Enhancement v.s. Restoration

- Image Enhancement:
  - A process which aims to improve bad images so they will "look" better.
- Image Restoration:
  - A process which aims to invert known degradation operations applied to images.

Enhancement vs. Restoration

"Better" visual representation	⟷	Remove effects of sensing environment
Subjective	⟷	Objective
No quantitative measures	⟷	Mathematical, model dependent quantitative measures

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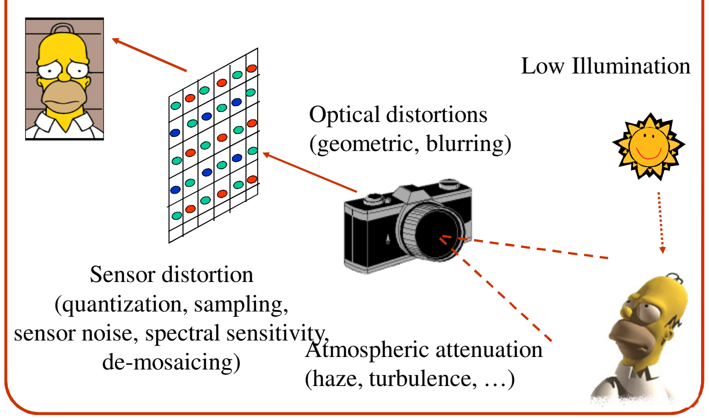
Typical Degradation Sources



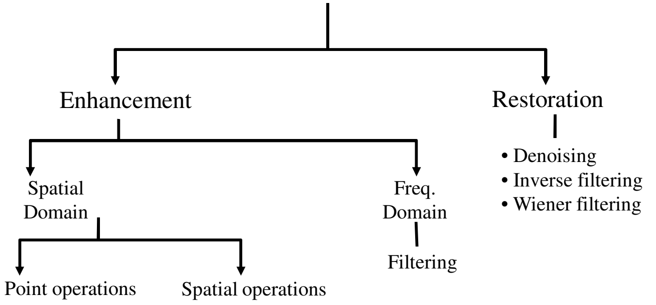
Low Illumination

Optical distortions (geometric, blurring)

Sensor distortion (quantization, sampling, sensor noise, spectral sensitivity, de-mosaicing)

Atmospheric attenuation (haze, turbulence, …)

Image Preprocessing



Enhancement

- Spatial Domain
  - Point operations
  - Spatial operations
- Freq. Domain
  - Filtering

Restoration

- Denoising
- Inverse filtering
- Wiener filtering

Examples

Hazing

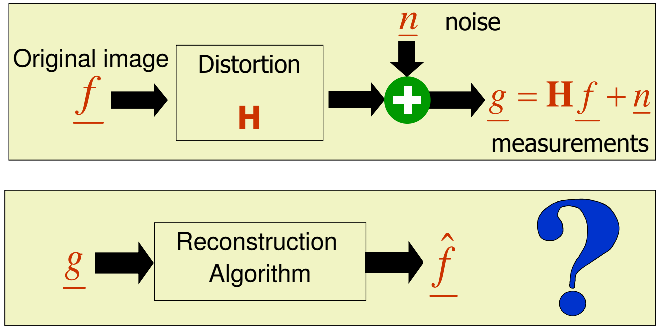
Echo image

Motion Blur

Blurred image

Blurred image + additive white noise

Reconstruction as an Inverse Problem



$$\underline{g} = \mathbf{H}\underline{f} + \underline{n}$$

So what is the problem?

$$\underline{g} \Rightarrow \boxed{H^{-1}\left(\underline{g}-\underline{n}\right)} \Rightarrow \underline{\hat{f}}$$

- Typically:
  - The distortion H is singular or ill-posed.
  - The noise n is unknown, only its statistical properties can be learnt.

Key point: Stat. Prior of Natural Images

MAP (Max A-posteriori) estimation:

$$\underline{\hat{f}} = \arg\max_{x} P\left(f \middle| g\right) \propto \arg\max_{x} \overset{\text{likelihood}}{P\left(g \middle| f\right)} \overset{\text{prior}}{P\left(f\right)}$$

Bayesian Denoising

- Assume an additive noise model :

$$g=f+n$$

- A MAP estimate for the original f:

$$\hat{f} = \arg\max_{f} P\left(f \mid g\right)$$

- Using Bayes rule and taking the log likelihood :

$$\hat{f} = \arg\max_{f} \frac{P(g \mid f)P(f)}{P(g)} = \arg\min_{f} \left\{-\log P(g \mid f) - \log P(f)\right\}$$

Bayesian Denoising

If noise component is white Gaussian distributed:

$g=f+n$ where n is distributed ~N(0,σ)

$$P\left(g \mid f\right) = e^{-(g-f)^2/\sigma}$$

$$-log\ P\left(g \mid f\right) = (g-f)^2/\sigma$$

$$\boxed{\hat{f} = \arg\min_{f} \left\{\underbrace{(g-f)^2}_{\text{data term}} + \underbrace{\lambda R(f)}_{\text{prior term}}\right\}}$$

R(f) is a penalty for non probable f

Inverse Filtering

- Degradation model:

$$\boxed{g(x,y) = h(x,y)*f(x,y)}$$

$$\updownarrow$$

$$G(u,v)=H(u,v)\cdot F(u,v)$$

$$\Rightarrow \hat{F}(u,v)=G(u,v)/H(u,v)$$

Inverse Filtering (Cont.)

Two problems with the above formulation:

1. H(u,v) might be zero for some (u,v).
2. In the presence of noise the noise might be amplified:

$$\hat{F}(u,v)=G(u,v)/H(u,v) + N(u,v)/H(u,v)$$

Solution: Use prior information

$$\boxed{\hat{F} = \arg\min_{F} \underbrace{(HF-G)^2}_{\text{data term}} + \underbrace{\lambda R(F)}_{\text{prior term}}}$$

Option 1: Prior Term

- Use penalty term that restrains high F values:

$$\hat{F} = \arg\min_{F} E(F)$$

where $E(F) = (HF-G)^2 + \lambda F^2$

- Solution:

$$\frac{\partial E(F)}{\partial F} = 2H^*(HF-G) + 2\lambda F = 0$$

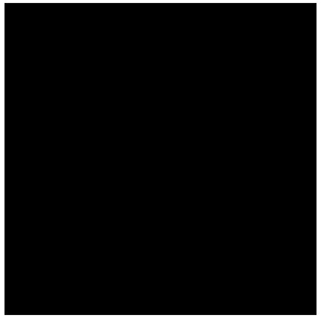
$$\boxed{\hat{F} = \frac{H^*}{H^*H+\lambda}G}$$

$H(u,v) >> 1 \;\Rightarrow\; \hat{F} = G/H$

$H(u,v) << 1 \;\Rightarrow\; \hat{F} = 0$

Degraded Image (echo)

$\hat{F}$=G/H



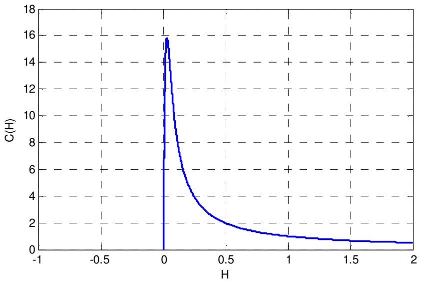
$$\hat{F} = \frac{H^*}{H^* H + \lambda} G$$

Degraded Image (echo+noise)

$$\hat{F} = \frac{H^*}{H^* H + \lambda} G$$

- The inverse filter is C(H)= $H^*/(H^*H+ \lambda)$
- At some range of (u,v):

  S(u,v)/N(u,v) < 1 ⇨ noise amplification.



$\lambda=10^{-3}$

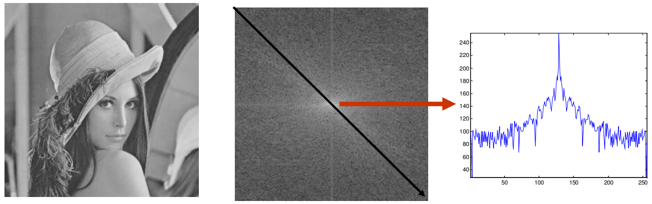
Option 2: Prior Term

1. Natural images tend to have low energy at high frequencies
2. White noise tend to have constant energy along freq.

$$\hat{F} = \arg\min_{F} E(F)$$

where

$$E(F) = (HF - G)^2 + \lambda(u^2 + v^2)F^2$$



- Solution: $\dfrac{\partial E(F)}{\partial F} = 2H^*(HF - G) + 2\lambda(u^2+v^2)F = 0$

$$\boxed{\hat{F} = \frac{H^*}{H^*H + \lambda(u^2+v^2)}G}$$

- This solution is known as the *Wienner Filter*
- Here we assume N(u,v) is constant.
- If N(u,v) is not constant:

$$\hat{F} = \frac{H^*}{H^*H + \lambda(u^2+v^2)\cdot N(u,v)}G$$

Degraded Image (echo+noise)

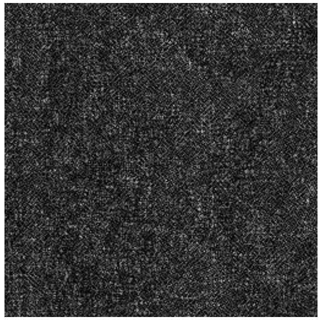
Wienner Filtering

Wienner

Previous

Degraded Image (blurred+noise)

Inverse Filtering



Using Prior (Option 1)

Wienner Filtering