# The Image Histogram

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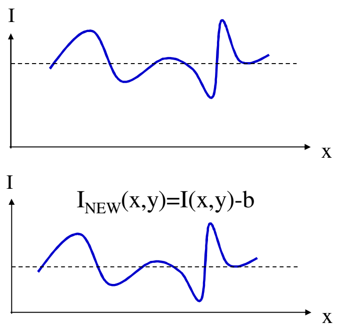
## Image Characteristics


## Image Mean

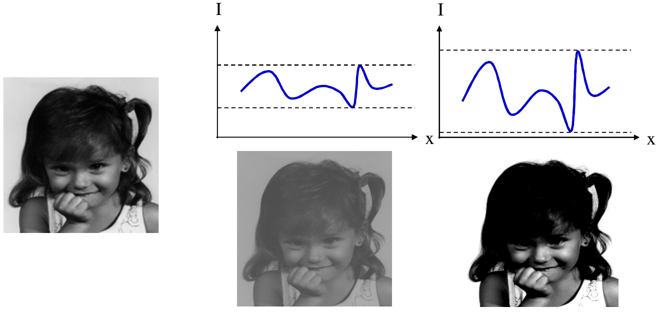
$$I_{av} = \frac{\sum_i \sum_j I(i,j)}{\sum_i \sum_j 1}$$




Changing the image mean

## Image Contrast

- The contrast definition of the entire image is ambiguous
- In general it is said that the image contrast is high if the image gray-levels fill the entire range




Low contrast High contrast

## Global Contrast

Global Contrast – Definition 1

$$\max\{ I(x,y)\} - \min\{ I(x,y)\}$$

Global Contrast – Definition 2

$$\mathrm{var}\{I(x,y)\} = \mathrm{mean}\{(I(x,y) - I_{av})^2\}$$

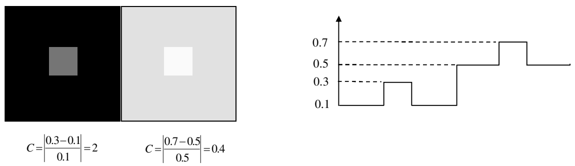
Global Contrast – Definition 3

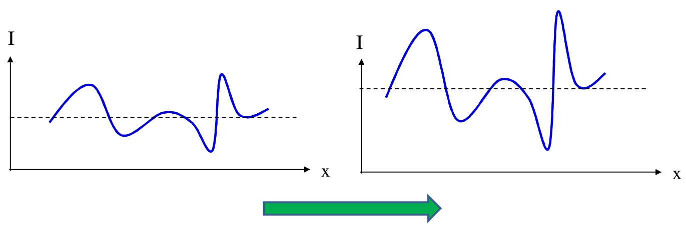
$$std\{ I(x,y)\} = \sqrt{var\{ I(x,y)\}}$$

## Local Image Contrast

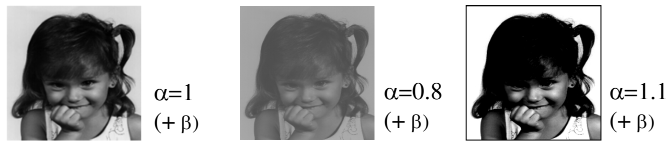
- The **local** contrast at an image point denotes the (relative) difference between the intensity of the point and the intensity of its neighborhood:

$$C = \left| \frac{I_p - I_n}{I_n} \right|$$



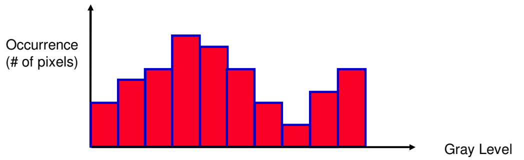




$$I_{NEW}(x,y) = \alpha \cdot I(x,y)$$

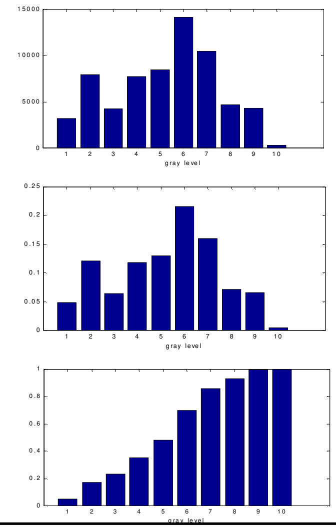



Q: How can we maximize the image contrast using linear operation on the image values? $I_{NEW}(x,y) = \alpha \cdot I(x,y) + \beta$

# The Image Histogram




- H(k) specifies the # of pixels with gray-value *k*
- Let N be the number of pixels: $N = \sum_k H(k)$
- P(k) = H(k)/N defines the normalized histogram
- $C(k)=\sum_{i=1}^{k} H(i)$ defines the accumulated histogram

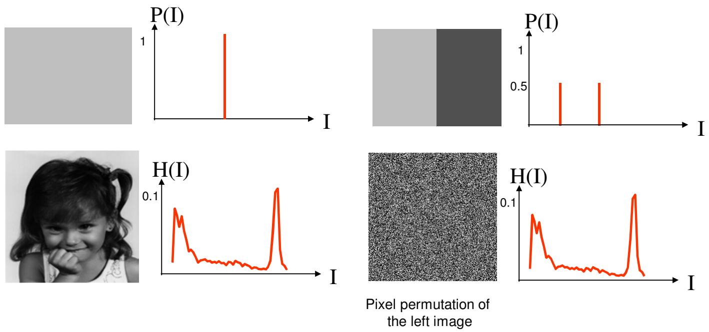



Histogram

Normalized Histogram

Accumulated Histogram

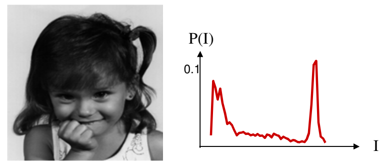
# Examples




Pixel permutation of the left image

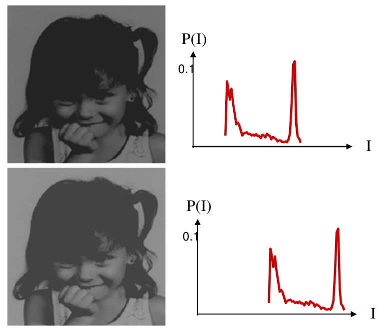
The image histogram does not fully represent the image

Original image




Decreasing contrast

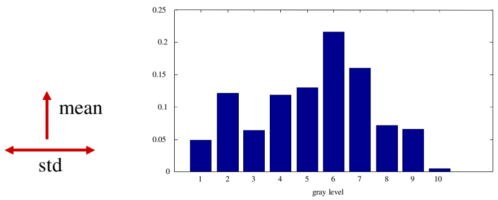
Increasing average



## Image Statistics

- The image mean: $E\{I\}=\frac{1}{N}\sum_{i,j} I(i,j)=\frac{1}{N}\sum_k k\,H(k)=\sum_k k\,P(k)$
- The image s.t.d. : $\sigma(I)=\sqrt{E\{(I-E\{I\})^2\}}=\sqrt{E(I^2)-E^2(I)}$

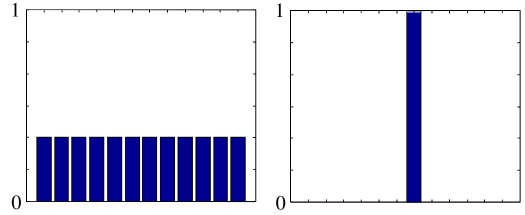
$$where\quad E\{I^2\}=\sum_k k^2 P(k)$$



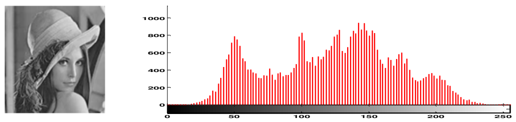

## Image Entropy

$$Entropy(I)=-\sum_k P(k)\log P(k)$$

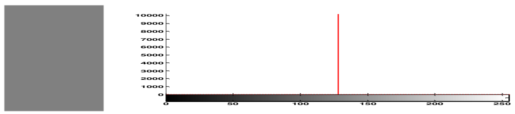
- The image entropy specifies the uncertainty in the image values.
- Measures the averaged amount of information required to encode the image values.




- An infrequent event provides more information than a frequent event
- Entropy is a measure of histogram dispersion



*entropy=7.4635*

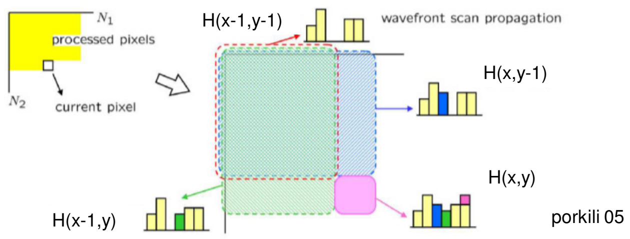


*entropy=0*

## Adaptive Histogram

- In many cases histograms are needed for local areas in an image
- Examples:
  - Pattern detection
  - adaptive enhancement
  - adaptive thresholding
  - tracking

## Implementation: Integral Histogram




- **Integral histogram**: H(x,y) represent the histogram of a window whose right-bottom corner is (x,y)
- Construct by scan order:

$$H(x,y) = H(x,y-1) + H(x-1,y) - H(x-1,y-1)$$

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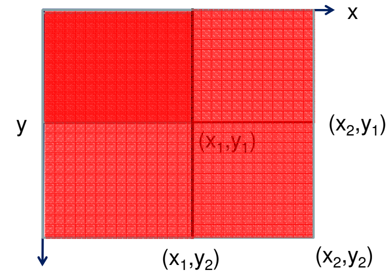
- Using integral histogram we can calculate local histograms of any window $H(x_1{:}x_2, y_1{:}y_2)$




$$H(x_1{:}x_2, y_1{:}y_2) = H(x_2,y_2) + H(x_1,y_1) - H(x_1,y_2) - H(x_2,y_1)$$

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## Histogram Usage

- Digitizing parameters
- Measuring image properties:
  - Average
  - Variance
  - Entropy
  - Contrast
  - Area (for a given gray-level range)
- Threshold selection
- Image distance
- Image Enhancement
  - Histogram equalization
  - Histogram stretching
  - Histogram matching

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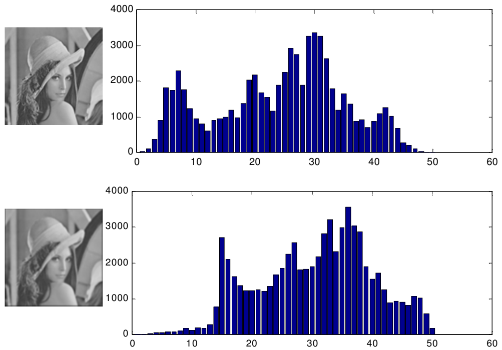
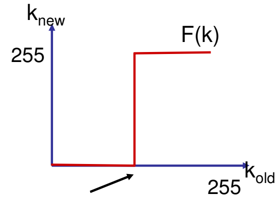
## Example: Auto-Focus

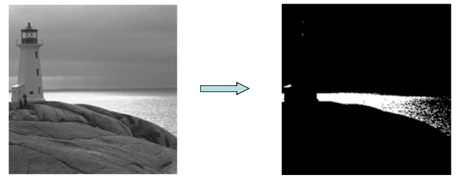
- In some optical equipment (e.g. slide projectors) inappropriate lens position creates a blurred (“out-of-focus”) image
- We would like to automatically adjust the lens
- How can we measure the amount of blurring?




- Image mean is not affected by blurring
- Image s.t.d. (entropy) is decreased by blurring
- Algorithm: Adjust lens according the changes in the histogram s.t.d.

## Thresholding

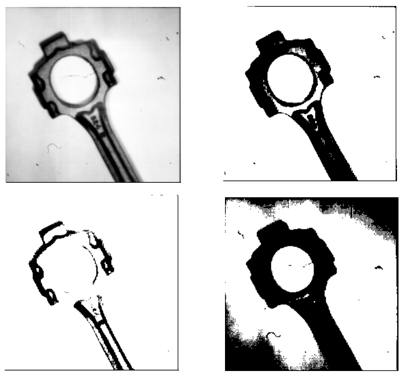


## Threshold Selection

Original Image

Binary Image



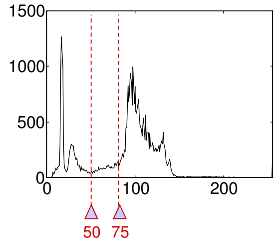
Threshold too low

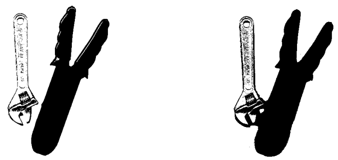
Threshold too high

## Segmentation using Thresholding

Original

Histogram



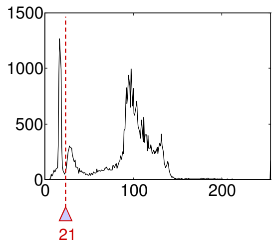
Threshold = 50

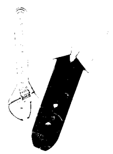
Threshold = 75

## Segmentation using Thresholding

Original

Histogram

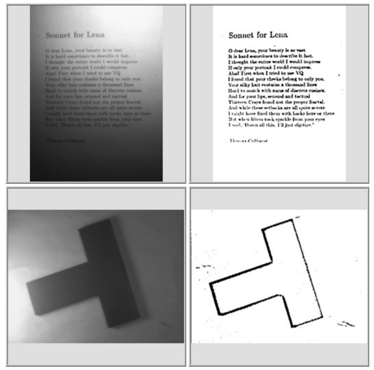


Threshold = 21

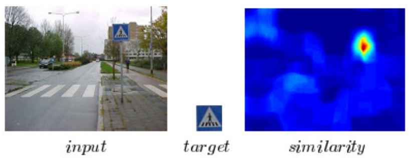
## Adaptive Thresholding

- Thresholding is space variant.
- How can we choose the local threshold values?



## Histogram based image distance

- **Problem**: Given two images A and B whose (normalized) histogram are $P_A$ and $P_B$ define the distance D(A,B) between the images.
- Example Usage:
  - Tracking
  - Image retrieval
  - Registration
  - Detection
  - Many more ...

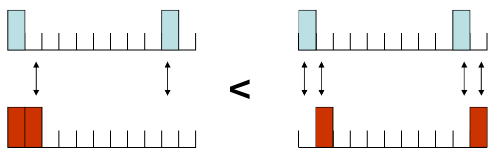



Porkili 05

## Option 1: Minkowski Distance

$$D_p(A,B) = \left[ \sum_k \left| P_A(k) - P_B(k) \right|^p \right]^{1/p}$$

- **Problem**: distance may not reflect the perceived dissimilarity:



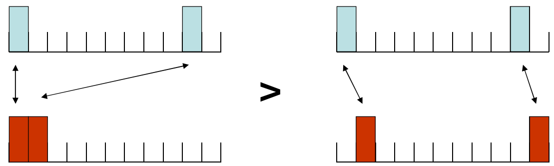
## Option 2: Kullback-Leibler (KL) Distance

$$D_{KL}(A \parallel B) = \sum_k P_A(k) \log \frac{P_A(k)}{P_B(k)}$$

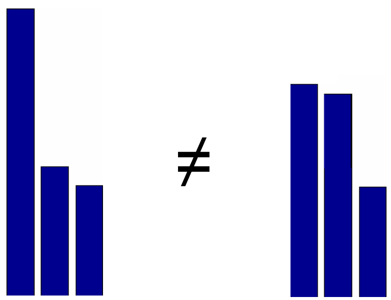
- Measures the amount of added information needed to encode image A based on the histogram of image B.
- Non-symmetric: $D_{KL}(A||B) \neq D_{KL}(B||A)$
- Suffers from the same drawback of the Minkowski distance.

## Option 3: The Earth Mover Distance (EMD)

- Suggested by Rubner & Tomasi 98
- Defines as the minimum amount of "work" needed to transform histogram $H_A$ towards $H_B$
- The term $d_{ij}$ defines the "ground distance" between gray-levels i and j.
- The term $F=\{f_{ij}\}$ is an admissible flow from $H_A(i)$ to $H_B(j)$

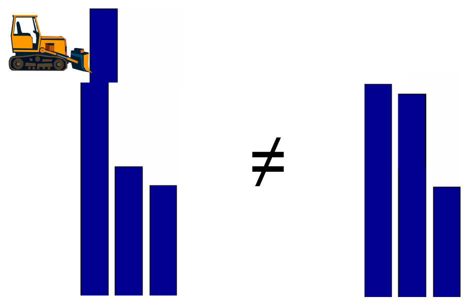


## Option 3: The Earth Mover Distance (EMD)



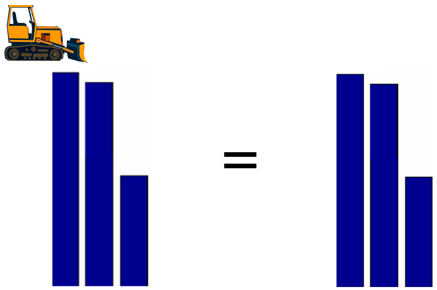
From: Pete Barnum

## Option 3: The Earth Mover Distance (EMD)



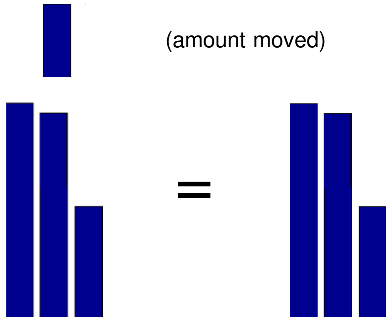
From: Pete Barnum

## Option 3: The Earth Mover Distance (EMD)



From: Pete Barnum

## Option 3: The Earth Mover Distance (EMD)




From: Pete Barnum

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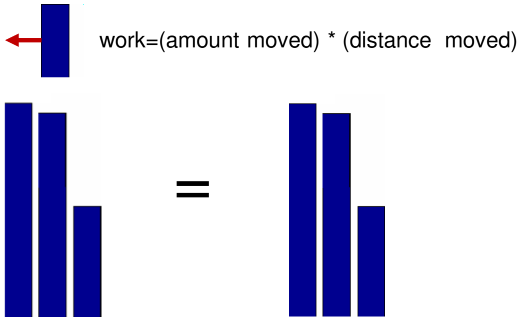



From: Pete Barnum

## Option 3: The Earth Mover Distance (EMD)

$$D_{EMD}(A,B) = \min_F \sum_i \sum_j f_{ij} \cdot d_{ij}$$

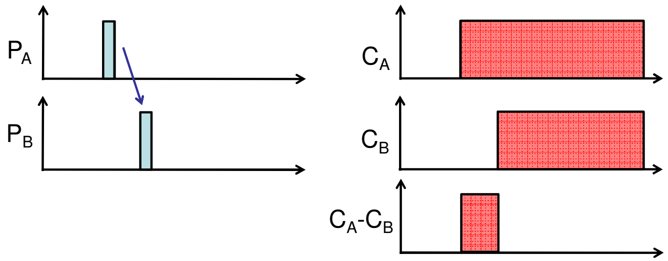
$$s.t. \quad f_{ij} \geq 0 \; ; \; P_B(k) = \sum_i f_{ik} \quad ; \quad P_A(k) \geq \sum_i f_{ki}$$

- Constraints:
  - Move earth only from A to B
  - After move $P_A$ will be equal to $P_B$
  - Cannot send more “earth” than there is
- Can be solved using Linear Programming
- Can be applied in high dim. histograms (color).

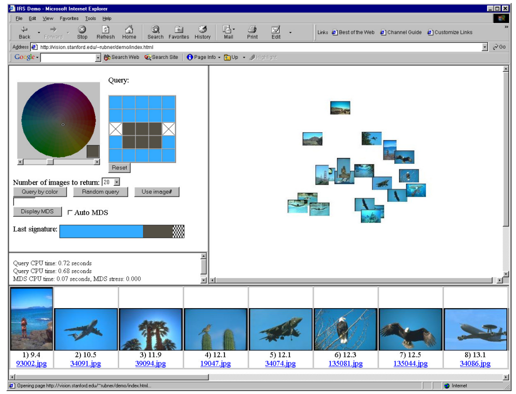
## Special case: EMD in 1D

- Define $C_A$ and $C_B$ as the accumulated histograms of image A and B respectively:

$$D_{EMD}(A,B) = \sum_k \left| C_A(k) - C_B(k) \right|$$




## Special case: EMD in 3D



Color Based Image Retrieval

Rubner & Tomasi 98