

Computational tools for logic-based grammar formalisms

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Abstract

A well-known slogan in language technology is ‘parsing-as-deduction’: syntax and meaning analysis of a text takes the form of a mathematical proof/derivation. Developers of language technology (and students of computational linguistics) want to visualize these mathematical objects in a variety of formats.

We discuss a language engineering environment for two ‘logic-based’ frameworks: type-logical grammar and ‘derivational’ minimalism. The kernel is a general theorem prover for the relevant framework, implemented in the logic-programming language Prolog. The kernel produces ‘proof objects’ for its internal computations. The front-end displays these proof objects in a number of user-defined formats. Local interaction with the kernel is via a tcl/tk GUI. Alternatively, one can call the kernel remotely from dynamic PDF documents, using the form features of Sebastian Rahtz’ `hyperref` package.

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1. The project

- ▶ Aim: teaching materials/courseware for logic-based NLP
- ▶ Frameworks: type-logical grammar, ‘derivational’ minimalism
- ▶ Architecture:
 - ▷ Kernel: logic programming implementation of the general algorithmic proof theory for TLG/MG
 - ▷ Interaction with the kernel:
 - ★ tcl/tk GUI
 - ★ dynamic PDF documents
- ▶ Inspiration: the fold/unfold concept of the Mathematica notebook. Discourse interleaved with mathematical expressions that can be evaluated, visualized on demand.

2. The team

Michael Moortgat Coordinating the computational linguistics curriculum at Utrecht University. Research interests: math and language, type-logical grammar.

Richard Moot Author of the type-logical grammar development environment GRAIL. PhD thesis on proof nets for NLP.

Dick Oehrle Formerly: Linguistics, U Arizona. Then: YY. Now: ...

Willemijn Vermaat Currently: PhD project on the logical perspective on minimalist grammars. Before: IT in the computational linguistics curriculum.

3. Current uses of the material

- ▶ Graduate level courses
 - ▷ various ESSLLI installments
 - ▷ European Master School on Language and Speech (Leuven, 2002).
- ▶ Undergraduate level courses
 - ▷ CKI (CogScience/AI) Utrecht University
 - ▷ Linguistics, Utrecht University
- ▶ Secondary school
 - ▷ β workshops, Utrecht University
 - ▷ Adriatic coast (Raffaella Bernardi)
- ▶ ...

4. Type-logical grammar

Background reading for the technical set-up:

Moortgat, Moot & Oehle (2001) T_EX in Teaching. Proceedings EuroT_EX 2001 Conference, Kerkrade.

General project info and course pages:

<http://grail.let.uu.nl/tour.pdf>

<http://www.let.uu.nl/~ctl/docenten/moortgat.html>

5. Type-logical grammar

Think of type-logical grammar as a functional **programming language** (cf Haskell), customized for NLP (analysis, generation).

Functional programming

- ▶ basic types: integers, booleans, ...
- ▶ functional types $T \rightarrow T'$
- ▶ using/constructing types $T \rightarrow T'$: application/abstraction

Curry-Howard-de Bruyn Perspective shift logic/computation.

- ▶ functional types/implicational formulas,
- ▶ type computations/logical derivations.

6. Illustration: square

A simple example: constructing a **square** function out of a built-in **times** function:

$$\frac{\frac{\text{times} : \text{Int} \rightarrow (\text{Int} \rightarrow \text{Int}) \quad x : \text{Int}}{(\text{times } x) : \text{Int} \rightarrow \text{Int}} \quad (Elim \rightarrow) \quad x : \text{Int}}{(\text{times } x \ x) : \text{Int}} \quad (Elim \rightarrow)}{\lambda x. (\text{times } x \ x) : \text{Int} \rightarrow \text{Int}} \quad (Intro \rightarrow)$$

- ▶ $(Elim \rightarrow)$: use of a function, application
- ▶ $(Intro \rightarrow)$: construction of a function, abstraction

7. The logic of grammar

Let us write $A \bullet B$ for the combination of an expression A with an expression B . We obtain a grammar logic by dropping all ‘structural rules’ for the product:

- ▶ Resource sensitivity: no duplication/waste of material
- ▶ Structure sensitivity: linear order/grouping
 - ▷ Drop **Commutativity**: $A \bullet B = B \bullet A$
Would imply that linear order doesn’t affect well-formedness
But: compare *man bites dog* and *dog bites man*.
 - ▷ Drop **Associativity**: $(A \bullet B) \bullet C = A \bullet (B \bullet C)$
Destroys structural information:
I had completely forgotten how good beer tastes.

8. Parsing as deduction

Two implications in the absence of Commutativity!

Algebra: residuation laws

$$A \rightarrow C/B \quad \text{iff} \quad A \bullet B \rightarrow C \quad \text{iff} \quad B \rightarrow A \setminus C$$

Logic: inference rules (elimination/introduction)

$$\frac{X \vdash B \quad Y \vdash B \setminus A}{X \circ Y \vdash A} \setminus E \qquad \frac{X \vdash A/B \quad Y \vdash B}{X \circ Y \vdash A} /E$$

$$\frac{B \circ X \vdash A}{X \vdash B \setminus A} \setminus I \qquad \frac{X \circ B \vdash A}{X \vdash A/B} /I$$

9. The structural module

To capture **variation**, structural rules can be reintroduced in a controlled form. Control operations \diamond, \square in addition to the composition operations $/, \bullet, \backslash$.

► **Logical rules**

$$\diamond A \rightarrow B \quad \text{iff} \quad A \rightarrow \square B$$

► **Structural rules:** under \diamond control. For example:

$$\begin{aligned} \diamond A \bullet B &\rightarrow B \bullet \diamond A \\ (A \bullet B) \bullet \diamond C &\rightarrow A \bullet (B \bullet \diamond C) \end{aligned}$$

10. The GRAIL system

Richard Moot's unix-based **GRAIL** system offers a general development environment for type-logical grammars. Software components:

- ▶ SICStus Prolog: the programming language for the kernel;
- ▶ Tcl/Tk for the graphical user interface;
- ▶ a standard teTeX environment for the visualization/export of derivations.

The system is available under the GNU General Public License agreement from

<ftp.let.uu.nl/pub/users/moot>

11. A session

The user designs a grammar fragment, using the following tools:

- ▶ **Lexicon tool:** graphical editor to assign formulas (and meaning programs) to words in the lexicon or edit lexical entries,
- ▶ **Postulate tool:** graphical editor to add or modify structural rewrite rules,
- ▶ **Parsing/debugging:** run the theorem prover on sample expressions; interactive mode using proof net technology.

12. Export formats

User-defined L^AT_EX output formats.

Prawitz style Derivations in tree format, using Tatsuta's **proof.sty** package.

$$\frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{surpassed}}{(np \setminus s) / np} \quad \frac{\text{himself}}{((np \setminus s) / np) \setminus (np \setminus s)}}{\text{surpassed} \circ \text{himself} \vdash np \setminus s} [\setminus E]}{\text{knuth} \circ (\text{surpassed} \circ \text{himself}) \vdash s} [\setminus E]$$

Fitch style Linear format, handy when meaning assembly is included.

- | | | |
|----|---|----------------------|
| 1. | knuth : $np - \text{knuth}$ | <i>Lex</i> |
| 2. | surpassed : $(np \setminus s) / np - \text{surpass}$ | <i>Lex</i> |
| 3. | himself : $((np \setminus s) / np) \setminus (np \setminus s) - \lambda z_2. \lambda x_3. ((z_2 \ x_3) \ x_3)$ | <i>Lex</i> |
| 4. | surpassed \circ himself : $np \setminus s - \lambda x_3. ((\text{surpass} \ x_3) \ x_3)$ | $\setminus E$ (2, 3) |
| 5. | knuth \circ (surpassed \circ himself) : $s - ((\text{surpass} \ \text{knuth}) \ \text{knuth})$ | $\setminus E$ (1, 4) |

13. Dynamic derivations

The core notion of ‘proof’ is inherently dynamic:

‘a sequence of inference steps, leading from axioms to the desired conclusion’

↪ dynamic display format

Tools for the implementation (thanks to Bernhard Fisseni):

- ▶ an expanded version of `\infer` from [proof.sty](#), taking advantage of
- ▶ the `\stepwise` family of commands from Lehmke’s [texpower.sty](#) package
- ▶ the kernel computes the sequencing order from the internal proof object, with bottom-up or top-down options

14. Dynamic derivations: bottom up

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Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$\frac{\text{the}}{np/n}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

book
 $\frac{\text{the}}{np/n}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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the book
 np/n

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

that

$\frac{\text{the}}{np/n}$ $\frac{\text{book}}{n}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$\frac{\text{the}}{np/n}$ $\frac{\text{book}}{n}$ $\frac{\text{that}}{\quad}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

knuth

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

knuth

$\frac{\text{the}}{np/n}$ $\frac{\text{book}}{n}$ $\frac{\text{that}}{(n \setminus n)/(s/np)}$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\text{knuth}}{np}$$
$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

wrote

$\frac{\text{knuth}}{np}$

$\frac{\text{the}}{np/n}$ $\frac{\text{book}}{n}$ $\frac{\text{that}}{(n \setminus n)/(s/np)}$

Meaning. $\iota(\lambda y_3. (\text{write}(\text{knuth}, y_3) \wedge \text{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that}} \quad \frac{\text{knuth}}{np}}{\text{wrote}}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\text{knuth}}{np} \quad \frac{\text{wrote}}{(np \setminus s)/np}$$

$$\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$$\frac{\text{knuth}}{np} \quad \frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1$$

$$\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$$\frac{\text{knuth}}{np} \quad \frac{\text{wrote}}{(np \setminus s) / np} \quad \frac{[p_1 \vdash np]^1}{[/ E]}$$

$$\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/ E]$$

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{}$$

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$$\frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s) / np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/ E]}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [\setminus E]$$

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)}}{\text{the} \text{ book that}} \quad [/ E]$$

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$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)}}{\text{the} \circ (\text{book} \circ \text{that}) \vdash s}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s) / np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} [\setminus E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [P2]}{[\setminus E]} [E]$$

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)}}{}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that}} \quad \frac{(n \setminus n)/(s/np)}{\text{knuth} \quad \frac{\frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/\!E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} \quad [\! \setminus E]}{\text{(knuth} \circ \text{wrote)} \circ p_1 \vdash s} \quad [P2]}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that}} \quad \frac{(n \setminus n) / (s / np)}{\text{knuth} \quad \frac{\frac{\text{wrote}}{(np \setminus s) / np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/\!E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} \quad [/\!E]}{\frac{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} \quad [P2]} \quad [/\!I]^1$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} [/\!E]}{\frac{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s}[(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s] [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/\!I]^1} [/\!E]$$

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$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad \frac{\frac{\frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/\!E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} \quad [/\!E]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} \quad [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} \quad [/\!I]^1}{\text{knuth} \circ \text{wrote} \vdash s/np} \quad [/\!E]^1$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E] \quad \frac{\frac{\frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s)/np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} [/\!E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/\!I]^1} [/\!E]}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

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$$\frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{\text{book}}}{n}}{\text{that}}}{(n \setminus n) / (s / np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{\text{wrote}}}{(np \setminus s) / np}}{[p_1 \vdash np]^1}}{\text{wrote} \circ p_1 \vdash np \setminus s}}{[\setminus E]}}{[\setminus E]}}{[\setminus E]}}{[\setminus E]}}{[\setminus E]}}{\frac{\frac{\frac{\frac{\frac{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s}{[P2]}}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s}}{[I]^1}}{\text{knuth} \circ \text{wrote} \vdash s / np}}{[E]}}{[\setminus E]}}{[\setminus E]}}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [E]}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

14. Dynamic derivations: bottom up

$$\begin{array}{c}
 \frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)} \quad \frac{\frac{\text{knuth}}{np} \quad \frac{\frac{\text{wrote}}{(np \setminus s) / np} \quad [p_1 \vdash np]^1}{\text{wrote} \circ p_1 \vdash np \setminus s} [/ E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{\frac{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [\text{wrote} \circ p_1 \vdash np \setminus s]}{\text{knuth} \circ \text{wrote} \vdash s / np} [/ I]^1} [/ E]}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [/ E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/ E]}
 \end{array}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$



15. Dynamic derivations: top down

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15. Dynamic derivations: top down

$\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))} \vdash np \quad [/ E]$$

Meaning. $\iota(\lambda y_3. (\text{write}(\text{knuth}, y_3) \wedge \text{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{np/n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))} \vdash np \quad [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\overline{np/n}}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))} \vdash np \quad [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n}}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))} \vdash np \quad [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [\wedge E]}{[\wedge E]}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{n}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\backslash E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\overline{n}}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\backslash E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\backslash E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad \frac{[\backslash E]}{[\backslash E]} \quad \frac{[\backslash E]}{[\backslash E]}}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad \frac{[\backslash E]}{[\backslash E]}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{(n \setminus n)/(s/np)}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad [/\!E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad [\! \setminus E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad [/\!E]$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \overline{(n \setminus n) / (s / np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad [/ E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad [\setminus E]} \quad [/ E]$$
$$\frac{}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad [/ E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad [/ E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad [\setminus E]} \quad [/ E]$$

$$\frac{}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad [/ E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\frac{\text{that}}{(n \setminus n)/(s/np)} \quad \text{knuth} \circ \text{wrote} \vdash s/np}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [/\!E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\! \setminus E]} [/\!E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/\!E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad \frac{\frac{\text{that}}{(n \setminus n)/(s/np)} \quad \text{knuth} \circ \text{wrote} \vdash s/np}{[I]^1}}{[E]} \quad [E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\wedge E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\wedge E]} \quad \frac{\frac{\frac{\text{that}}{(n \setminus n)/(s/np)} \quad \frac{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s}[\wedge I]^1}{\text{knuth} \circ \text{wrote} \vdash s/np} [\wedge E]}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\wedge E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [\wedge E]$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad \frac{\frac{\text{that}}{(n \setminus n)/(s/np)} \quad \frac{\frac{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s}{\text{knuth} \circ \text{wrote} \vdash s/np} [P2]}{[I]^1}}{[E]} \quad [E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{n \setminus n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]} \quad \frac{\frac{\frac{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/I]^1}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n}}{n \setminus n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]} \quad \frac{\frac{\frac{\frac{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s}{} [\setminus E]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s / np} [/I]^1}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [/E]}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{\text{book}}{n}}{\text{that}}{(n \setminus n)/(s/np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{\text{knuth} \circ \text{wrote} \vdash s/np} [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/I]^1} [/E]}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{n}}{\text{book}}}{(n \setminus n) / (s / np)}{\text{that}}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]}
 \frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{s}}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s / np} [/I]^1}
 \text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash s / np} [/E]}
 \text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{n}}{\text{book}}}{(n \setminus n) / (s / np)}{\text{that}}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]}
 \frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{wrote \circ p_1 \vdash np \setminus s}}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [P2]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [I]^1}}{\text{knuth} \circ \text{wrote} \vdash s / np} [/E]}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}
 \frac{}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/E]$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\begin{array}{c}
 \frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)} \quad \frac{\frac{\text{knuth}}{np} \quad \frac{\text{wrote} \circ p_1 \vdash np \setminus s}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} \quad [/\!E]}{\frac{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s}{\text{knuth} \circ \text{wrote} \vdash s / np} \quad [/\!I]^1} \quad [/\!E]}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad [/\!E]} \\
 \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad [/\!E]
 \end{array}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\begin{array}{c}
 \frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n) / (s / np)} \quad \frac{\frac{\text{knuth}}{np} \quad \frac{(np \setminus s) / np}{\text{wrote} \circ p_1 \vdash np \setminus s} \quad [/ E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} \quad [\setminus E]}{\frac{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s}[\text{P2}]}{\text{knuth} \circ \text{wrote} \vdash s / np} \quad [/ I]^1} \quad [/ E]}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} \quad [\setminus E]}{\frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} \quad [/ E]}
 \end{array}$$

Meaning. $\iota(\lambda y_3. (\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{n}}{\text{book}}}{\text{that}}}{(n \setminus n) / (s / np)}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]} \\
 \frac{\frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{(np \setminus s) / np}}{\text{wrote} \circ p_1 \vdash np \setminus s} [/ E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{(\text{knuth} \circ \text{wrote}) \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s / np} [/ I]^1}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [/ E]} \\
 \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np}{} [/ E]
 \end{array}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{n}}{\text{book}}}{(n \setminus n)/(s/np)}{\text{that}}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]} \\
 \frac{\frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{(np \setminus s)/np}}{\text{wrote}}}{\text{wrote} \circ p_1 \vdash np \setminus s} [/ E]}{\text{knuth} \circ (\text{wrote} \circ p_1) \vdash s} [\setminus E]}{\text{(knuth} \circ \text{wrote)} \circ p_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/ I]^1} \\
 \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} [/ E]
 \end{array}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

15. Dynamic derivations: top down

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\frac{\frac{\text{the}}{np/n}}{n}}{\text{book}}}{(n \setminus n)/(s/np)}{\text{that}}}{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n} [\setminus E]}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} [\setminus E]} \\
 \frac{\frac{\frac{\frac{\frac{\frac{\frac{\text{knuth}}{np}}{(np \setminus s)/np} [\text{p}_1 \vdash np]^1}{\text{wrote} \circ \text{p}_1 \vdash np \setminus s} [\setminus E]}{\text{knuth} \circ (\text{wrote} \circ \text{p}_1) \vdash s} [\setminus E]}{(\text{knuth} \circ \text{wrote}) \circ \text{p}_1 \vdash s} [P2]}{\text{knuth} \circ \text{wrote} \vdash s/np} [/I]^1} \\
 \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np} {} [/E]
 \end{array}$$

Meaning. $\iota(\lambda y_3.(\mathbf{write}(\mathbf{knuth}, y_3) \wedge \mathbf{book}(y_3)))$

17. Web interaction with the kernel

To realize web interaction with the kernel, we move through the following stages.

- ▶ Command line interaction (Prolog)
- ▶ Shell interaction (Unix)
- ▶ Web interaction (Cgi)



18. Prolog command line interaction

Consulting a fragment, parsing test sentences, producing L^AT_EX output `eg.tex` to be processed by a wrapper file `proofs.tex`

```
SICStus 3.8.5 (sparc-solaris-5.7): Fri Oct 27 10:12:22 MET DST 2000
Licensed to let.uu.nl
| ?- consult('notcl2000.pl'). % the kernel without tcl/tk GUI
{consulting notcl2000.pl...}
yes
| ?- consult('knuth.pl').
{consulting knuth.pl...}
{consulted knuth.pl in module user, 20 msec 6952 bytes}
yes
| ?- tex([knuth,surpassed,himself],s).
===
Lookup: 0, Max # links: 12
===
Telling LaTeX output directory eg.tex
1 solution found. CPU Time used: 0.200 ... .. latex ready
```

19. From shell interaction to web interaction

Unix \rightsquigarrow shell interaction. The SICStus `save_program` predicate saves a state of the run of the program that can be resumed with the `-r` flag. In addition, arguments can be passed from the unix command line using the `-a` flag.

```
% sicstus -r wwwgrailstate
-a knuth yes yes yes inactive nd s knuth surpassed himself
{restoring wwwgrailstate...}
{wwwgrailstate restored in 80 msec 513808 bytes}
{consulting knuth.pl...}
```

Cgi \rightsquigarrow web interaction. The `sicstus -r ... -a ...` call is realized via a cgi program, using the html or pdf form facilities.

20. Using Rahtz' hyperref package

We can use the `\href` command of the `hyperref` package to call a cgi script:

```
...  
\hyperbaseurl{http://grail.let.uu.nl/cgi-bin/grail/  
\newcommand{\parsescript}[4]  
{\href{wwwgrail.cgi?  
frag=#1&struct=yes&sem=no&lexsem=yes&unary=inactive&mode=nd&goal=#2&test=#3}{#4}}  
...  

```

The parameters for `\parsescript` are a fragment name (#1), a goal formula (#2), and the test expression, in cgi (#3) and print (#4) format. Sample sentences can now be evaluated/parsed on-line. `wwwgrail.cgi` sends back the typeset derivation, and the source file.

```
...  
\parsescript{whleft}{np}{de+soepschildpad+die+alice+wil+plagen}  
{de Soepschildpad die Alice wil plagen} $\vdash np$  

```

21. Fragment libraries

The next step in the direction of the Mathematica ‘notebook’ concept:

The kernel transforms Prolog source code into a typeset fragment, with evaluable examples, and form interaction.

- ▶ **Static library.** A directory of annotated fragments used for didactic purposes. For example

<http://www.let.uu.nl/~Michael.Moortgat/personal/Courses/fragments/>

- ▶ **Dynamic library.** Users submit their individual fragments, which the Perl LWP module fetches from a specified URL.

<http://grail.let.uu.nl/submitfragment-e.html>



22. hyperref form interaction

```
\section{Interactive session}
\renewcommand{\LayoutTextField}[2]{\makebox[2in][l]{#1}#2}
\renewcommand{\LayoutChoiceField}[2]{\makebox[1.5in][l]{#1}#2}
\renewcommand{\LayoutCheckField}[2]{#1\makebox[1.5in][l]{#2}}
\renewcommand{\DefaultWidthofCheckBox}{12pt}
\renewcommand{\DefaultHeightofCheckBox}{12pt}

\begin{Form}[action=http://grail.let.uu.nl/cgi-bin/grail/wwwgrail.cgi,
            encoding=html,method=post]

\subsection*{Test example}
\TextField[width=3in,name=test]{Type in an example:}
\TextField[width=3in,name=goal]{Goal formula:}
...
\subsection*{Display options}
\ChoiceMenu[radio,default=yes,name=struct]{Structure labels:}{Yes=yes,No=no}
...
\Submit{\textsf{Submit}}\quad\Reset{\textsf{Reset}}
\end{Form}
```


23. Future work/worries

- ▶ **Work.** Extend the form interaction to allow for
 - ▷ Lexicon editing/updating
 - ▷ Postulate editing/updating
 - ▷ Proof net unfolding
- ▶ **Worry.** Can we depend on Acrobat? Dynamic PDF features might change, disappear ...
- ▶ **Alternative?** Is a switch to Hans Hagen's `conTEXt` environment an option?
 - ▷ lets ask him ...

References

Type-logical grammar

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