Foundation of Cryptography, Introduction

Adminstration + Introduction

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Part I

Administration and Course Overview

Section 1

Administration

Important Details

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2. Course website: Can be reached from Ronen's homepage.

Course Prerequisites

1. Computational Models
2. Probability theory.

Course Material

1. Books:
   1.1 Oded Goldreich. Foundations of Cryptography.
   1.2 Jonathan Katz and Yehuda Lindell. An Introduction to Modern Cryptography.
2. Lecture notes
   2.1 Ran Canetti `www.cs.tau.ac.il/~canetti/f08.html`
   2.2 Yehuda Lindell `u.cs.biu.ac.il/~lindell/89-856/main-89-856.html`
   2.3 Luca Trevisan `www.cs.berkeley.edu/~daw/cs276/`
   2.4 Salil Vadhan `people.seas.harvard.edu/~salil/cs120/`
   2.5 Benny Applebaum and Iftach Haitner `http://moodle.tau.ac.il/2016/course/view.php?id=368416201`

Section 2

Course Topics

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Basic primitives in cryptography (i.e., one-way functions, pseudorandom generators and zero-knowledge proofs).

- Focus on *formal* definitions and *rigorous* proofs.
- The goal is not studying some list, but to understand cryptography.
- Start with “what is security?”
- Only then do we ask how to achieve it.
- Start from the bottom and work our way up.

Part II

Foundation of Cryptography

Some stories and motivation

- Encryption (symmetric and public-key).
- Coin tossing over the phone (impossible information theoretically, but possible against poly-time adversaries).

Section 3

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5. One-way functions: an efficiently computable function that no efficient algorithm can invert.

Part III

Notation

Notation I

- For $t \in \mathbb{N}$, let $[t] := \{1, \ldots, t\}$.
- Given a string $x \in \{0,1\}^*$ and $0 \leq i < j \leq |x|$, let $x_{i,\ldots,j}$ stands for the substring induced by taking the $i, \ldots, j$ bit of x (i.e., $x[i] \ldots, x[j]$).
- Given a function f defined over a set $\mathcal{U}$, and a set $\mathcal{S} \subseteq \mathcal{U}$, let $f(\mathcal{S}) := \{f(x) \colon x \in \mathcal{S}\}$, and for $y \in f(\mathcal{U})$ let $f^{-1}(y) := \{x \in \mathcal{U} \colon f(x) = y\}$.
- poly stands for the set of all polynomials.
- The worst-case running-time of a *polynomial-time algorithm* on input x, is bounded by $p(|x|)$ for some $p \in \mathrm{poly}$.
- A function is *polynomial-time computable*, if there exists a polynomial-time algorithm to compute it.
- PPT stands for probabilistic polynomial-time algorithms.
- A function $\mu \colon \mathbb{N} \mapsto [0,1]$ is negligible, denoted $\mu(n) = \mathrm{neg}(n)$, if for any $p \in \mathrm{poly}$ there exists $n' \in \mathbb{N}$ with $\mu(n) \leq 1/p(n)$ for any $n > n'$.

Distribution and random variables I

- The support of a distribution P over a finite set $\mathcal{U}$, denoted $\mathsf{Supp}(P)$, is defined as $\{u \in \mathcal{U} \colon P(u) > 0\}$.
- Given a distribution P and en event E with $\Pr_P[E] > 0$, we let $(P \mid E)$ denote the conditional distribution P given E (i.e., $(P \mid E)(x) = \frac{P(x) \wedge E}{\Pr_P[E]}$).
- For $t \in \mathbb{N}$, let let U_t denote a random variable uniformly distributed over $\{0,1\}^t$.
- Given a random variable X, we let $x \leftarrow X$ denote that x is distributed according to X (e.g., $\Pr_{x \leftarrow X}[x = 7]$).
- Given a final set $\mathcal{S}$, we let $x \leftarrow \mathcal{S}$ denote that x is uniformly distributed in $\mathcal{S}$.
- We use the convention that when a random variable appears twice in the same expression, it refers to a *single* instance of this random variable. For instance, $\Pr[X = X] = 1$ (regardless of the definition of X).

Distribution and random variables II

- Given distribution P over $\mathcal{U}$ and $t \in \mathbb{N}$, we let P^t over $\mathcal{U}^t$ be defined by $D^t(x_1, \ldots, x_t) = \Pi_{i \in [t]} D(x_i)$.
- Similarly, given a random variable X, we let X^t denote the random variable induced by t independent samples from X.