Enhancement v.s. Restoration

- Image Enhancement:
  - A process which aims to improve bad images so they will “look” better.
- Image Restoration:
  - A process which aims to invert known degradation operations applied to images.

Enhancement vs. Restoration

• "Better" visual representation	↔	• Remove effects of sensing environment
• Subjective	↔	• Objective
• No quantitative measures	↔	• Mathematical, model dependent quantitative measures

Typical Degradation Sources

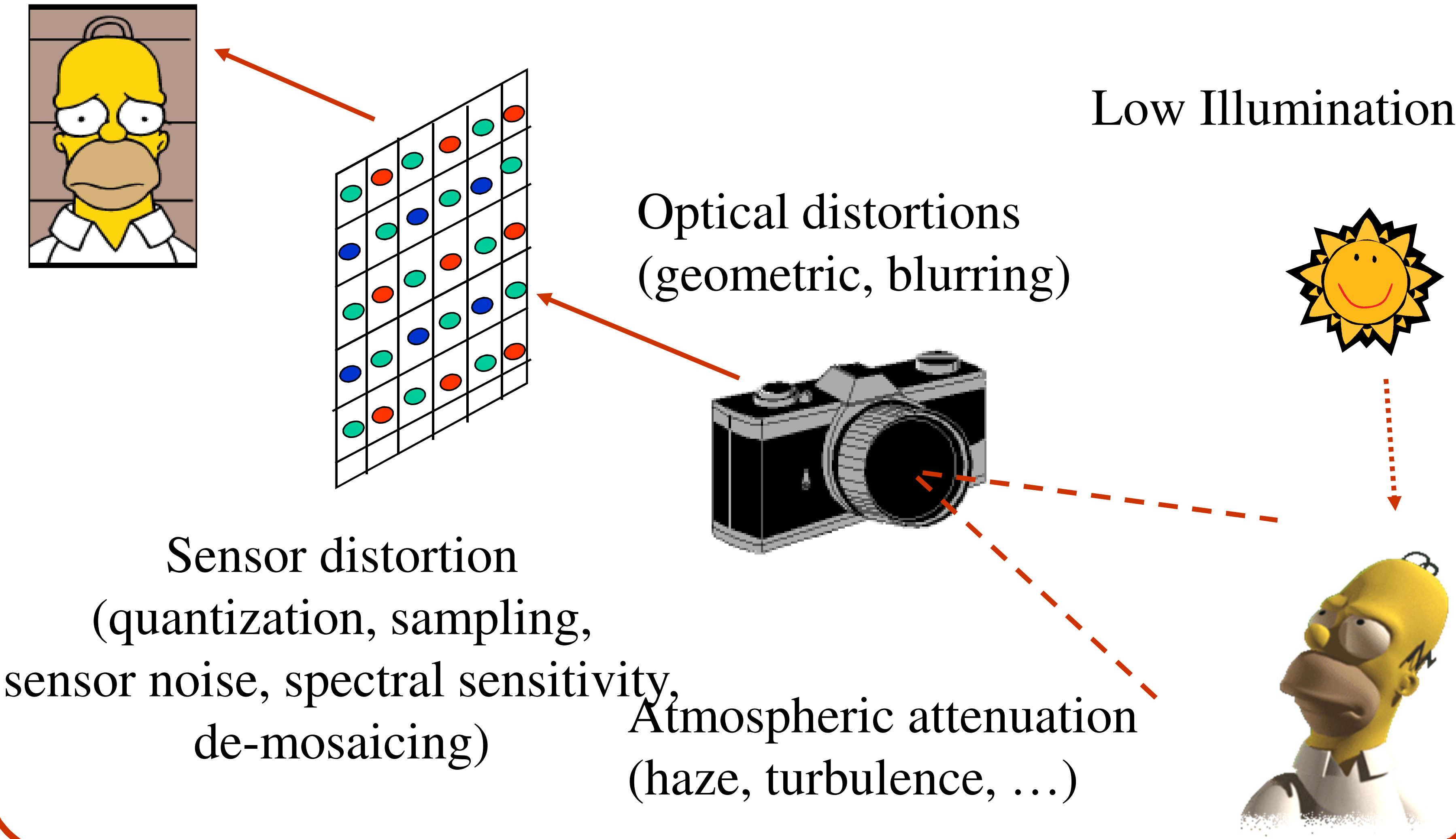


Image Preprocessing

- Enhancement
  - Spatial Domain
    - Point operations
    - Spatial operations
  - Freq. Domain
    - Filtering
- Restoration
  - Denoising
  - Inverse filtering
  - Wiener filtering

Examples

Hazing

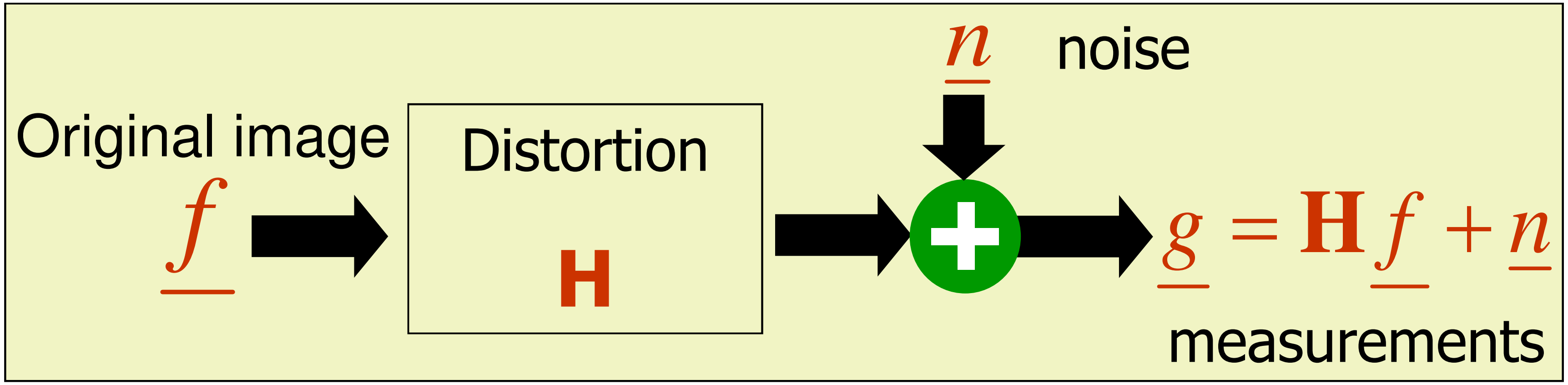
Echo image

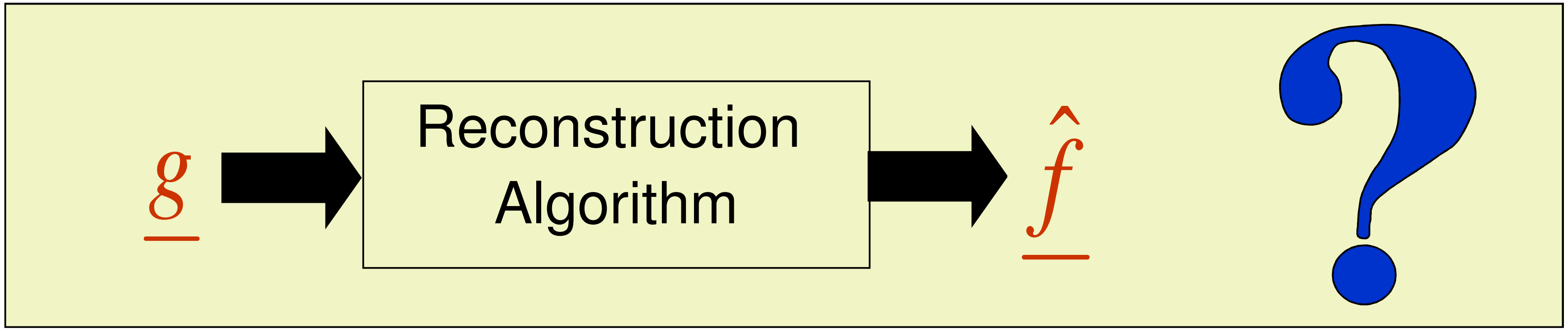
Motion Blur

Blurred image

Blurred image + additive white noise

Reconstruction as an Inverse Problem





So what is the problem?

$$\underline{g} \Rightarrow \boxed{\mathrm{H}^{-1}\left(\underline{g}-\underline{n}\right)} \Rightarrow \underline{\hat{f}}$$

- Typically:
  - The distortion H is singular or ill-posed.
  - The noise n is unknown, only its statistical properties can be learnt.

Key point: Stat. Prior of Natural Images

MAP (Max A-posteriori) estimation:

likelihood prior

$$\underline{\hat{f}} = \arg\max_{x} P\left(f \middle| g\right) \propto \arg\max_{x} \underbrace{P\left(g \middle| f\right)}_{\text{likelihood}} \underbrace{P\left(f\right)}_{\text{prior}}$$

Bayesian Denoising

- Assume an additive noise model :

g=f + n

- A MAP estimate for the original f:

$$\hat{f} = \arg\max_{f} P(f \mid g)$$

- Using Bayes rule and taking the log likelihood :

$$\hat{f} = \arg\max_{f} \frac{P(g \mid f)P(f)}{P(g)} = \arg\min_{f} \{-\log P(g \mid f) - \log P(f)\}$$

Bayesian Denoising

If noise component is white Gaussian distributed:

g=f + n   where n is distributed ~N(0,σ)

$$P(g \mid f) = e^{-(g-f)^2/\sigma}$$

$$-\log P(g \mid f) = (g-f)^2/\sigma$$

$$\boxed{\hat{f} = \arg\min_{f} \{ \underbrace{(g-f)^2}_{\text{data term}} + \underbrace{\lambda R(f)}_{\text{prior term}} \}}$$

R(f) is a penalty for non probable f

Inverse Filtering

- Degradation model:

$$g(x,y) = h(x,y)*f(x,y)$$

$$\updownarrow$$

$$G(u,v)=H(u,v)\cdot F(u,v)$$

$$\Rightarrow \hat{F}(u,v)=G(u,v)/H(u,v)$$

Inverse Filtering (Cont.)

Two problems with the above formulation:

1. H(u,v) might be zero for some (u,v).
2. In the presence of noise the noise might be amplified:

$$\hat{F}(u,v)=G(u,v)/H(u,v) + N(u,v)/H(u,v)$$

Solution: Use prior information

$$\hat{F} = \arg\min_{F} \underbrace{\left(HF - G\right)^2}_{\text{data term}} + \underbrace{\lambda R\left(F\right)}_{\text{prior term}}$$

Option 1: Prior Term

- Use penalty term that restrains high F values:

$$\hat{F} = \arg\min_F E(F)$$

where $$E(F) = (HF - G)^2 + \lambda F^2$$

- Solution:

$$\frac{\partial E(F)}{\partial F} = 2H^*(HF - G) + 2\lambda F = 0$$

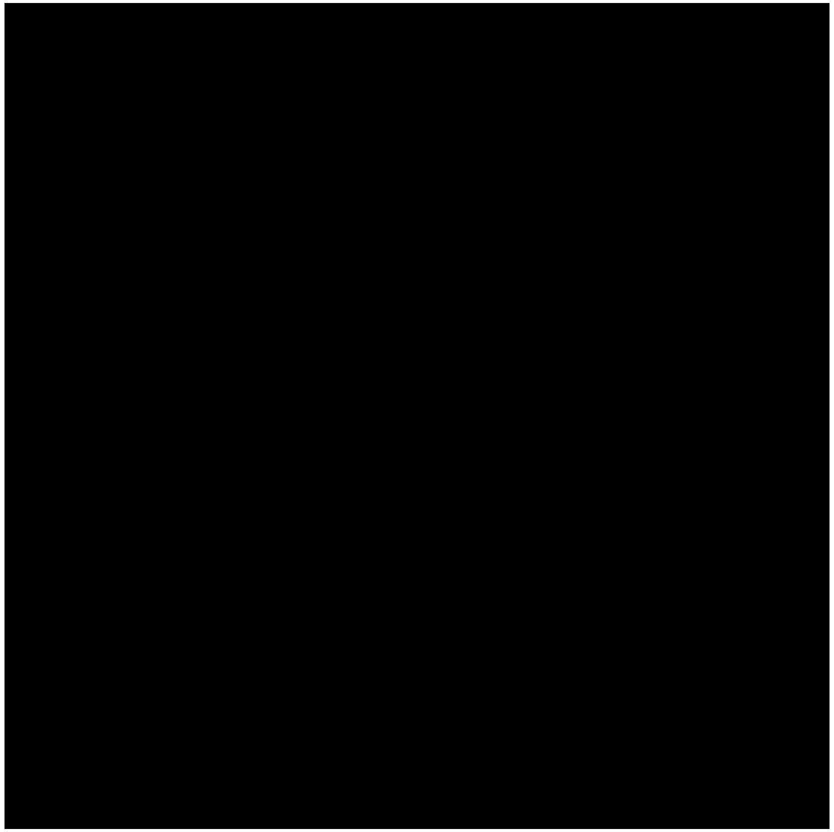
$$\boxed{\hat{F} = \frac{H^*}{H^*H + \lambda} G}$$

$$H(u,v) >> 1 \;\Rightarrow\; \hat{F} = G/H$$

$$H(u,v) << 1 \;\Rightarrow\; \hat{F} = 0$$

Degraded Image (echo)

$$\hat{F} = G/H$$



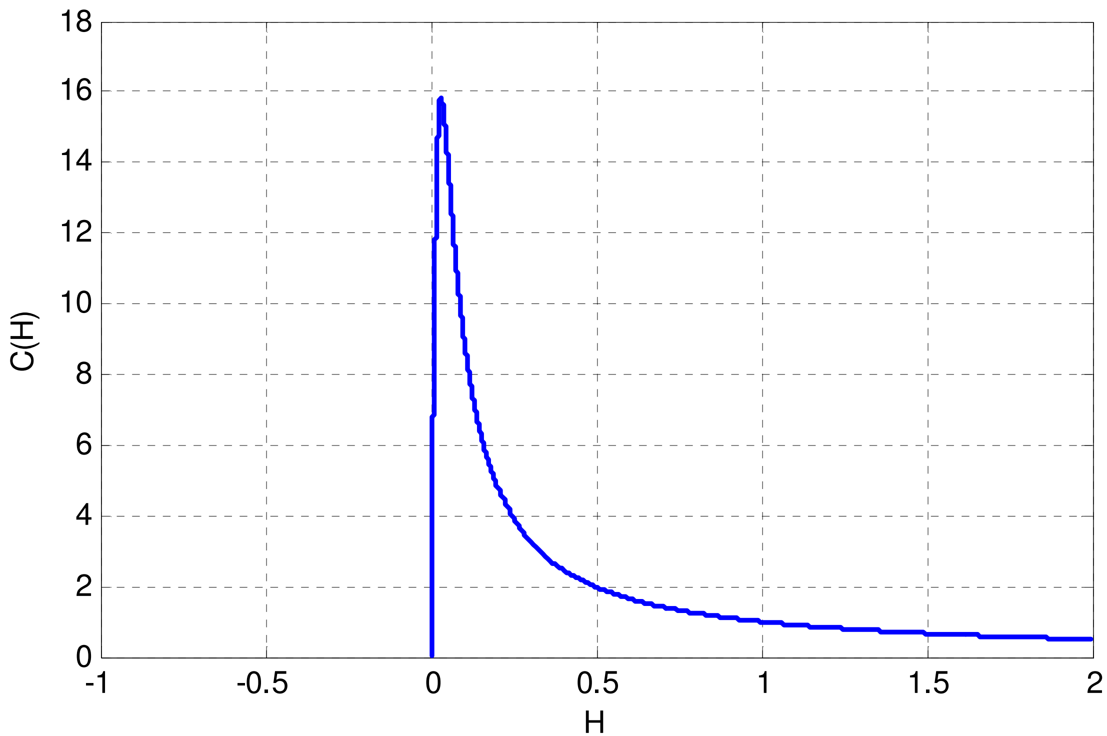
$$\hat{F} = \frac{H^*}{H^*H + \lambda} G$$

Degraded Image (echo+noise)

$$\hat{F} = \frac{H^*}{H^* H + \lambda} G$$

- The inverse filter is $C(H) = H^*/(H^*H + \lambda)$
- At some range of (u,v):

  $S(u,v)/N(u,v) < 1 \Rightarrow$ noise amplification.



$\lambda = 10^{-3}$

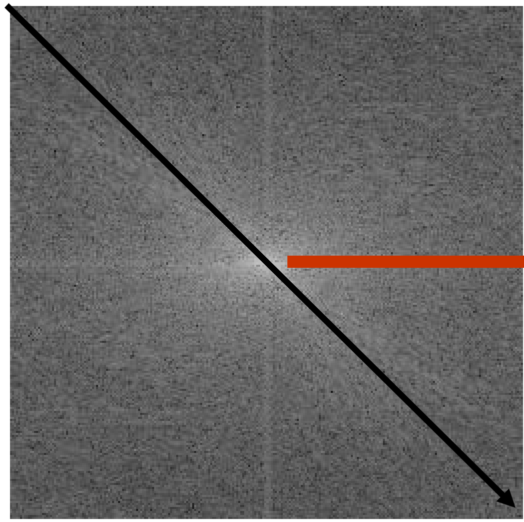
Option 2: Prior Term

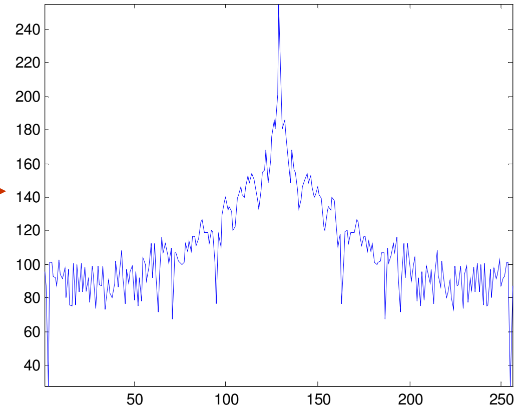
1. Natural images tend to have low energy at high frequencies
2. White noise tend to have constant energy along freq.

$$\hat{F} = \arg\min_{F} \; E(F)$$

where

$$E(F) = (HF - G)^2 + \lambda\left(u^2 + v^2\right)F^2$$





- Solution: $\frac{\partial E(F)}{\partial F} = 2H^*(HF - G) + 2\lambda(u^2 + v^2)F = 0$

$$\boxed{\hat{F} = \frac{H^*}{H^*H + \lambda(u^2 + v^2)} G}$$

- This solution is known as the *Wienner Filter*
- Here we assume N(u,v) is constant.
- If N(u,v) is not constant:

$$\hat{F} = \frac{H^*}{H^*H + \lambda(u^2 + v^2) \cdot N(u,v)} G$$

Degraded Image (echo+noise)

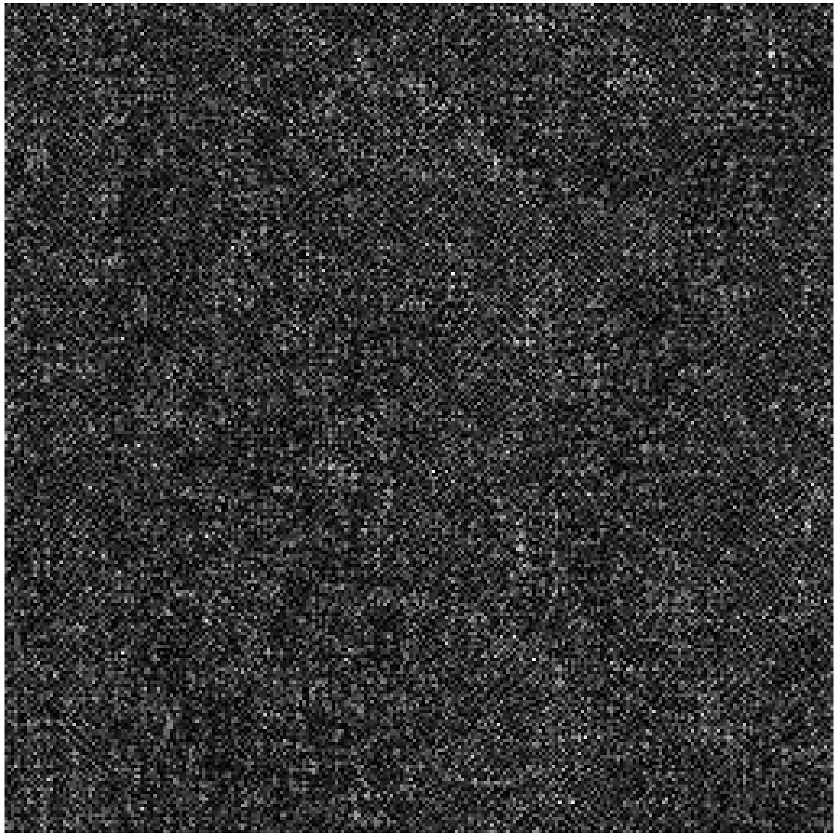
Wienner Filtering

Wienner

Previous

Degraded Image (blurred+noise)

Inverse Filtering



Using Prior (Option 1)

Wienner Filtering