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A robust and efficient, highly scalable parallel solution of the Helmholtz equation with large wave numbers

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Abstract

Numerical solution of the Helmholtz equation is a challenging computational task, particularly when the wave number is large. Recent years have seen great improvements in the finite difference approach to this problem through enhancements of the shifted Laplacian preconditioner, originally introduced in [1]. For a recent survey and some new results, see [4]. In some cases, this approach may be difficult to implement due to the fact that each iteration of the solver uses a multigrid solution of the preconditioner. More recently, a new approach, based on an algebraic multilevel preconditioner, was introduced in [3]. This approach uses symmetric maximum weight matchings and an inverse-based pivoting strategy.

This work presents numerical experiments with the block-parallel CARP-CG algorithm, applied to the Helmholtz equation with large wave numbers. CARP-CG was recently shown to be a very robust and efficient parallel solver of linear systems with very large off-diagonal elements and discontinuous coefficients, obtained from strongly convection dominated elliptic PDEs in heterogeneous media [7]. CARP-CG is simple to implement even on unstructured grids, or when subdomain boundaries are complicated. CARP-CG is a CG acceleration of CARP [5], which is a block-parallel version of the Kaczmarz algorithm (which is SOR on the normal equations). On one processor, CARP-CG is identical to the CGMN algorithm [2,6].

Numerical experiments were carried out on the domain $[0,1] \times [0,1]$, with wave number $k = 75, 150, 300$. A Dirichlet boundary condition with a discontinuity at $(0.5, 0)$ was prescribed on the side $y = 0$, and a first order absorbing boundary condition was prescribed on the other sides. A second order finite difference discretization scheme was used, leading to a complex, nonsymmetric and indefinite linear system. For each wave number, the domain was discretized so as to obtain 6, 8, and 10 grid points per wavelength. Three different convergence goals were prescribed for the relative residual: 10^{-4}, 10^{-7}, and 10^{-10}. Experiments were carried out with 1, 4, 8, 16 and 32 processors. A fixed relaxation parameter of 1.7 was used in all cases.

The results demonstrate the robustness and runtime efficiency of CARP-CG on the tested problems. A most significant result of these experiments is that as k increases, the scalability of CARP-CG improves: for $k = 300$, the number of iterations on 32 processors was only about 15% more than required on one processor, and there was very little variance in this result when the convergence goal or the number of grid points per wavelength were changed. This places CARP-CG as a very viable parallel solver for the Helmholtz equation with large wave numbers.

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