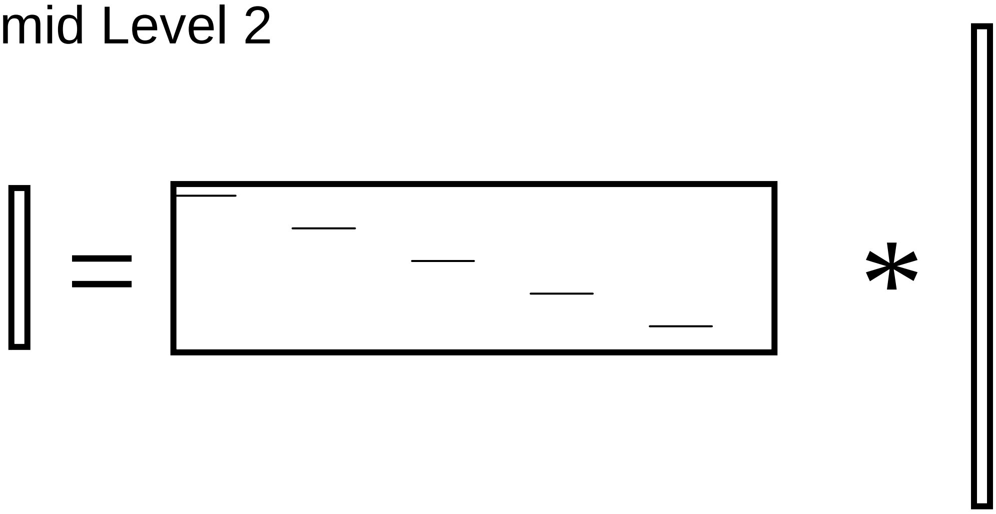


Pyramid =
Convolution + Sampling

Pyramid Level 2



Pyramid Level 2



Pyramid as Matrix Computation - Example

U1 =

1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	6	4	1	0

- Next pyramid level

U2 =

1	4	6	4	1	0	0	0
0	0	1	4	6	4	1	0
0	0	0	0	1	4	6	4
0	0	0	0	0	0	1	4

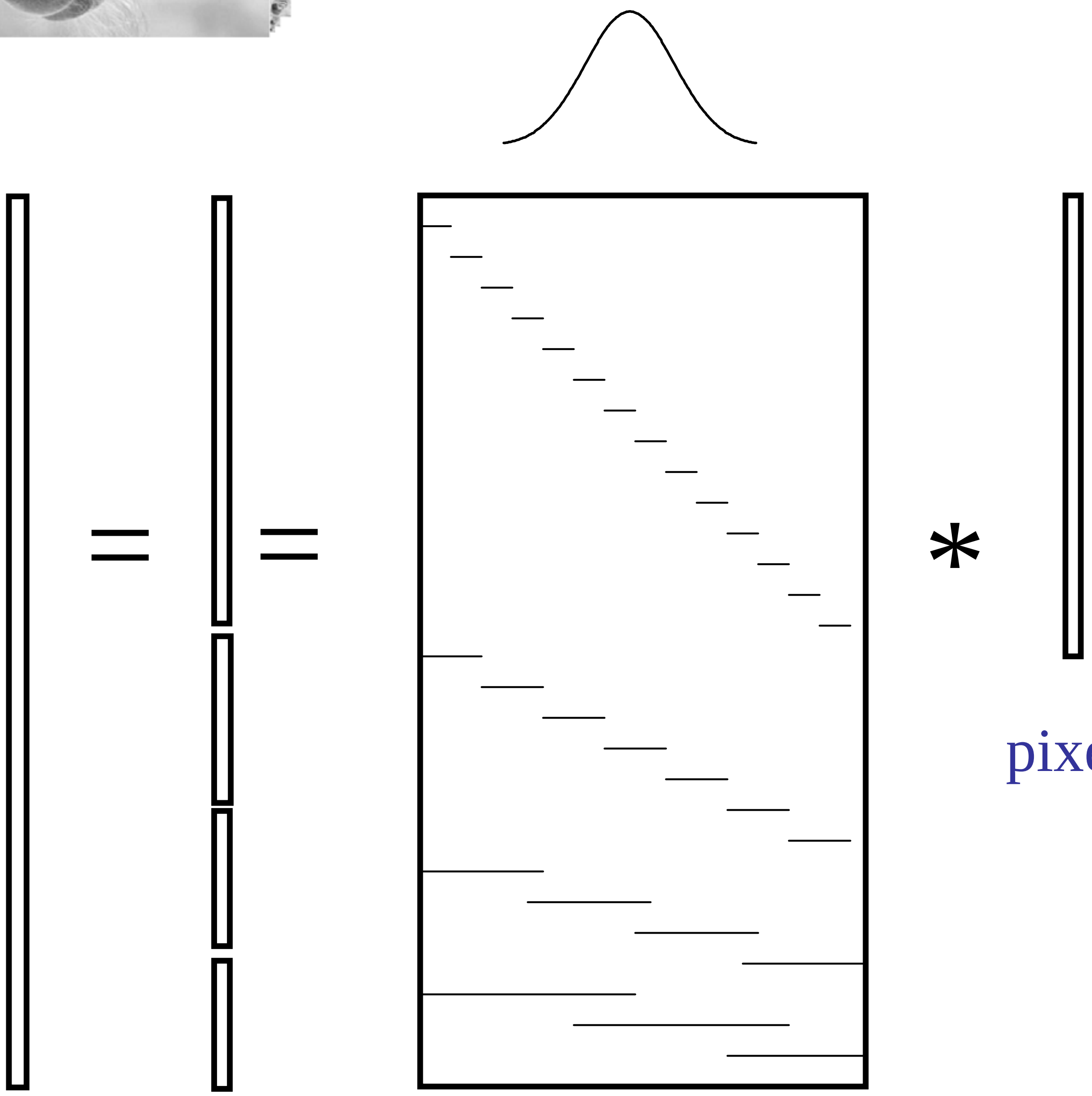
- The combined effect of the two pyramid levels

U2 * U1 =

1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0	0	0	0	0
0	0	0	0	1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0
0	0	0	0	0	0	0	0	1	4	10	20	31	40	44	40	30	16	4	0
0	0	0	0	0	0	0	0	0	0	0	0	1	4	10	20	25	16	4	0

from: B.Freeman

Gaussian Pyramid



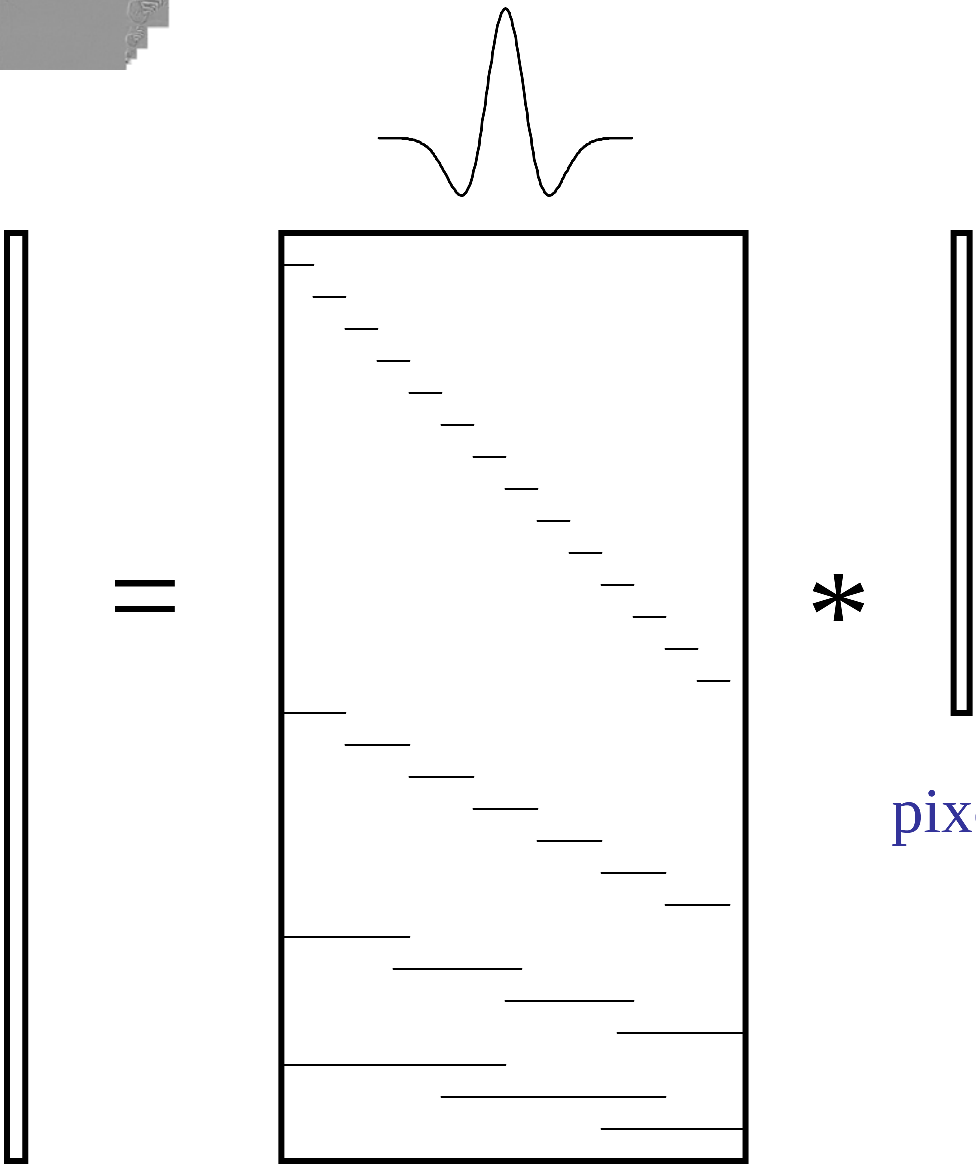
pixel image

Gaussian
pyramid

Overcomplete representation.
Low-pass filters, sampled
appropriately for their blur.

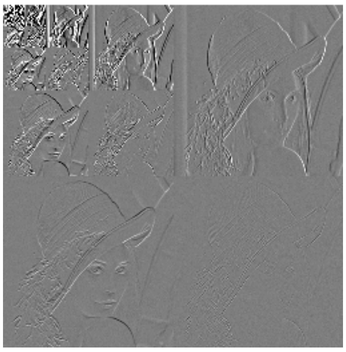
From: B. Freeman

Laplacian Pyramid

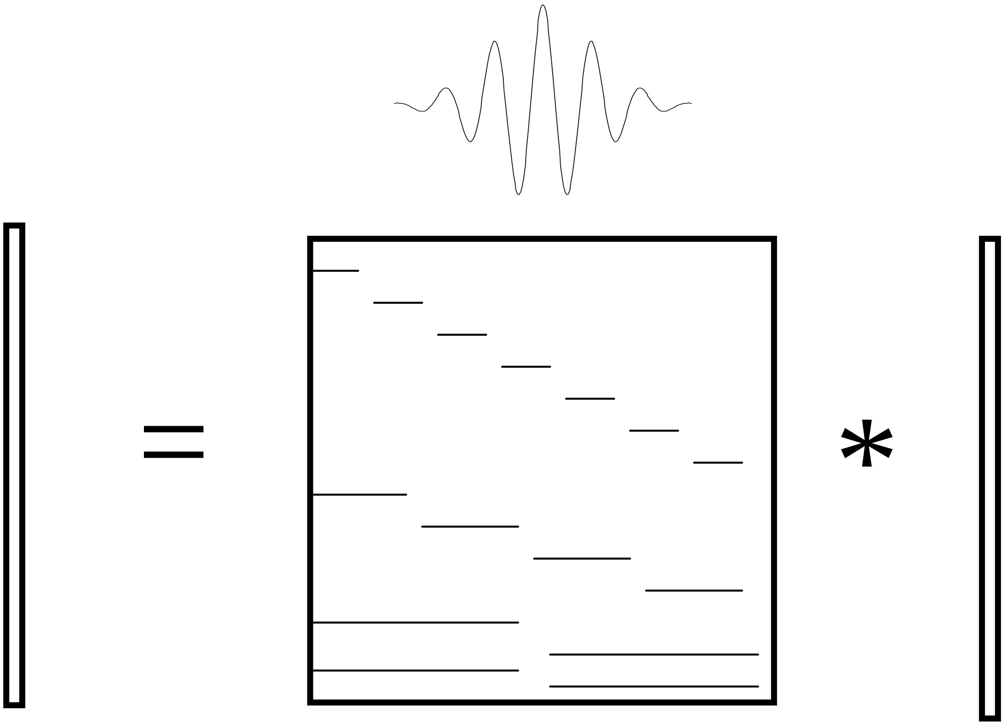


Overcomplete representation.
Transformed pixels represent
bandpassed image information.

From: B. Freeman



Wavelet Transform



Wavelet pyramid

$=$

Ortho-normal transform (like Fourier transform), but with localized basis functions.

$*$

pixel image

From: B. Freeman

The End