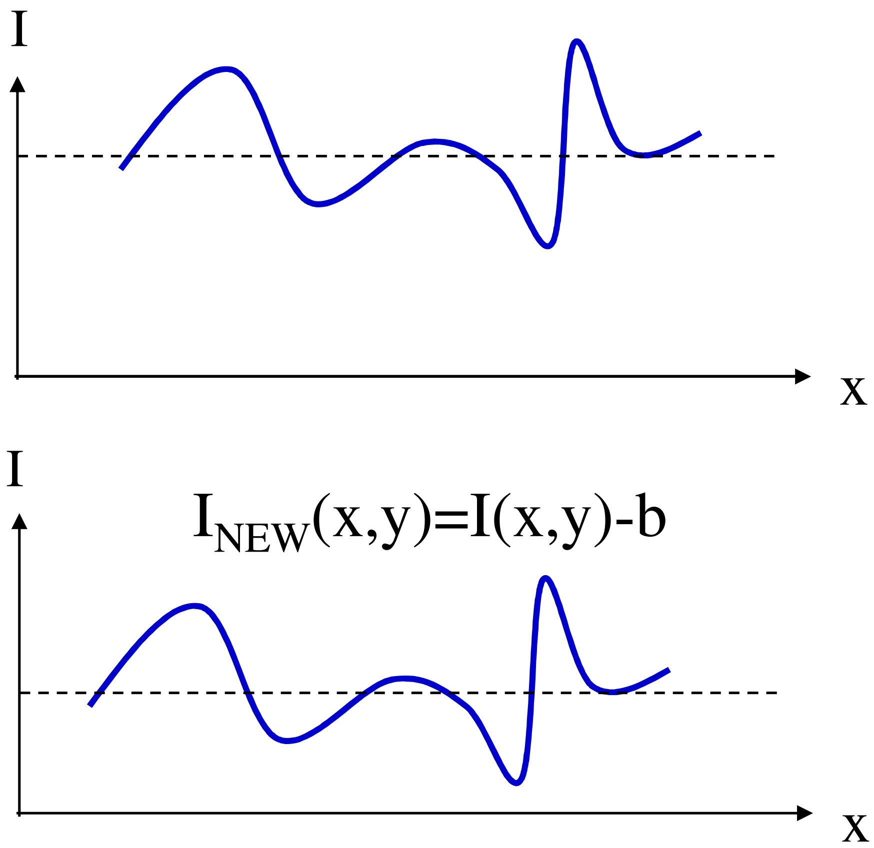
The Image Histogram

Image Characteristics

Image Mean

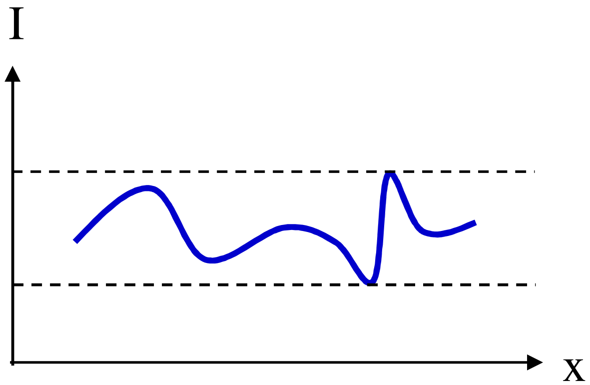
$$I_{av} = \frac{\sum_i \sum_j I(i,j)}{\sum_i \sum_j 1}$$

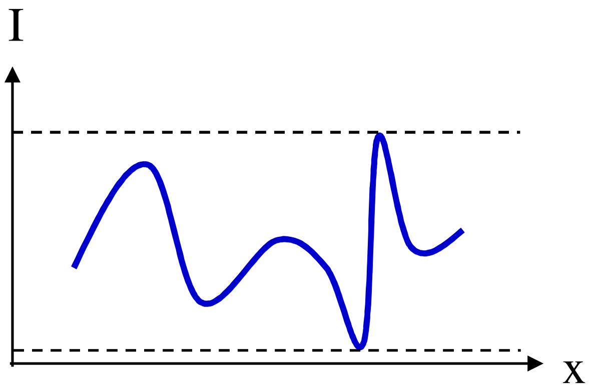


Changing the image mean

Image Contrast

- The contrast definition of the entire image is ambiguous
- In general it is said that the image contrast is high if the image gray-levels fill the entire range





Low contrast

High contrast

Global Contrast

Global Contrast – Definition 1

$$\max\{ I(x,y)\} - \min\{ I(x,y)\}$$

Global Contrast – Definition 2

$$\mathrm{var}\{ I(x,y)\} = \mathrm{mean}\{(I(x,y) - I_{av})^2\}$$

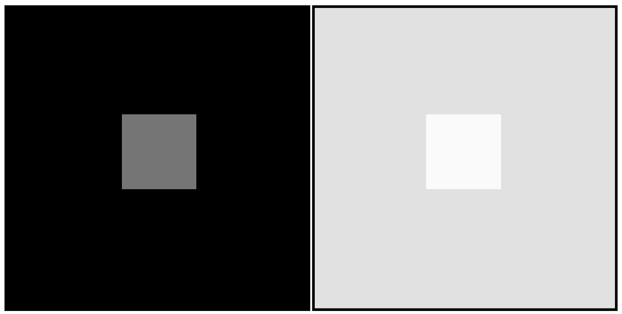
Global Contrast – Definition 3

$$std\{ \mathrm{I}(x, y)\} = \sqrt{var\{ \mathrm{I}(x, y)\}}$$

Local Image Contrast

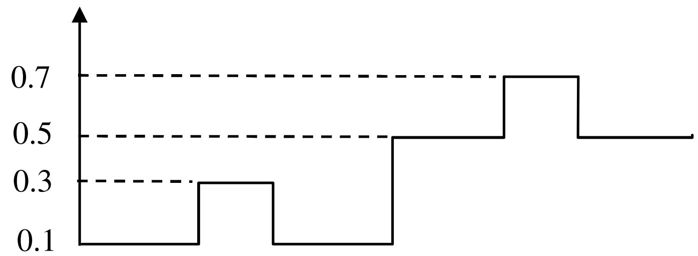
- The **local** contrast at an image point denotes the (relative) difference between the intensity of the point and the intensity of its neighborhood:

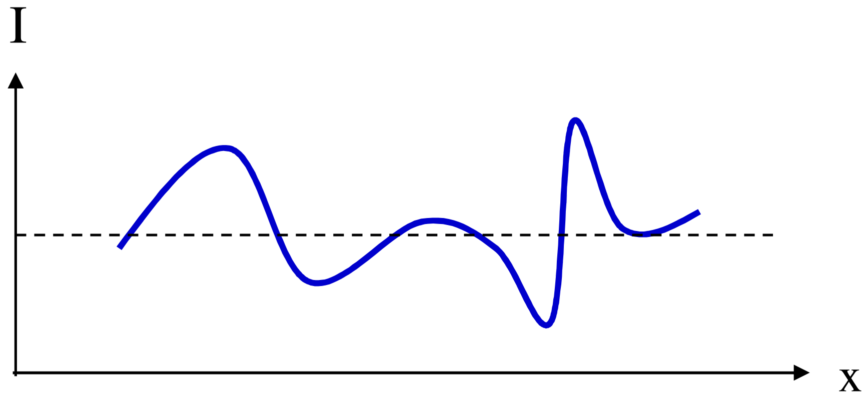
$$C = \left| \frac{I_p - I_n}{I_n} \right|$$

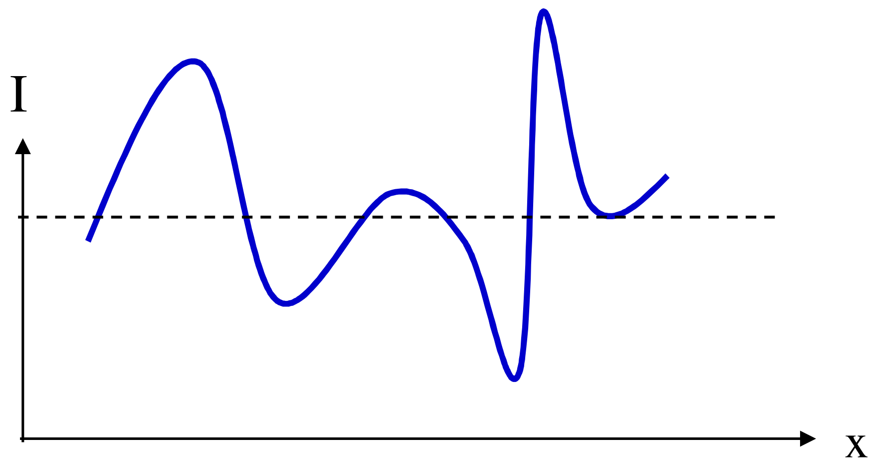


$C = \left| \frac{0.3 - 0.1}{0.1} \right| = 2$

$C = \left| \frac{0.7 - 0.5}{0.5} \right| = 0.4$







$$I_{NEW}(x,y)=\alpha\cdot I(x,y)$$

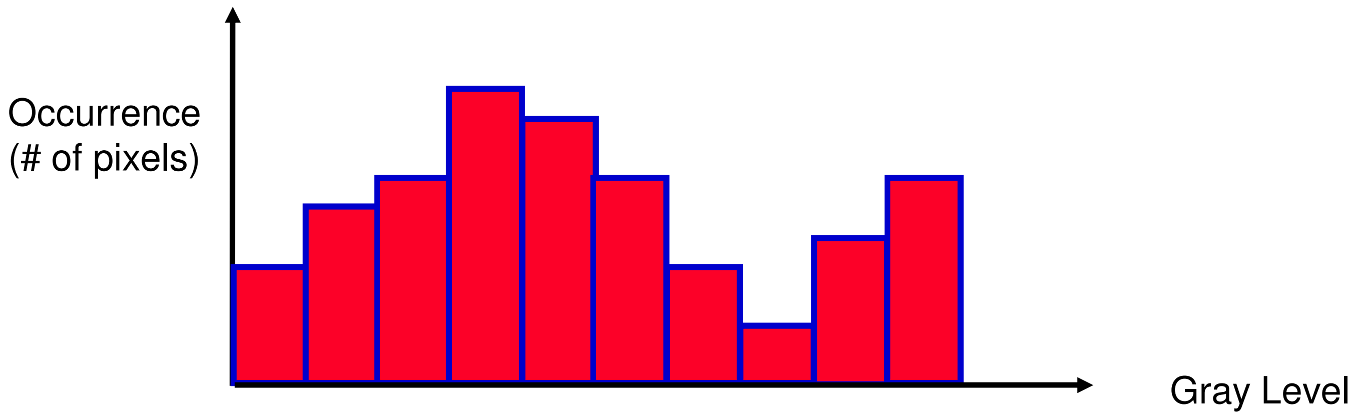
$\alpha=1$
$(+\ \beta)$

$\alpha=0.8$
$(+\ \beta)$

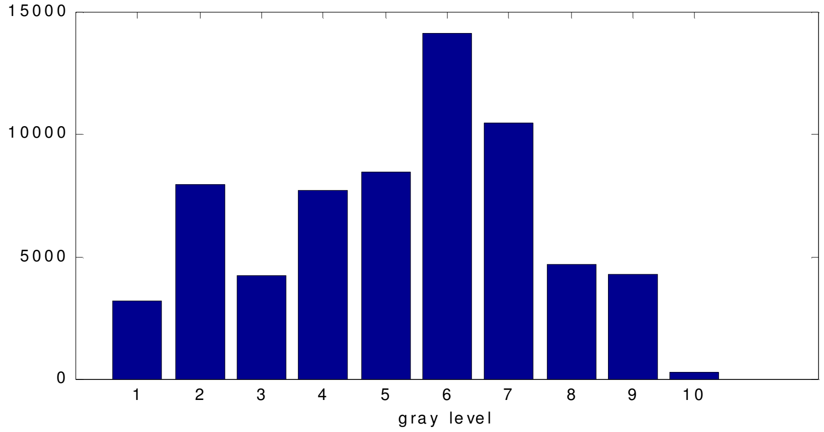
$\alpha=1.1$
$(+\ \beta)$

Q: How can we maximize the image contrast using linear operation on the image values? $I_{NEW}(x,y)=\alpha\cdot I(x,y)+\beta$

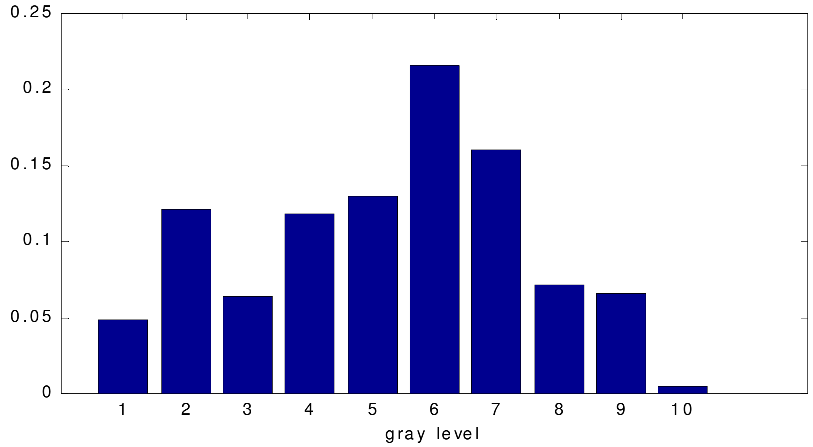
The Image Histogram



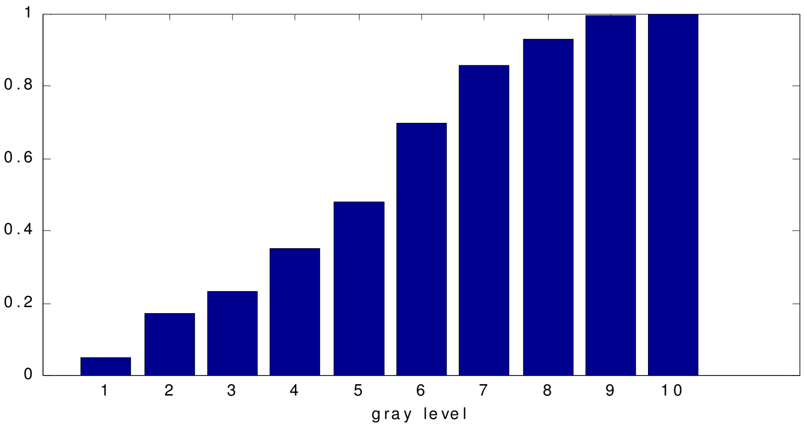
- H(k) specifies the # of pixels with gray-value *k*
- Let N be the number of pixels: $N = \sum_{k} H(k)$
- P(k) = H(k)/N defines the normalized histogram
- $C(k) = \sum_{i=1}^{k} H(i)$ defines the accumulated histogram



Histogram

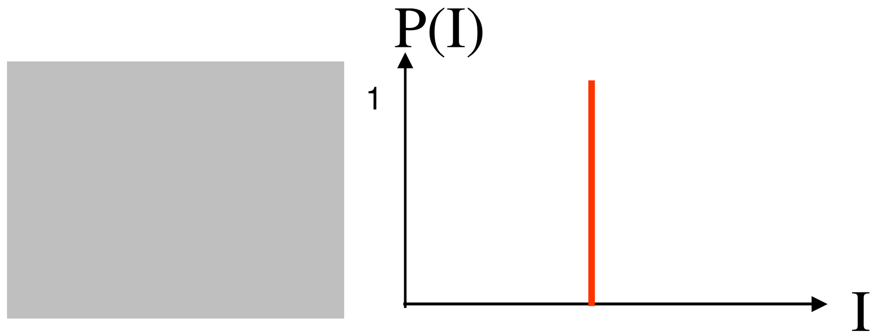


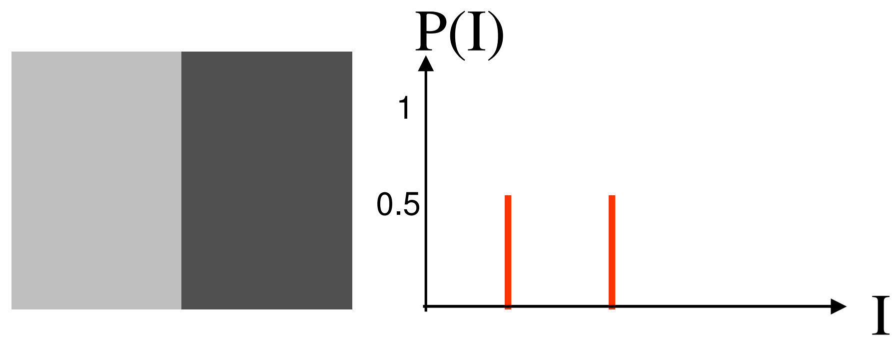
Normalized Histogram

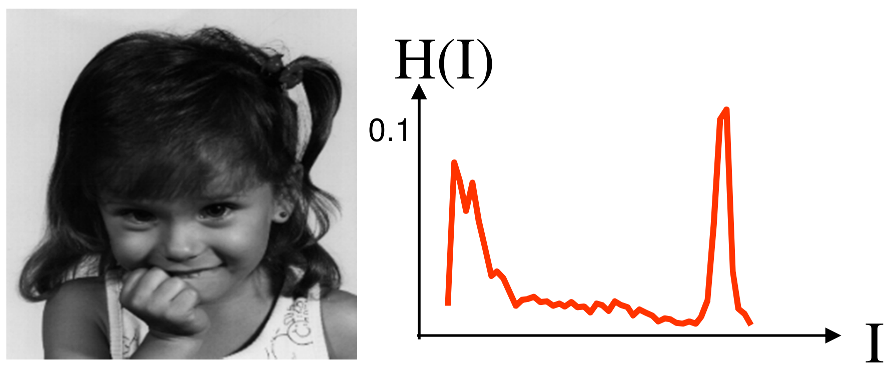


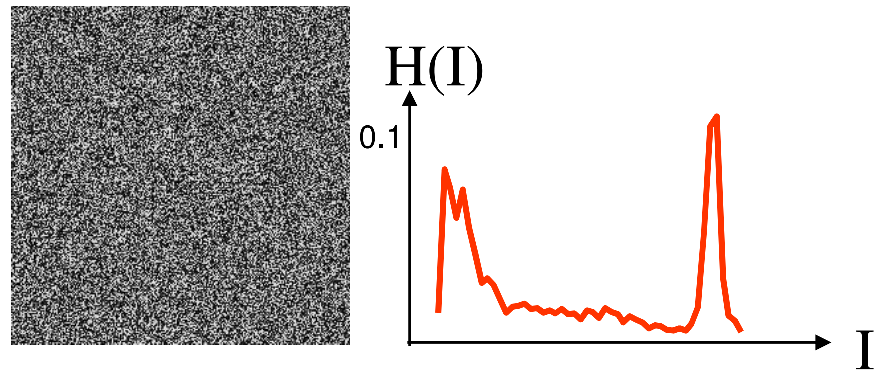
Accumulated Histogram

Examples





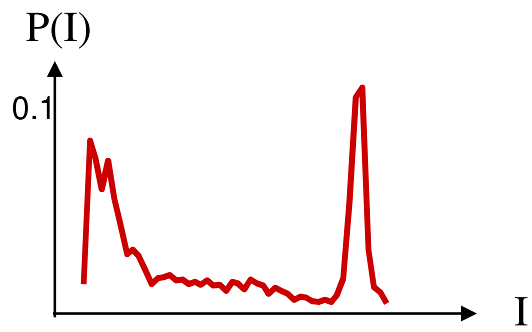




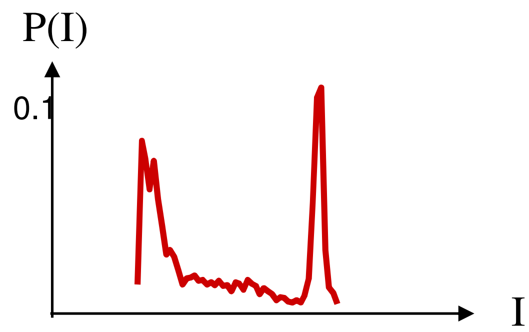
Pixel permutation of the left image

The image histogram does not fully represent the image

Original image



Decreasing contrast



Increasing average

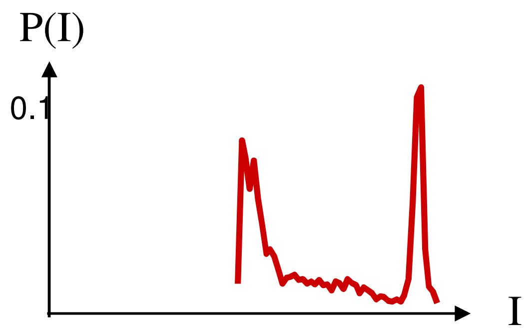
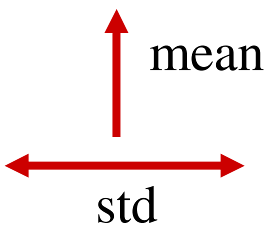


Image Statistics

- The image mean: $E\{I\} = \frac{1}{N}\sum_{i,j} I(i,j) = \frac{1}{N}\sum_{k} k\,H(k) = \sum_{k} k\,P(k)$

- The image s.t.d. : $\sigma(I) = \sqrt{E\left\{\left(I - E\{I\}\right)^2\right\}} = \sqrt{E\left(I^2\right) - E^2(I)}$

$$where \quad E\left\{I^2\right\} = \sum_{k} k^2 P(k)$$



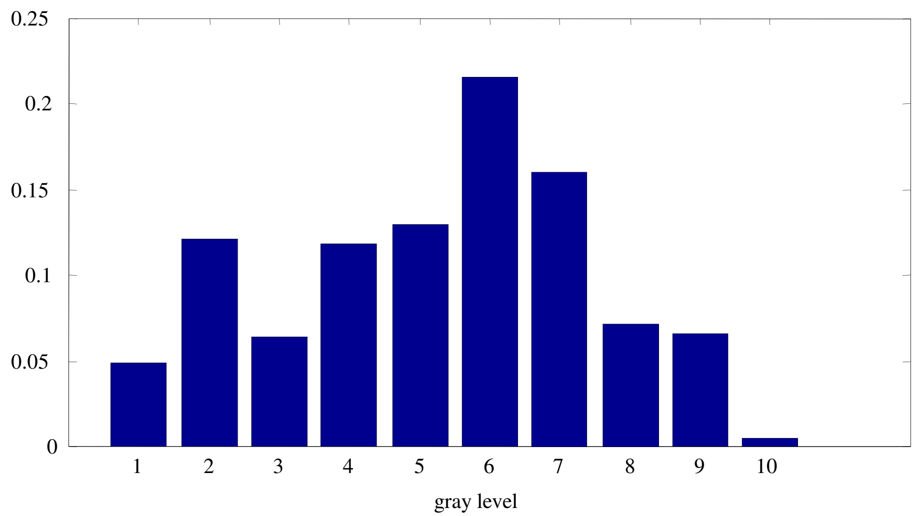
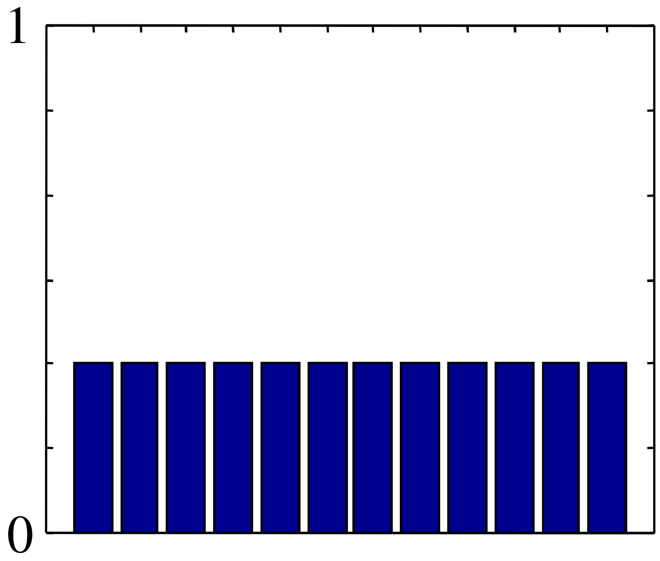


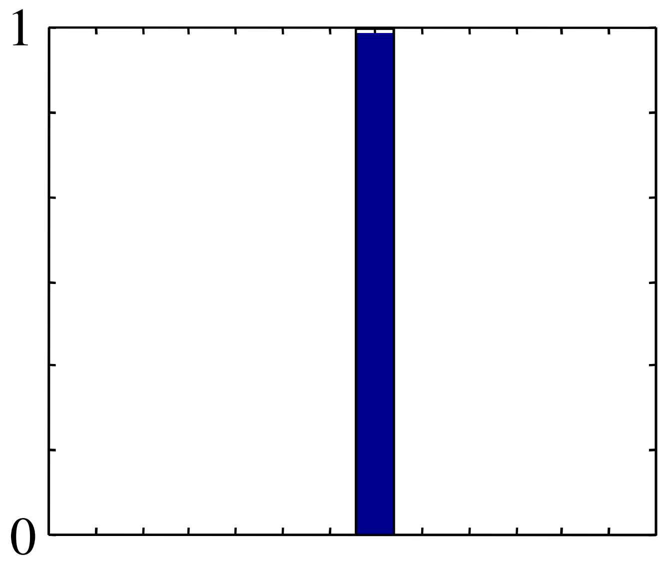
Image Entropy

$$Entropy(I) = -\sum_k P(k) \log P(k)$$

- The image entropy specifies the uncertainty in the image values.
- Measures the averaged amount of information required to encode the image values.

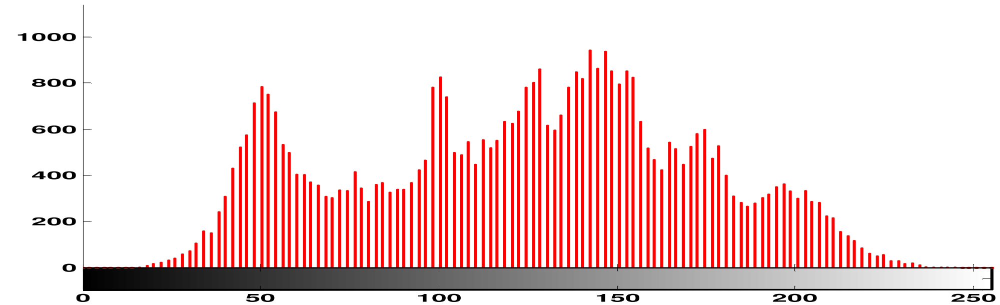


Max Entropy

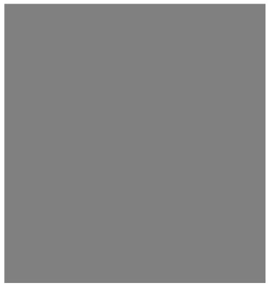


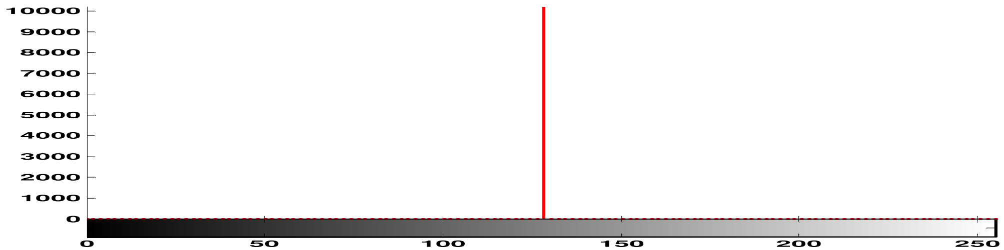
Min Entropy

- An infrequent event provides more information than a frequent event
- Entropy is a measure of histogram dispersion



entropy=7.4635



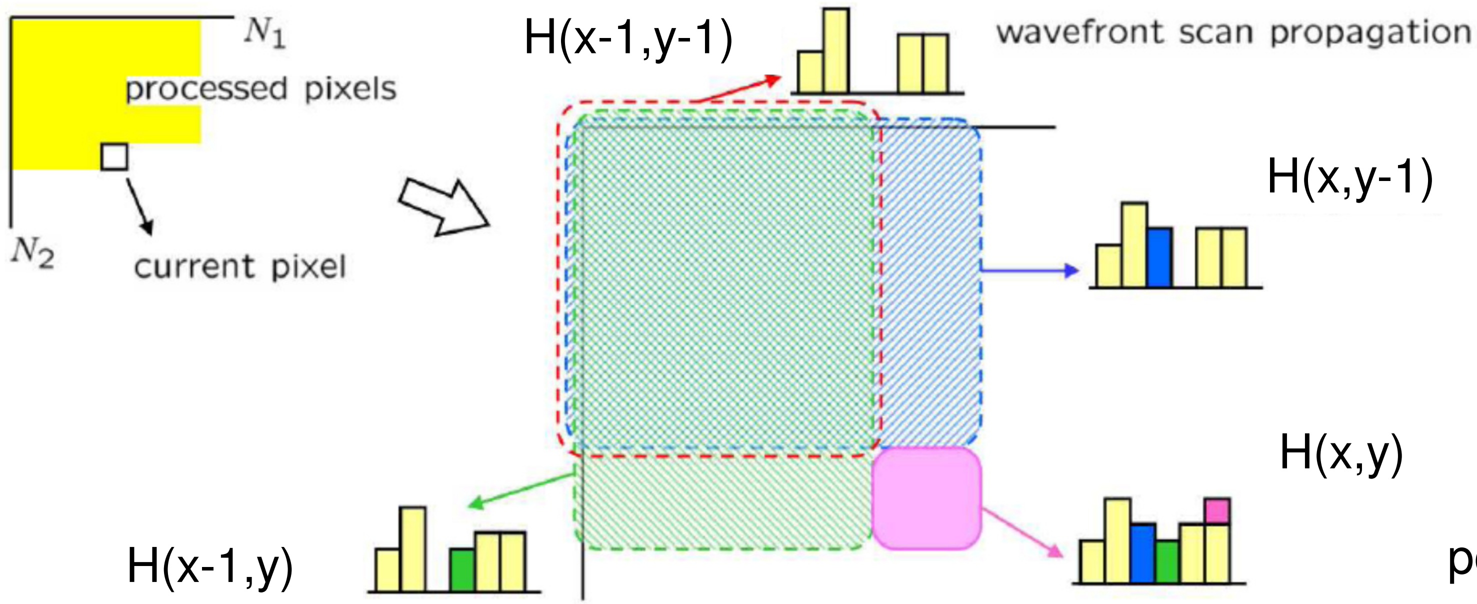


entropy=0

Adaptive Histogram

- In many cases histograms are needed for local areas in an image
- Examples:
  - Pattern detection
  - adaptive enhancement
  - adaptive thresholding
  - tracking

Implementation: Integral Histogram



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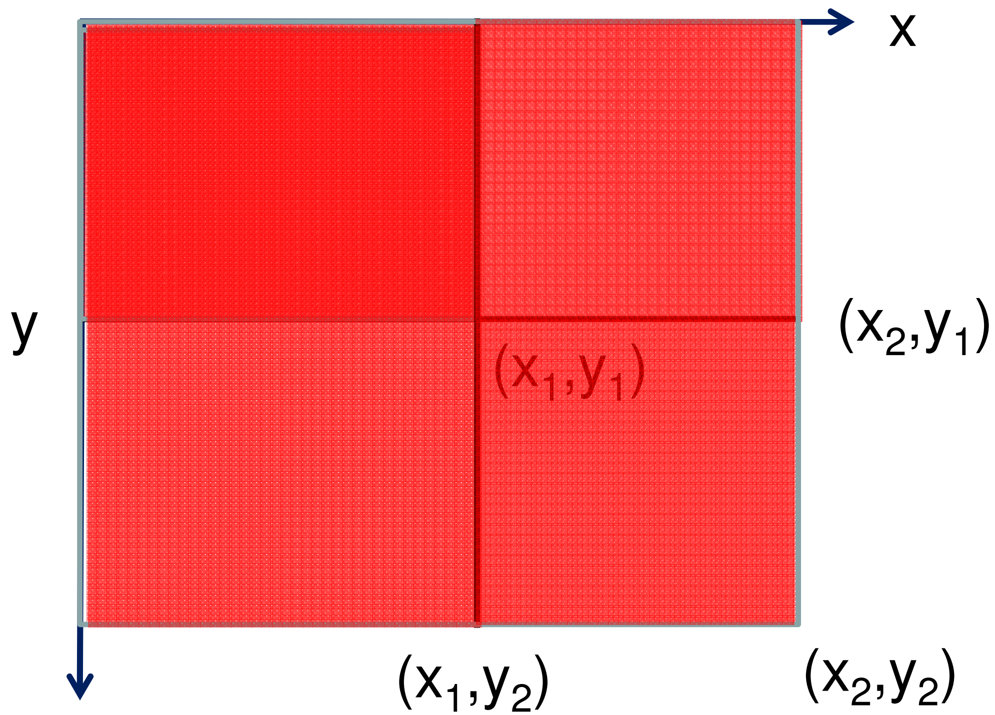
- **Integral histogram**: H(x,y) represent the histogram of a window whose right-bottom corner is (x,y)
- Construct by scan order:

$$H(x,y)= H(x,y-1)+H(x-1,y) - H(x-1,y-1)$$

- Using integral histogram we can calculate local histograms of any window $H(x_1:x_2, y_1:y_2)$



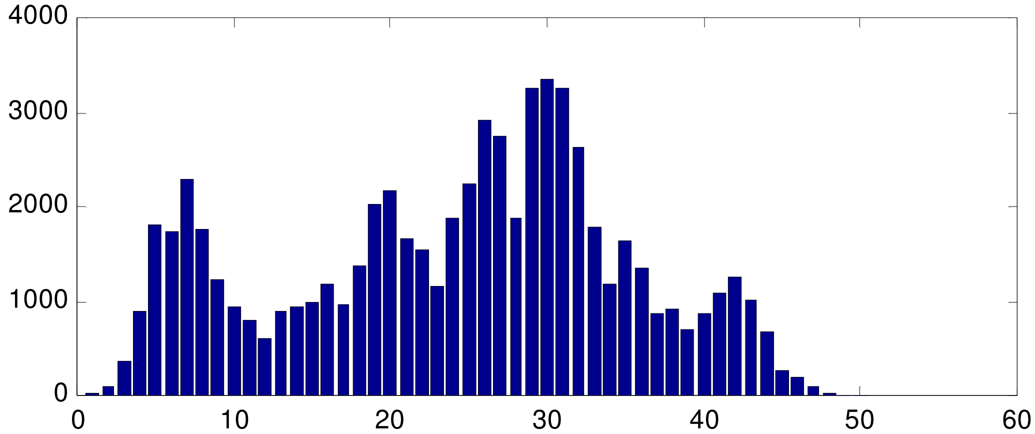
$$H(x_1:x_2, y_1:y_2) = H(x_2,y_2) + H(x_1,y_1) - H(x_1,y_2) - H(x_2,y_1)$$

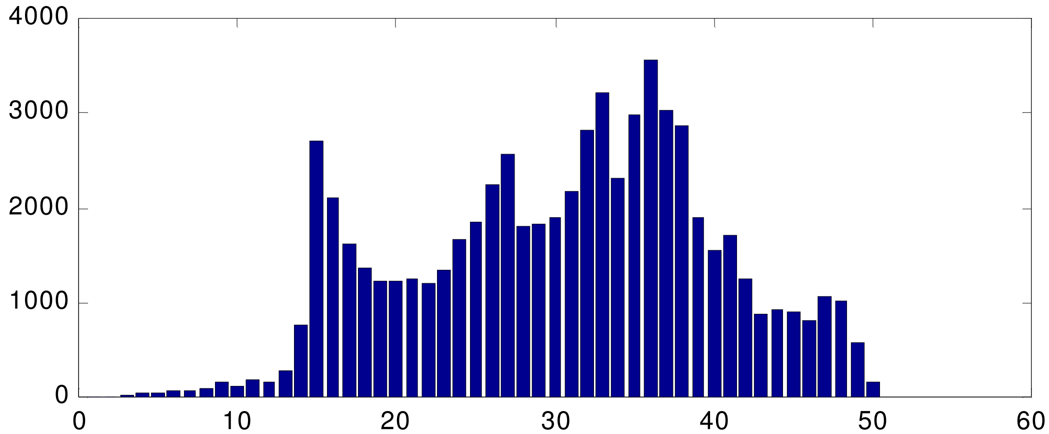
Histogram Usage

- Digitizing parameters
- Measuring image properties:
  - Average
  - Variance
  - Entropy
  - Contrast
  - Area (for a given gray-level range)
- Threshold selection
- Image distance
- Image Enhancement
  - Histogram equalization
  - Histogram stretching
  - Histogram matching

Example: Auto-Focus

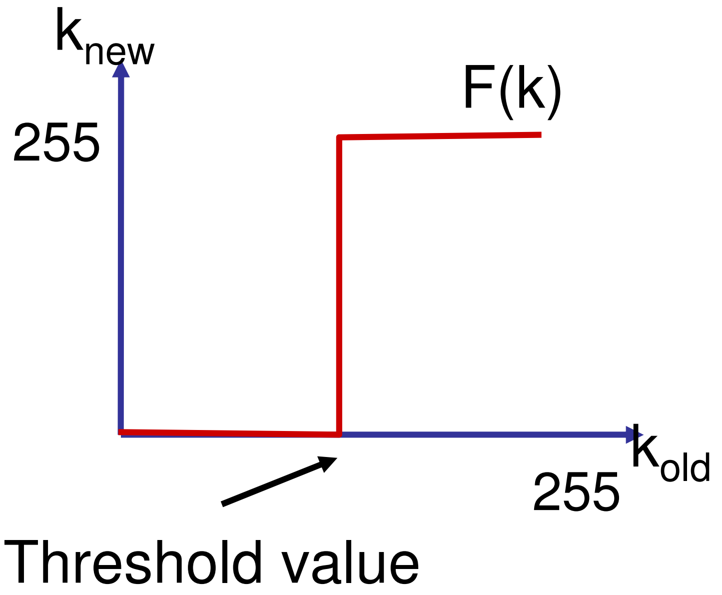
- In some optical equipment (e.g. slide projectors) inappropriate lens position creates a blurred (“out-of-focus”) image
- We would like to automatically adjust the lens
- How can we measure the amount of blurring?

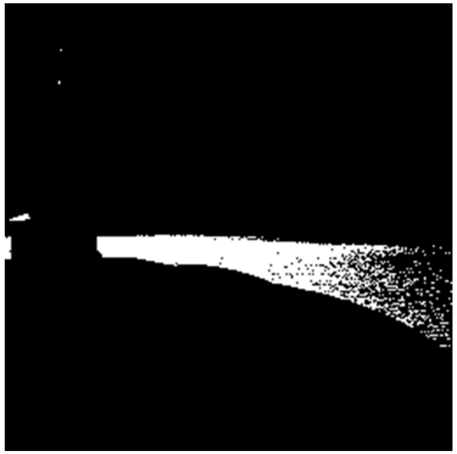




- Image mean is not affected by blurring
- Image s.t.d. (entropy) is decreased by blurring
- Algorithm: Adjust lens according the changes in the histogram s.t.d.

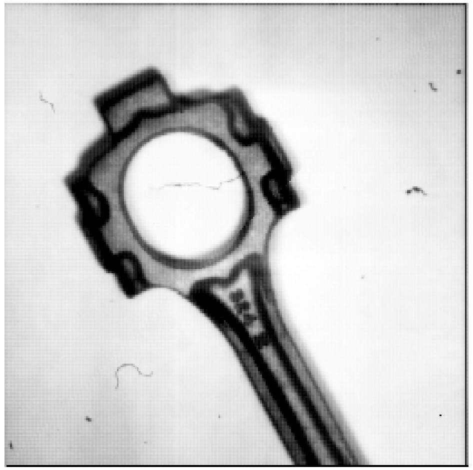
Thresholding



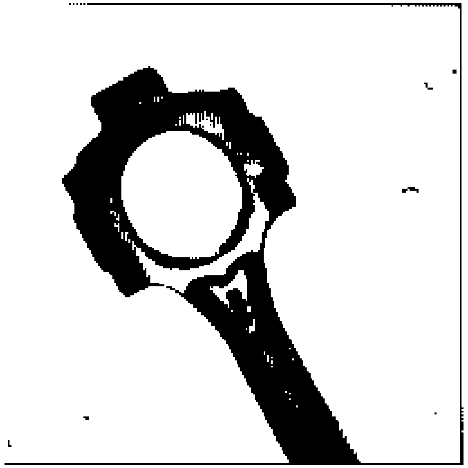


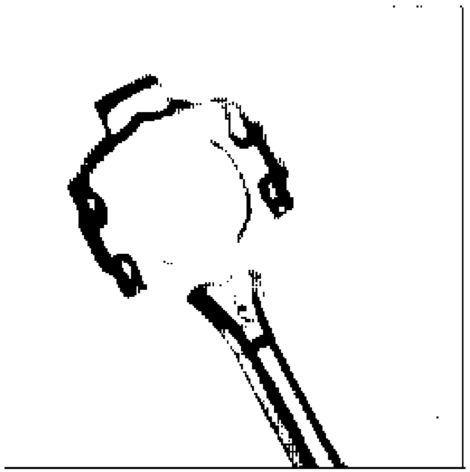
Threshold Selection

Original Image

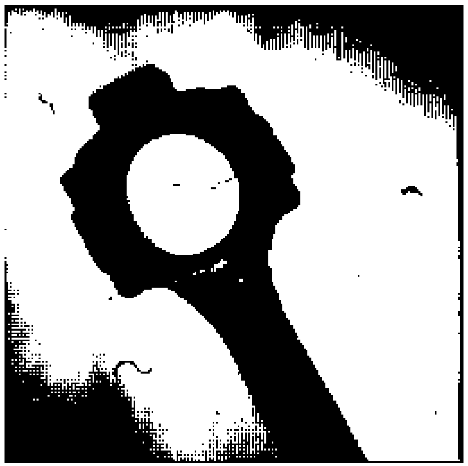


Binary Image





Threshold too low

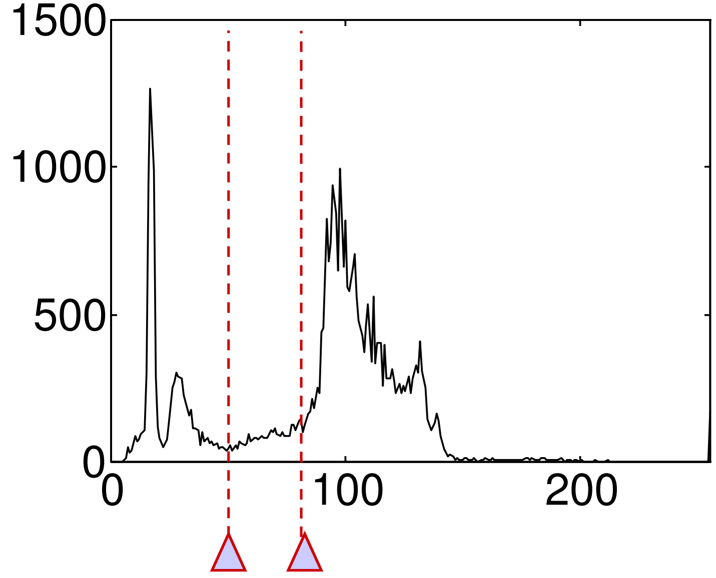


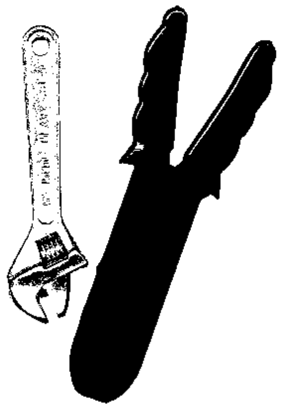
Threshold too high

Segmentation using Thresholding

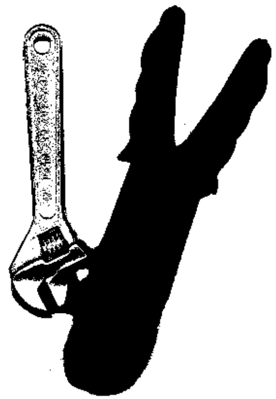
Original

Histogram





Threshold = 50

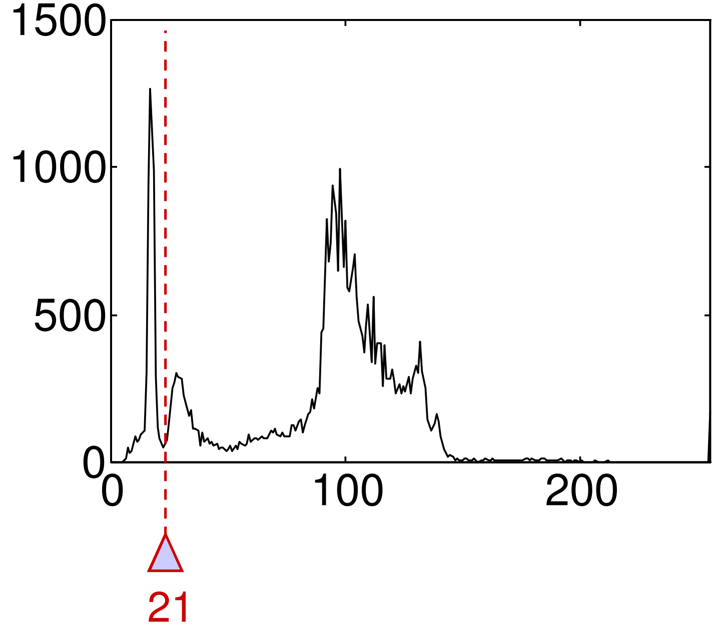


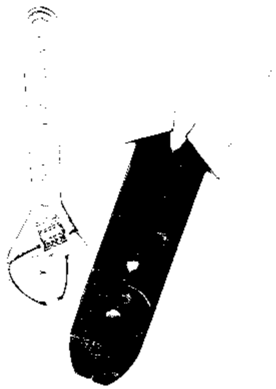
Threshold = 75

Segmentation using Thresholding

Original

Histogram

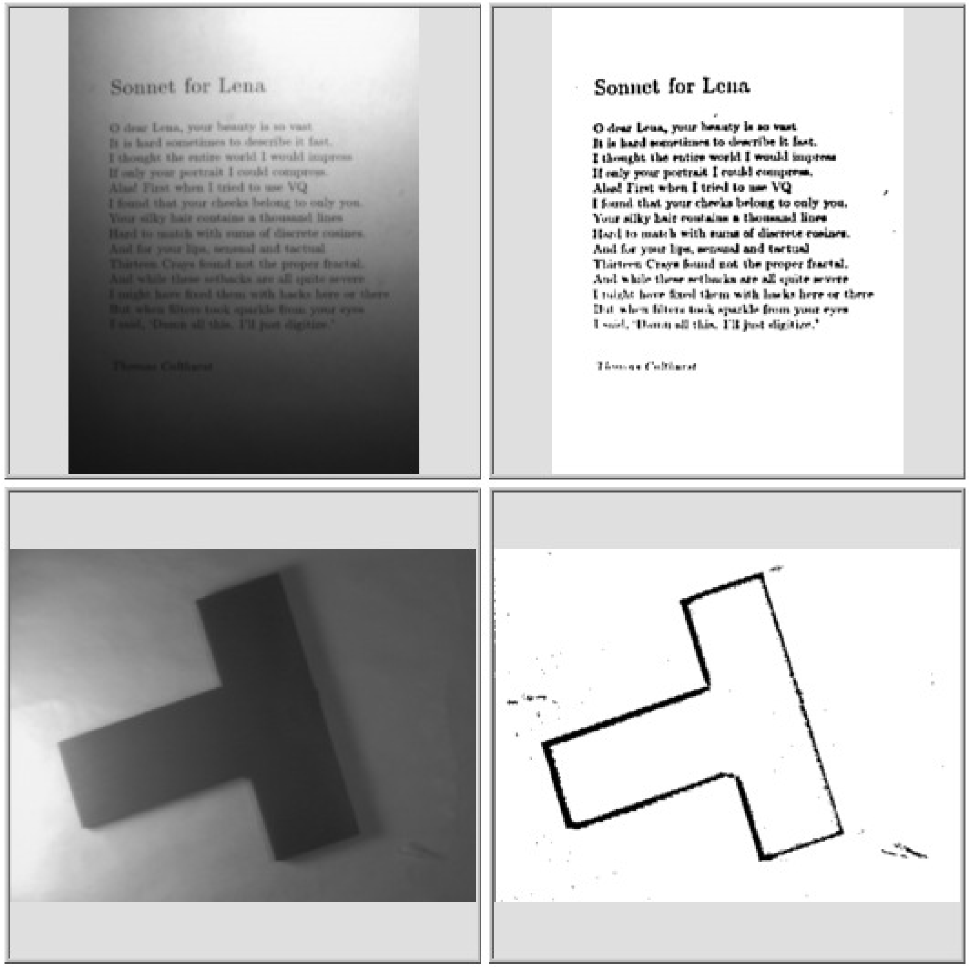




Threshold = 21

Adaptive Thresholding

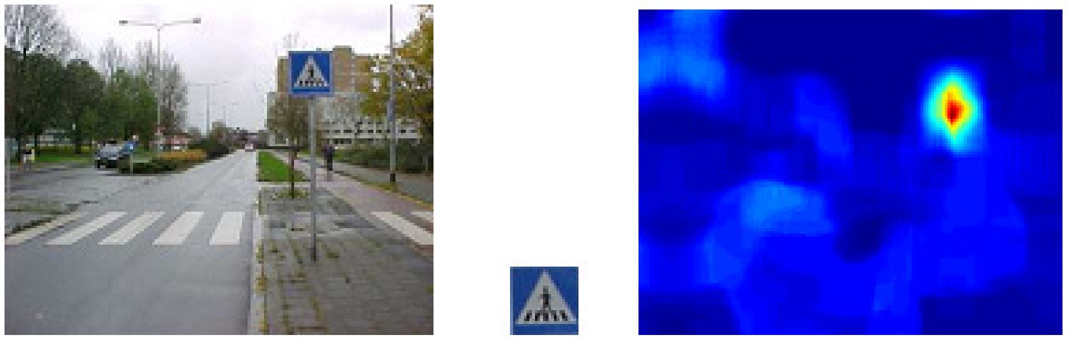
- Thresholding is space variant.
- How can we choose the local threshold values?



Histogram based image distance

- **Problem**: Given two images A and B whose (normalized) histogram are P_A and P_B define the distance D(A,B) between the images.

- Example Usage:
  - Tracking
  - Image retrieval
  - Registration
  - Detection
  - Many more ...



input *target* *similarity*

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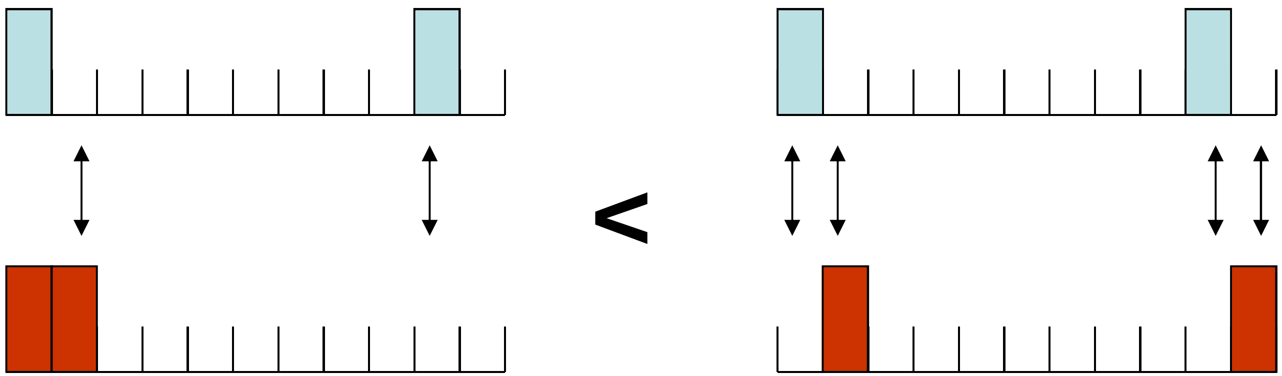
Option 1: Minkowski Distance

$$D_p(A,B)=\left[\sum_k \left|P_A(k)-P_B(k)\right|^p\right]^{1/p}$$

- **Problem**: distance may not reflect the perceived dissimilarity:



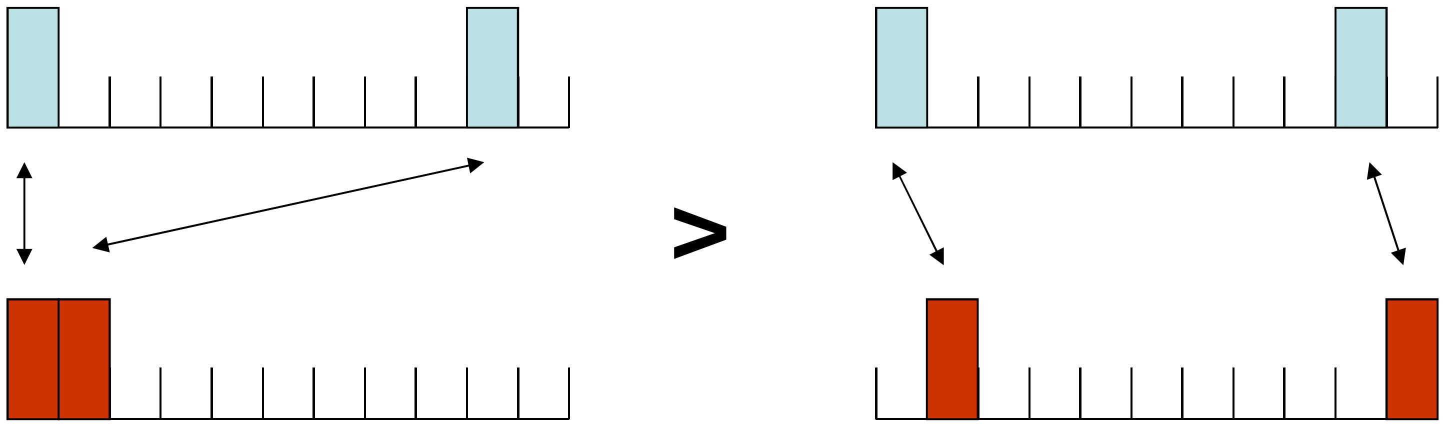
Option 2: Kullback-Leibler (KL) Distance

$$D_{KL}(A \parallel B) = \sum_k P_A(k) \log \frac{P_A(k)}{P_B(k)}$$

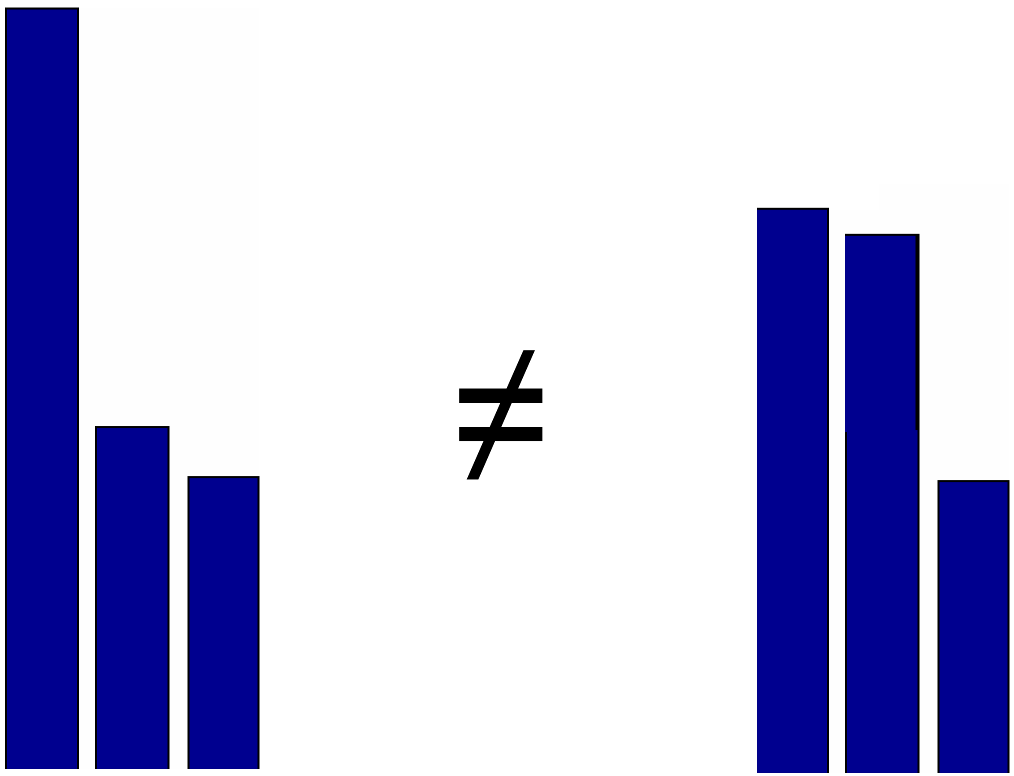
- Measures the amount of added information needed to encode image A based on the histogram of image B.
- Non-symmetric: $D_{KL}(A||B) \neq D_{KL}(B||A)$
- Suffers from the same drawback of the Minkowski distance.

Option 3: The Earth Mover Distance (EMD)

- Suggested by Rubner & Tomasi 98
- Defines as the minimum amount of "work" needed to transform histogram H_A towards H_B
- The term d_{ij} defines the "ground distance" between gray-levels i and j.
- The term $F=\{f_{ij}\}$ is an admissible flow from $H_A(i)$ to $H_B(j)$

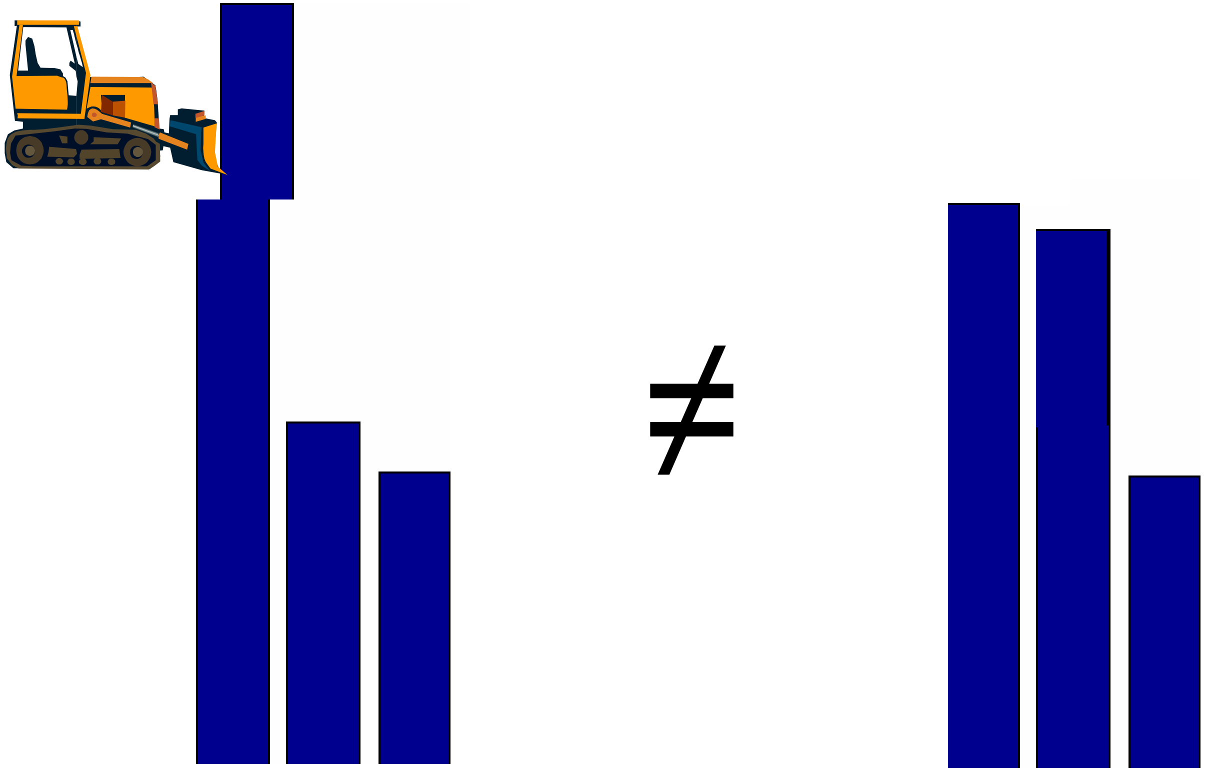


Option 3: The Earth Mover Distance (EMD)



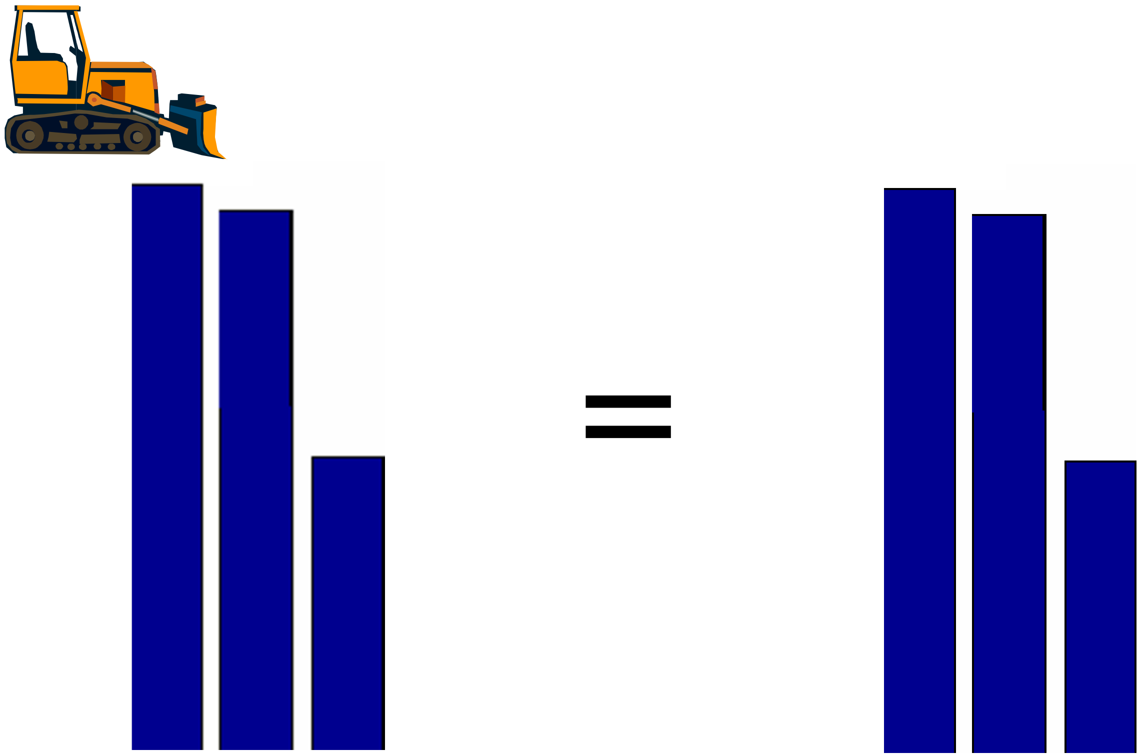
From: Pete Barnum

Option 3: The Earth Mover Distance (EMD)



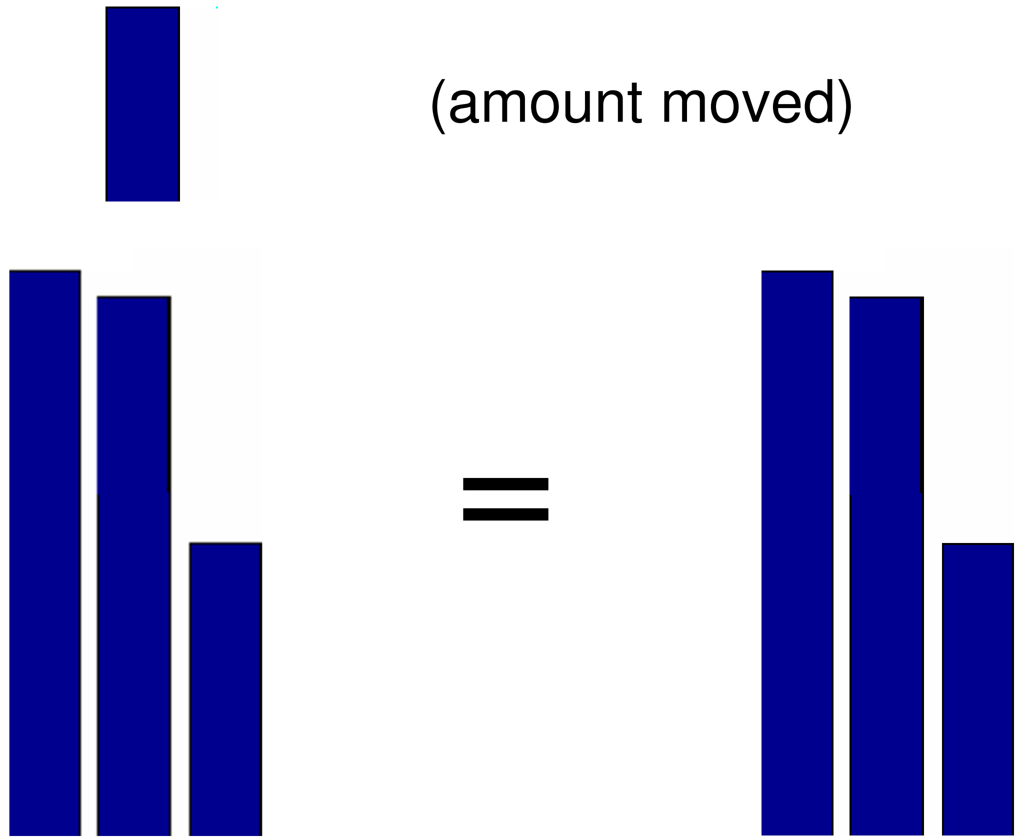
From: Pete Barnum

Option 3: The Earth Mover Distance (EMD)



From: Pete Barnum

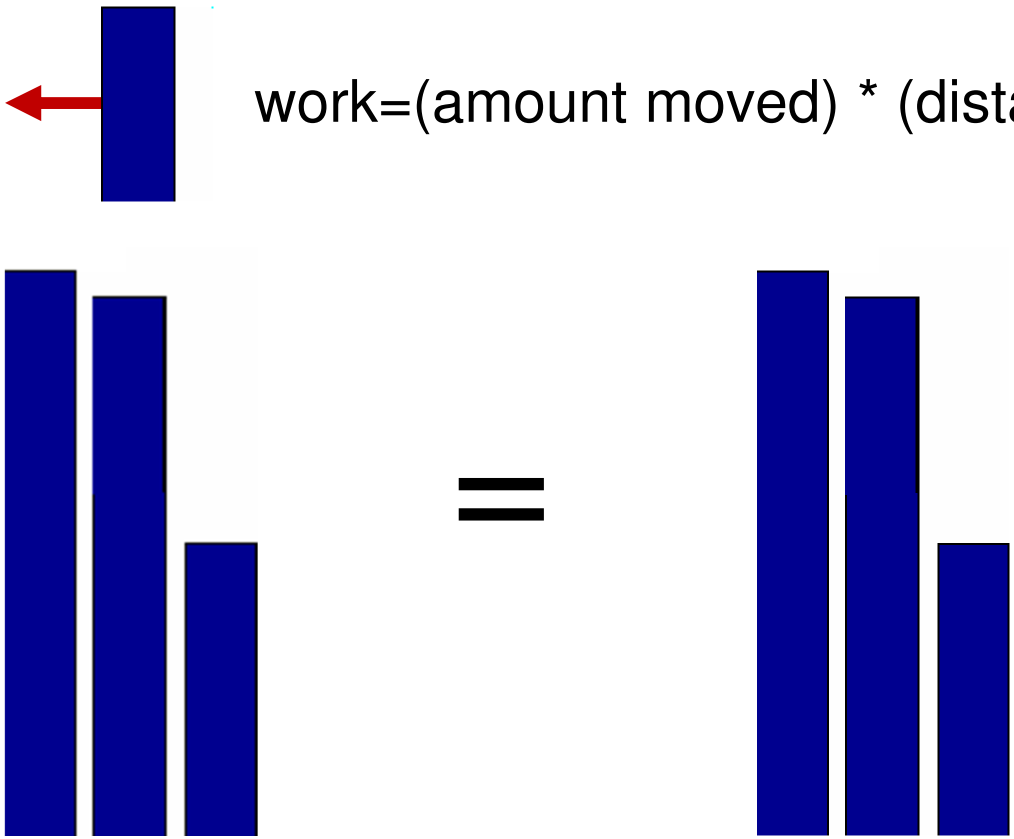
Option 3: The Earth Mover Distance (EMD)



From: Pete Barnum

Option 3: The Earth Mover Distance (EMD)

work=(amount moved) * (distance moved)



From: Pete Barnum

Option 3: The Earth Mover Distance (EMD)

$$D_{EMD}(A,B) = \min_F \sum_i \sum_j f_{ij} \cdot d_{ij}$$

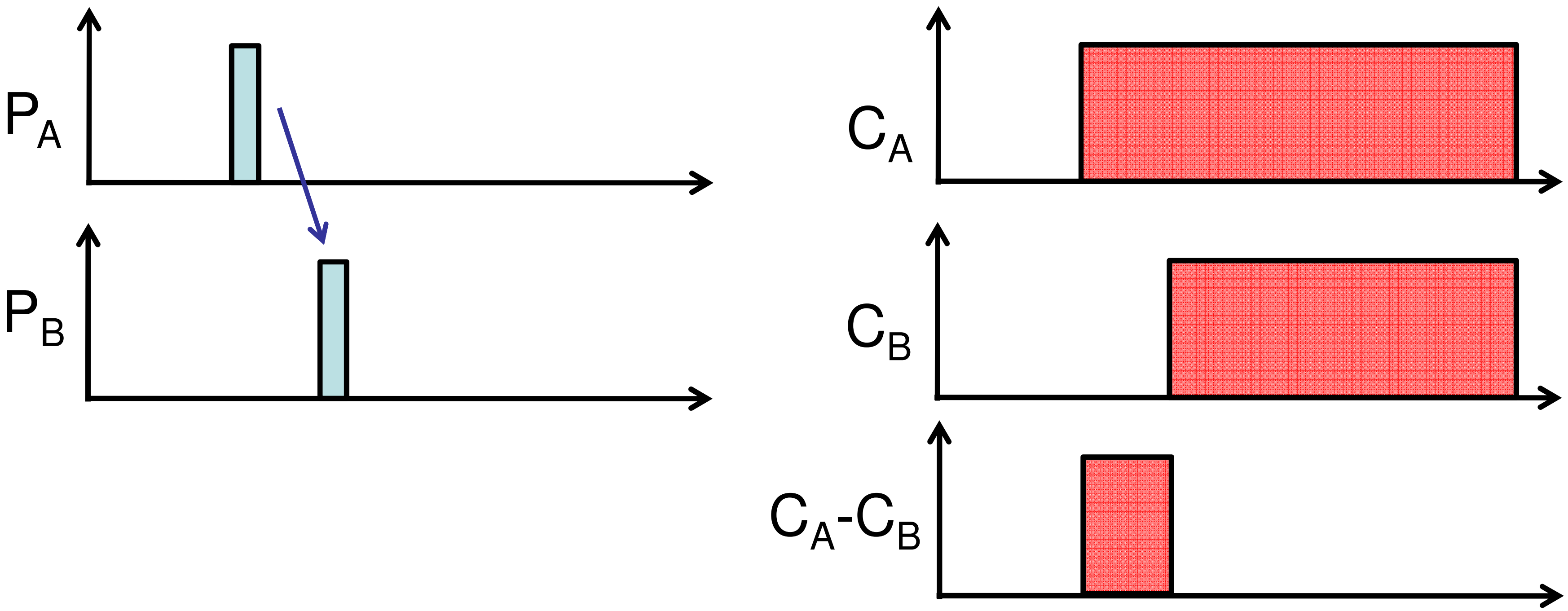
$$s.t. \quad f_{ij} \geq 0 \; ; \quad P_B(k) = \sum_i f_{ik} \quad ; \quad P_A(k) \geq \sum_i f_{ki}$$

- Constraints:
  - Move earth only from A to B
  - After move P_A will be equal to P_B
  - Cannot send more "earth" than there is
- Can be solved using Linear Programming
- Can be applied in high dim. histograms (color).

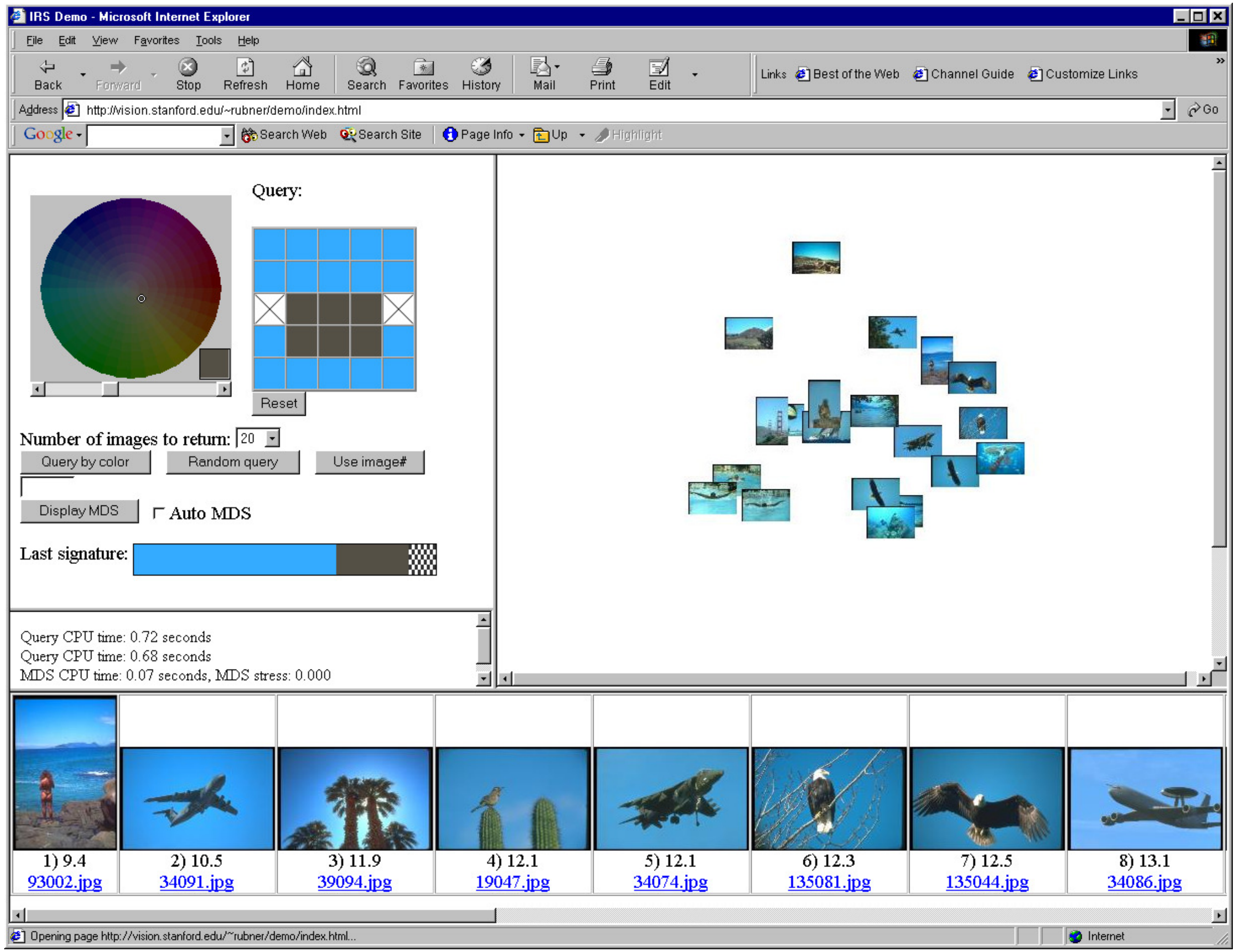
Special case: EMD in 1D

- Define C_A and C_B as the accumulated histograms of image A and B respectively:

$$D_{EMD}(A,B)=\sum_k \left|C_A(k)-C_B(k)\right|$$



Special case: EMD in 3D



Color Based Image Retrieval

Rubner & Tomasi 98