Complexity Theory : Exercise 2

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1. (Diagonalization) Let $coNTIME(t(n)) = \left\{\bar{L} : L \in NTIME(t(n))\right\}$. Show that there is a language $L \in coNTIME(n^5)$ such that $L \notin NTIME(n^2)$. Give a direct proof.

2. (Padding arguments) Let L be a language and $t : N \to N$ be a function such that $t(n) > n$. We define $L_t = \left\{1^{t(|x|-|x|-1)}0 \circ x : x \in L\right\}$ and call it the padded version of L using t.

   (a) Show that if $t(n)$ is a valid time function then $L \in TIME(t(n))$ implies $L_t \in TIME(n)$.

   (b) Show that if $NTIME(n) \subseteq TIME(n^2)$ then $NTIME(n^5) \subseteq TIME(n^{10})$.

   (c) Show that if $NTIME(n) \subseteq TISP(n^{1.2}, n^{0.2})$ then $NTIME(n^{10}) \subseteq TISP(n^{12}, n^2)$.

   (d) Let $NEXP = \cup_{c=1}^{\infty} NTIME(2^{n^c})$. Show that if $NEXP \neq EXP$ then $NP \neq P$. (Hint: use padding).

3. (Oracles)

   (a) Show that $NP^{PSPACE} = PSPACE$. Conclude that $P^{PSPACE} = NP^{PSPACE}$.

   (b) Let $Exact{-}Clique = \{(G, k) :$ G is a graph, k is an integer, and the largest clique in G is of size $k\}$. Show that $Exact - Clique \in P^{NP}$.

4. (Definition of the polynomial time hierarchy using oracles) Show that $\Sigma_2^p = NP^{NP}$. (Hint: the hard containment is that $NP^{NP} \subseteq \Sigma_2^p$. If you can't solve the general case try to prove the special case in which the NP^{NP} machine makes only one call to its oracle.)